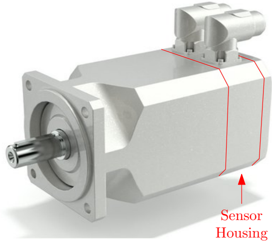
Motivations

Sensorless operations are interesting for:

- Reducing the electric drive cost,
- Shrinking the motor frame,
- Increasing the system reliability,
- Implementing plausibility algorithms.

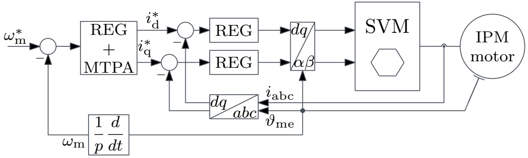


Problem Statement

We would like to estimate the electrical rotor position from measured currents (and voltages).

Motor voltage equations:

$$u_\mathrm{d} = R_\mathrm{s} i_\mathrm{d} + \frac{d\lambda_\mathrm{d}}{dt} - \omega_\mathrm{me}\lambda_\mathrm{q}$$
$$u_\mathrm{q} = R_\mathrm{s} i_\mathrm{q} + \frac{d\lambda_\mathrm{q}}{dt} + \omega_\mathrm{me}\lambda_\mathrm{d}$$



Requirement:

- Position estimation at standstill and low speed region

  $\Longrightarrow$ Fundamental signals are zero or their signal-to-noise ratio are too small for ensuring a reliable estimation.

Solution Idea

To excite the system $\Longrightarrow$ Additional high frequency (HF) signals are injected.

HF signals must be detectable, i.e., they must be isolable from fundamental signals.

HF voltage injection:
- $\Rightarrow$ Stationary reference frame $\alpha\beta$
- $\Rightarrow$ Estimated synchronous reference frame axis $\widehat{d}$-axis

The most promising technique is the latter, since it induces a smaller torque ripple, exhibits a reduced frequency-dependent estimation error and is easier to solve observer convergence issues.

Fundamental Elements

Motor hypothesis (ideal anisotropic motor):

- L_{dd} and L_{qq} are constant,
- $L_{\mathrm{dd}} > L_{\mathrm{qq}}$ (anisotropy is detectable),
- L_{dq} is zero.

NB: all inductances are differential

$$\frac{d\lambda_{\mathrm{d}}}{dt} = \underbrace{\frac{d\lambda_{\mathrm{d}}(i_{\mathrm{d}}, i_{\mathrm{q}})}{di_{\mathrm{d}}}}_{L_{\mathrm{dd}}} \frac{di_{\mathrm{d}}}{dt} + \underbrace{\frac{d\lambda_{\mathrm{d}}(i_{\mathrm{d}}, i_{\mathrm{q}})}{di_{\mathrm{q}}}}_{L_{\mathrm{dq}}} \frac{di_{\mathrm{q}}}{dt}$$

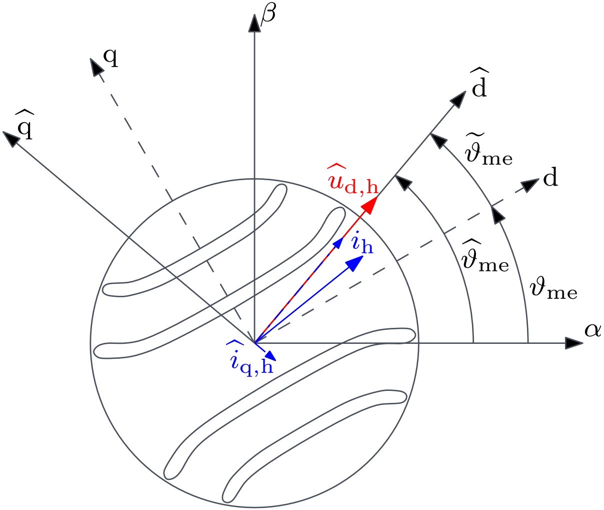
Notation:

- $\widehat{\cdot}$ stands for estimated quantities,
- $\widetilde{\cdot}$ stands for error quantities.

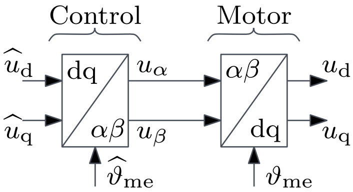
HF voltage injection:

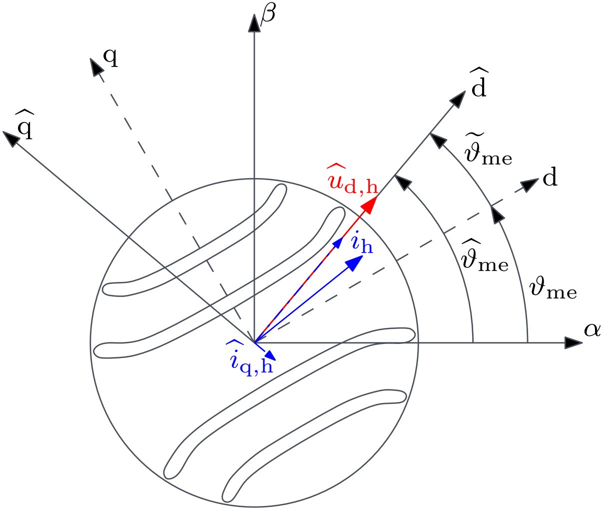
$$\widehat{u}_{\mathrm{d,h}} = U_{\mathrm{h}} \cos(\omega_{\mathrm{h}} t)$$
$$\widehat{u}_{\mathrm{q,h}} = 0$$

- $\widehat{u}_{\mathrm{d,h}}$ is injected,
- i_{h} pulsating current vector is induced,
- if $\widetilde{\vartheta}_{\mathrm{me}} \neq 0 \Rightarrow i_{\mathrm{h}}$ is not aligned with the injected voltage signal.



Idea





$$\begin{bmatrix} u_\mathrm{d} \\ u_\mathrm{q} \end{bmatrix} = \begin{bmatrix} \cos\widetilde{\vartheta}_\mathrm{me} & -\sin\widetilde{\vartheta}_\mathrm{me} \\ \sin\widetilde{\vartheta}_\mathrm{me} & \cos\widetilde{\vartheta}_\mathrm{me} \end{bmatrix} \begin{bmatrix} \widehat{u}_\mathrm{d} \\ \widehat{u}_\mathrm{q} \end{bmatrix}$$

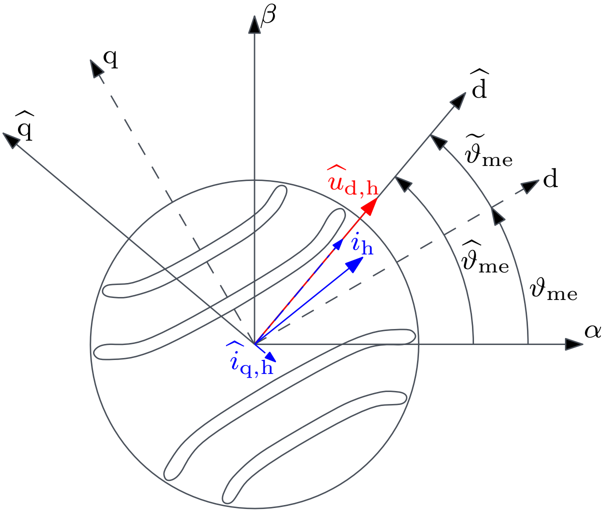
where $\widetilde{\vartheta}_\mathrm{me} = \widehat{\vartheta}_\mathrm{me} - \vartheta_\mathrm{me}$.

Idea

- $i_{\rm h}$ can be project on both estimated $\widehat{dq}$ axes,
- Hence, $\widehat{i}_{\rm q,h}$ appears.

If rotor position estimation error is not null $\Longrightarrow \widehat{i}_{\rm q,h} \neq 0$.

A control mechanism can adjust the estimated reference frame to nullify $\widehat{i}_{\rm q,h}$.



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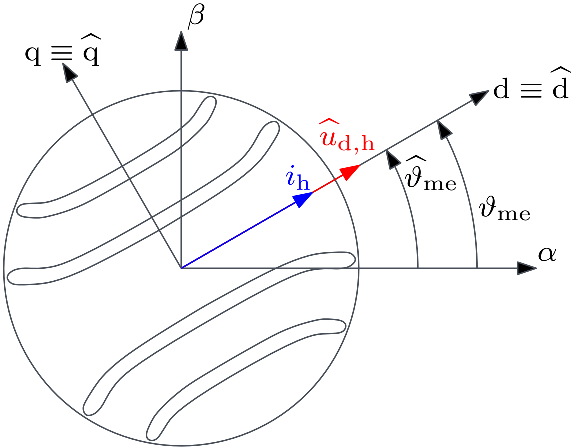
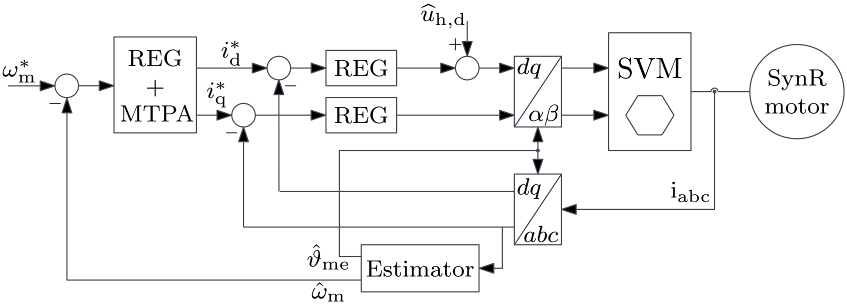


Position Observer



Voltage injection $\widehat{u}_{\mathrm{d,h}}$ acts a disturbance in current loops.

Current oscillation must be hidden to current regulators (i.e. using filters) or regulators must be blind and not see the disturbance (i.e. low bandwidth).

Rule of thumb: $\omega_{\mathsf{cc}} \leq \frac{\omega_{\mathsf{h}}}{10} \qquad \omega_{\mathsf{h}} \leq \frac{\omega_{\mathsf{sw}}}{10}$

Position Observer

Some equations:

Voltage Injection:

$$\widehat{u}_{\mathrm{d,h}} = U_{\mathrm{h}} \cos(\omega_{\mathrm{h}} t)$$

Motor voltage equations:
(Stator resistance voltage drops and motional terms are neglected)

$$u_{\mathrm{d}} \approx \frac{d\lambda_{\mathrm{d}}}{dt} = L_{\mathrm{dd}} \frac{di_{\mathrm{d}}}{dt}$$
$$u_{\mathrm{q}} \approx \frac{d\lambda_{\mathrm{q}}}{dt} = L_{\mathrm{qq}} \frac{di_{\mathrm{q}}}{dt}$$

Induced HF currents:

$$\widehat{i}_{\mathrm{d,h}} = I_{\mathrm{s}} \left[L_{\Sigma} - L_{\Delta} \cos(2\widetilde{\vartheta}_{\mathrm{me}}) \right] U_{\mathrm{h}} \sin(\omega_{\mathrm{h}} t)$$
$$\widehat{i}_{\mathrm{q,h}} = I_{\mathrm{s}} \left[L_{\Delta} \sin(2\widetilde{\vartheta}_{\mathrm{me}}) \right] U_{\mathrm{h}} \sin(\omega_{\mathrm{h}} t)$$

where:

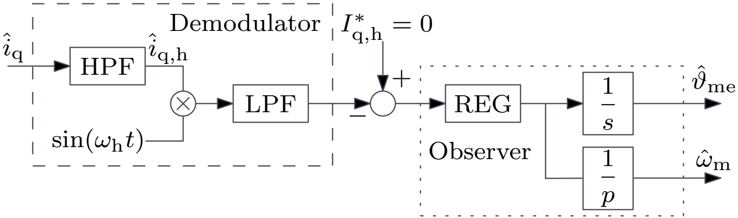
$$I_{\mathrm{s}} = \frac{1}{\omega_{\mathrm{h}} L_{\mathrm{dd}} L_{\mathrm{qq}}} \quad L_{\Sigma} = \frac{L_{\mathrm{dd}} + L_{\mathrm{qq}}}{2} \quad L_{\Delta} = \frac{L_{\mathrm{dd}} - L_{\mathrm{qq}}}{2}$$

Position Observer

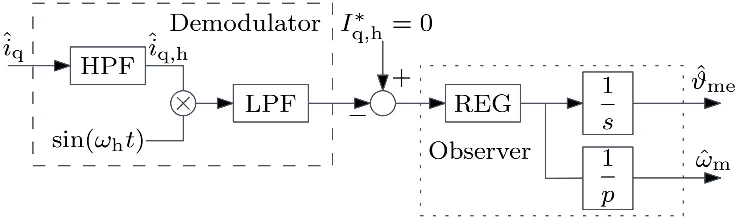
$$\widehat{i}_{\mathrm{q,h}} = \frac{U_{\mathrm{h}} L_{\Delta}}{\omega_{\mathrm{h}} L_{\mathrm{dd}} L_{\mathrm{qq}}} \sin(2\widehat{\vartheta}_{\mathrm{me}}) \sin(\omega_{\mathrm{h}} t) = I_{\mathrm{q,h}} \sin(\omega_{\mathrm{h}} t)$$

If $L_{\Delta} \neq 0$, $\quad I_{\mathrm{q,h}} = 0 \implies \widetilde{\vartheta}_{\mathrm{me}} = 0$

Observer:



Position Observer

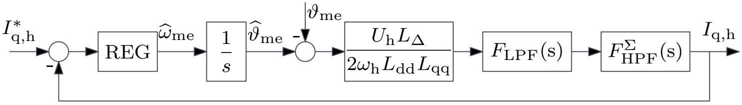


HF currents are extracted by means of: $\widehat{i}_{q,h}(t) = \mathrm{HPF}(s)\widehat{i}_q(t)$

$\widehat{i}_{q,h}$ demodulated as: $\widehat{i}_{q,h}\sin(\omega_h t) = I_{q,h}\sin^2(\omega_h t) = \frac{I_{q,h}}{2}\left[1 - \cos(2\omega_h t)\right]$

LPF + small estimation error hp: $\frac{I_{q,h}}{2} = \frac{U_h L_\Delta}{2\omega_h L_{dd} L_{qq}}\sin(2\widetilde{\vartheta}_{me}) \approx \frac{U_h L_\Delta}{\omega_h L_{dd} L_{qq}}\widetilde{\vartheta}_{me}$

Position Observer



- Motor parameters must be known to design the regulator.
- HPF transfer function differs to the implemented one.
- Actual position acts as a disturbance in the observer loop.
- L_{Δ} must be not null (motor anisotropy must be detectable).
- Two stable points in $\sin(2\widetilde{\vartheta}_{\mathrm{me}}) \implies \widehat{\vartheta}_{\mathrm{me}} = \widehat{\vartheta}_{\mathrm{me}} + \pi$.

Real Motor Model

Motor voltage equations:

(Stator resistance voltage drops and motional terms are neglected)

$$u_{\mathrm{d}} \approx \frac{d\lambda_{\mathrm{d}}}{dt} = L_{\mathrm{dd}}\frac{di_{\mathrm{d}}}{dt} + L_{\mathrm{dq}}\frac{di_{\mathrm{q}}}{dt}$$
$$u_{\mathrm{q}} \approx \frac{d\lambda_{\mathrm{q}}}{dt} = L_{\mathrm{dq}}\frac{di_{\mathrm{d}}}{dt} + L_{\mathrm{qq}}\frac{di_{\mathrm{q}}}{dt}$$

Induced HF currents:

$$\widehat{i}_{\mathrm{d,h}} = I_{\mathrm{s}}\left[L_{\Sigma} - L_{\Delta}\cos(2\widetilde{\vartheta}_{\mathrm{me}}) - L_{\mathrm{dq}}\sin(2\widetilde{\vartheta}_{\mathrm{me}})\right] U_{\mathrm{h}}\sin(\omega_{\mathrm{h}} t)$$
$$\widehat{i}_{\mathrm{q,h}} = I_{\mathrm{s}}\left[L_{\Delta}\sin(2\widetilde{\vartheta}_{\mathrm{me}}) - L_{\mathrm{dq}}\cos(2\widetilde{\vartheta}_{\mathrm{me}})\right] U_{\mathrm{h}}\sin(\omega_{\mathrm{h}} t)$$

where:

$$I_{\mathrm{s}} = \frac{1}{\omega_{\mathrm{h}}(L_{\mathrm{dd}}L_{\mathrm{qq}} - L_{\mathrm{dq}}^2)} \quad L_{\Sigma} = \frac{L_{\mathrm{dd}} + L_{\mathrm{qq}}}{2} \quad L_{\Delta} = \frac{L_{\mathrm{dd}} - L_{\mathrm{qq}}}{2}$$

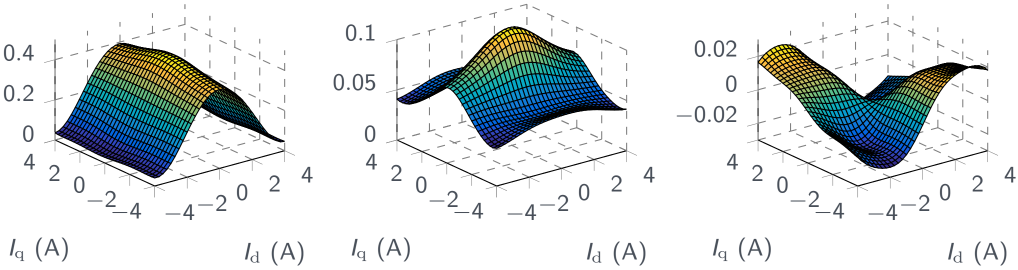
$$\widehat{i}_{\mathrm{q,h}} = \frac{U_{\mathrm{h}}\sqrt{L_\Delta^2 + L_{\mathrm{dq}}^2}}{\omega_{\mathrm{h}}(L_{\mathrm{dd}}L_{\mathrm{qq}} - L_{\mathrm{dq}}^2)} \sin(2\widehat{\vartheta}_{\mathrm{me}} + 2\bar{\vartheta}) \sin(\omega_{\mathrm{h}} t)$$

where: $\bar{\vartheta} \triangleq 0.5\mathrm{atan2}(-L_{\mathrm{dq}}, L_\Delta)$.

Steering to zero $\widehat{i}_{\mathrm{q,h}}$, an estimation error occurs.
The stable point of the observer is $\widehat{\vartheta}_{\mathrm{me}} = -\bar{\vartheta}$.

NB: the estimation error depends on the ratio between L_{dq} and L_Δ.
The higher the cross-differential inductance, the larger the estimation error.
The smaller the semi-difference inductance, the larger the estimation error.

Motor Inductances



(a) $L_{dd}(i_d, i_q)$ (H) (b) $L_{qq}(i_d, i_q)$ (H) (c) $L_{dq}(i_d, i_q)$ (H)

Motor saturates, so inductances change as a function of the operating point.

$\Longrightarrow$ Estimation error $\bar{\vartheta}$ depends on the operating point, i.e., the current level.

Estimator Issues with Real Motor

At a high current level, there may be no stable points for the observer.

In open loop, the observer shows only the estimation error $\bar{\vartheta}$.

In closed loop, the estimation error affects the control dynamics as well. Thus, the estimator accuracy deteriorates further.

Error Compensation

Compensation aims:

- achieving convergence at any torque level
- increasing stability margin
- increasing accuracy

Compensation techniques:

- HF flux linkages demodulation (instead of HF stator currents)
- observer compensation (angle and current)

In both methods, the magnetic model of the motor must be known accurately.