Model Predictive Control for Power Electronics

Electric motor drives applications

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Outline

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- Design
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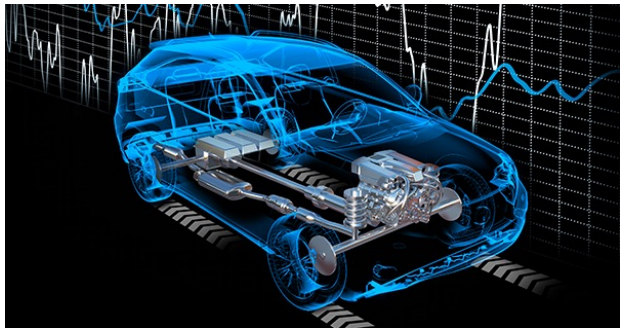
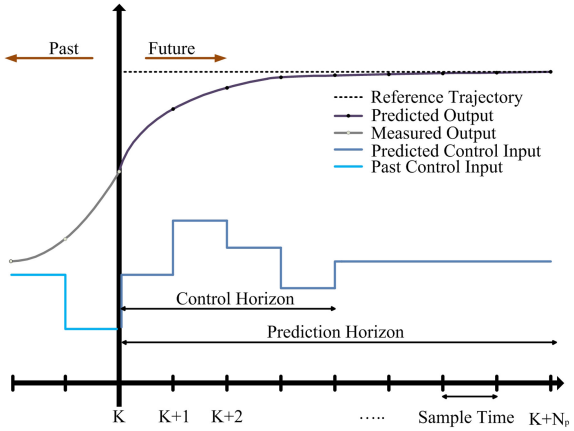
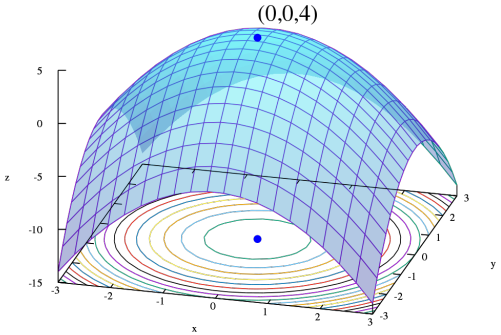
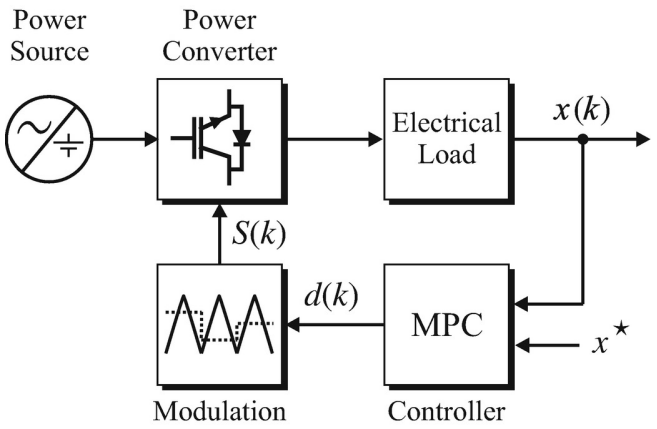
  - Prediction
  - Cost Function
- Case study: Finite-Set MPC
  - Design
  - Performances





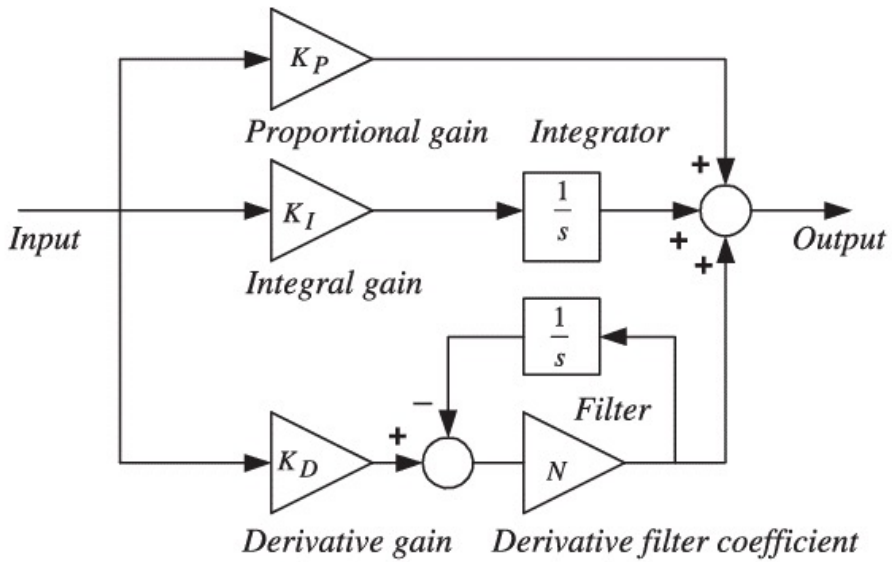




Motivation

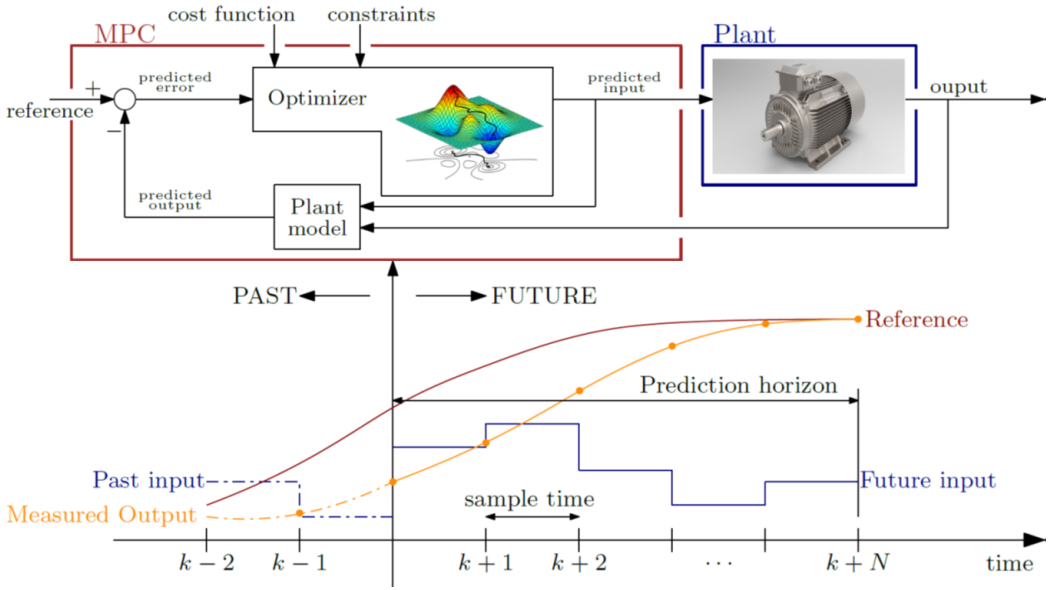
Motivation (I) PID vs MPC

Why PID?



- Simple
- Works in practice
- Straightforward design IF
  - Single-input-single-output plant
  - Linear dynamics
  - No direct handling of input/output constraints

Motivation (II) PID vs MPC



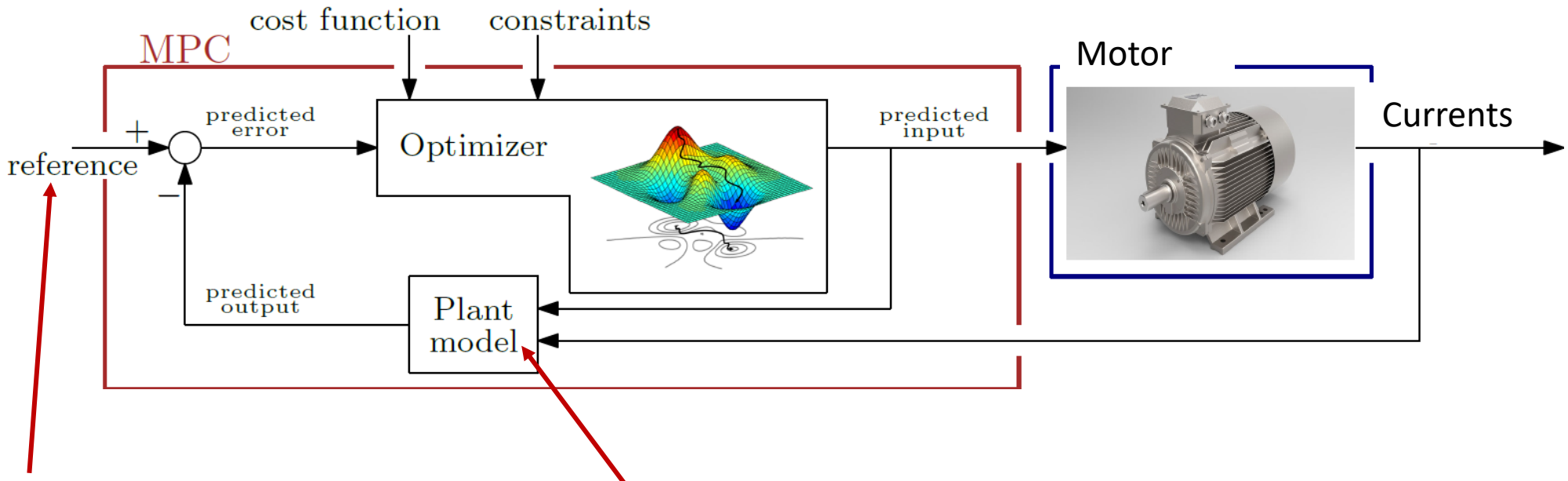
Why MPC?

- Manage multiple-input-multiple-output plant
- Powerful tools for handling nonlinear dynamics
- Include constraints in the control problem

Idea

Idea (I)

Write the control problem as an optimization problem

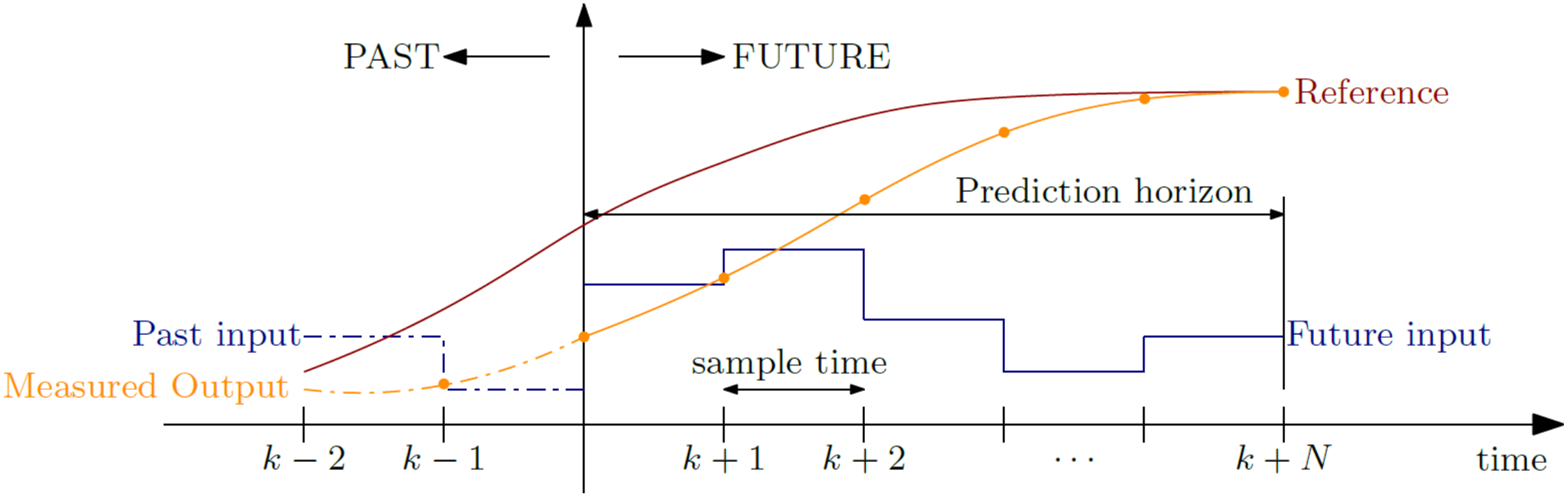


Current reference

dq voltage balance

Idea (II)

Prediction of the future currents



MPC Design

MPC design

3 main ingredients:

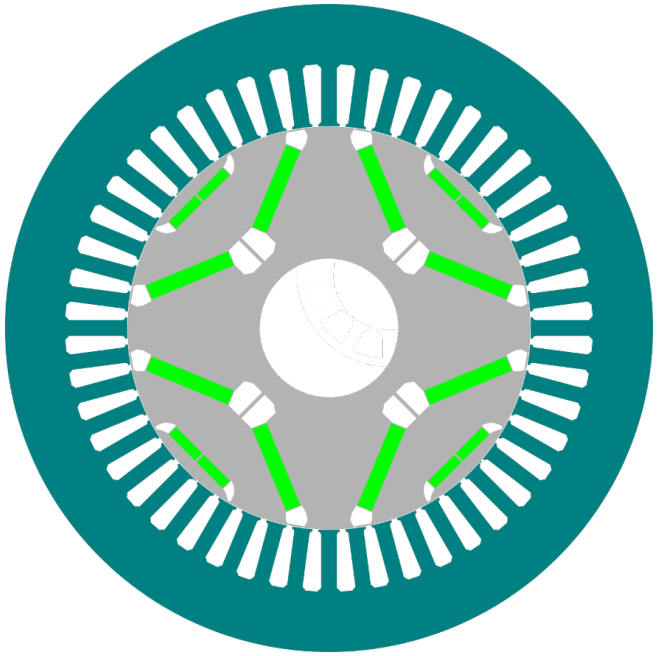
1) Model (dq voltage balance, approximations, linearization, discretization…)
2) Prediction (prediction horizon, control horizon,…)
3) Control Problem (cost function, constraints, receding horizon, tuning, optimization algorithms…)

MPC Design - Model

Model (I) – dq voltage balance equation

Model selection as trade-off between accurate description of the plant and simplicity

$$\begin{cases} \dfrac{di_d}{dt} & = -\dfrac{R}{L} i_d + \omega_e i_q + \dfrac{1}{L} u_d \\ \dfrac{di_q}{dt} & = -\omega_e i_d - \dfrac{R}{L} i_q + \dfrac{1}{L} u_q - \omega_e \dfrac{1}{L} \Lambda_{pm} \end{cases}$$



Model (II) – State-space form

State-space representation as general tool for control MIMO plant

$$\begin{bmatrix} \frac{di_d}{dt} \\ \frac{di_q}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega_e \\ -\omega_e & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} u_d \\ u_q \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} 0 \\ \omega_e \Lambda_{pm} \end{bmatrix}$$

$$\frac{dx}{dt} = A_c x(t) + B_c u(t) + w(t)$$

Model (III) – Discretization

Forward Euler approximation

$\frac{dx}{dt} \approx \frac{x(k+1)-x(k)}{T_s}$, being T_s the sampling time

Resulting:

$$x(k+1) = Ax(k) + Bu(k) + w(k),$$

being $B = T_s B_c$ and $A = I - T_s A_c$

-> other discretization methods could be used (exact, Tustin, Taylor, …)

Model (IV) – Approximations

Model selection involves always approximation:

- Nonlinear flux-current characteristic neglected
- No magnetic cross-saturation
- Thermal effect ignored
- No back-electromotive force harmonics
- Discretization errors
- …

MPC Design - Prediction

Prediction (I)

First of all select how far in the future to predict

-> selection of the prediction horizon N

How to predict?

Exploit state-space framework $x(k+1) = Ax(k) + Bu(k)$ and iterate:

$$x(k+2) = Ax(k+1) + Bu(k+1) = A\big(Ax(k) + Bu(k)\big) + Bu(k+1)$$

$$\rightarrow x(k+2) = A^2x(k) + ABu(k) + Bu(k+1)$$

Prediction (II)

And iterate….

$$
\begin{aligned}
x(k+3) &= Ax(k+2) + Bu(k+2) \\
&= A\big(Ax(k+1) + Bu(k+1)\big) + Bu(k+2) \\
&= A\Big(A\big(Ax(k) + Bu(k)\big) + Bu(k+1)\Big) + Bu(k+2)
\end{aligned}
$$

$$\rightarrow x(k+3) = A^3x(k) + A^2Bu(k) + ABu(k+1) + Bu(k+2)$$

Prediction (III) - Condensing

Condense the iterations in a compact matrix form

$x(k+1) = Ax(k) + Bu(k)$

$x(k+2) = A^2x(k) + ABu(k) + Bu(k+1)$

$x(k+3) = A^3x(k) + A^2Bu(k) + ABu(k+1) + Bu(k+2)$

$$\begin{bmatrix} x(k+1) \\ x(k+2) \\ x(k+3) \end{bmatrix} = \begin{bmatrix} A \\ A^2 \\ A^3 \end{bmatrix} x(k) + \begin{bmatrix} B & 0 & 0 \\ AB & B & 0 \\ A^2B & AB & B \end{bmatrix} \begin{bmatrix} u(k) \\ u(k+1) \\ u(k+2) \end{bmatrix}$$

Compact way to compute future sequences $\boldsymbol{X}(k) = \boldsymbol{A}x(k) + \boldsymbol{B}\boldsymbol{U}(k)$

MPC Design – Control problem

Control Problem (I) – Cost function

Control problem as an optimization with 2 conflicting targets

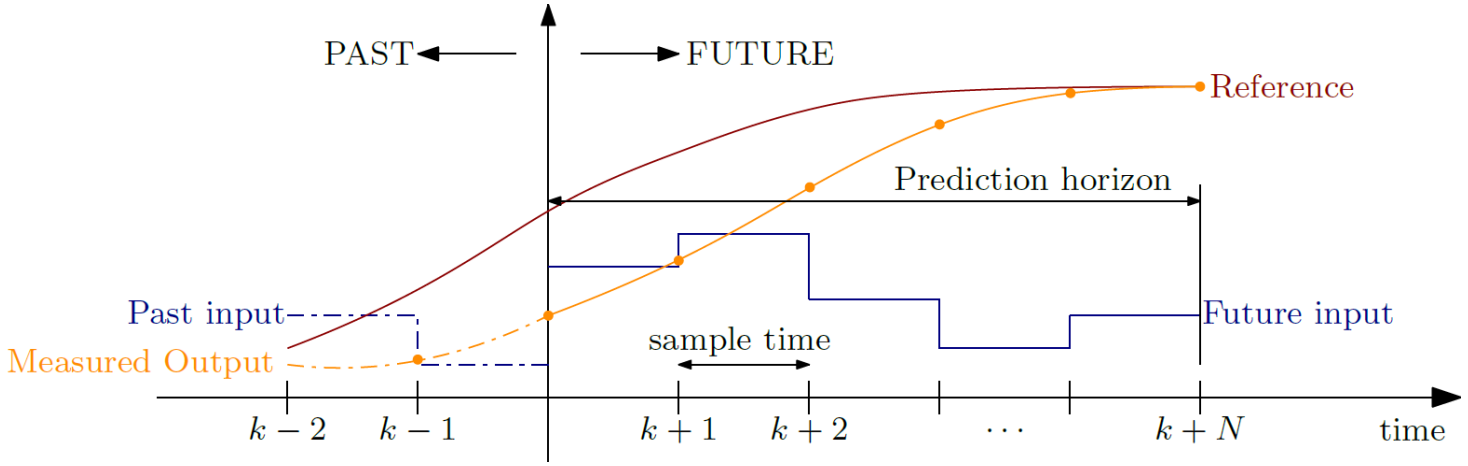
- Current reference tracking
- Input penalization (=penalize wide voltage variations, =minimize switching frequency)

Control Problem (II) – Cost function

This mathematically means

$$\min_{U} \boxed{\left(\boldsymbol{X}^{ref} - \boldsymbol{X}(k)\right)^T Q \left(\boldsymbol{X}^{ref} - \boldsymbol{X}(k)\right)} + \boxed{\boldsymbol{U}(k)^T R \boldsymbol{U}(k)}$$

Current reference tracking penalty on input



Control Problem (III) – Tuning

One/two parameters to tune

$$\min_{U}\left(\boldsymbol{X}^{ref} - \boldsymbol{X}(k)\right)^{T} \boxed{Q} \left(\boldsymbol{X}^{ref} - \boldsymbol{X}(k)\right) + \boldsymbol{U}(k)^{T} \boxed{R} \boldsymbol{U}(k)$$

Tuning matrices

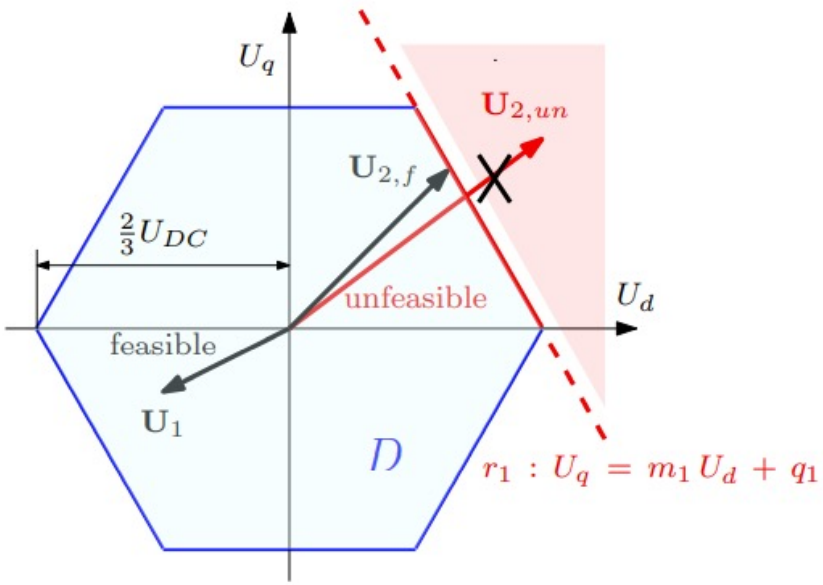
Rule of thumb -> Q identity and R according to desired performances

Control Problem (IV) – Inequality constraints

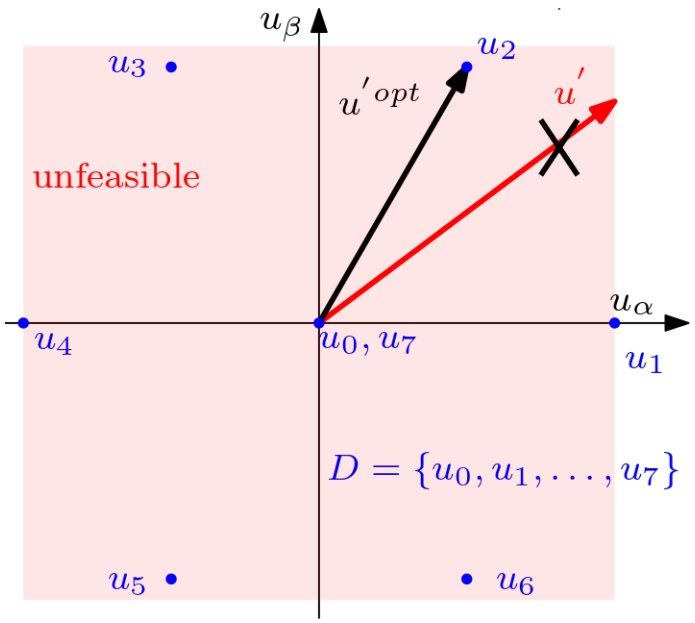
Two main categories of MPC in power electronics applications:

- MPC with PWM -> Continuous Set MPC
- MPC without PWM -> Finite Set

Control Problem (V) – Inequality constraints



Continuous Set

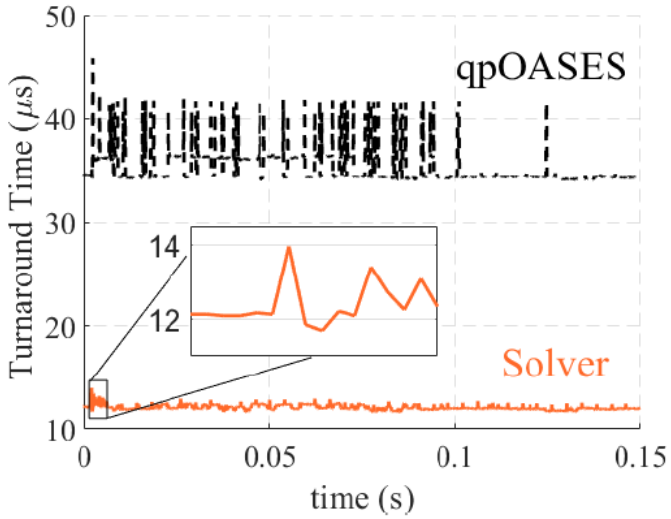


Finite Set

Control Problem (VI): Optimization tools

Continuous-Set:

- No voltage constraints (unconstrained)
  -> direct solution
- Voltage constraints
  -> quadratic problem
  (MATLAB "*quadprog*")



Finite-Set (FS):

- Enumeration
- Integer programming solver (branch and bound,…)

Case study: Finite Set MPC

Design FS MPC - Specifications

Simplest FS MPC

- Unitary prediction horizon
- Weight R=0?
- Enumeration Solver

Case study: Finite Set MPC - Design

Design FS MPC (I) – Enumeration Solver

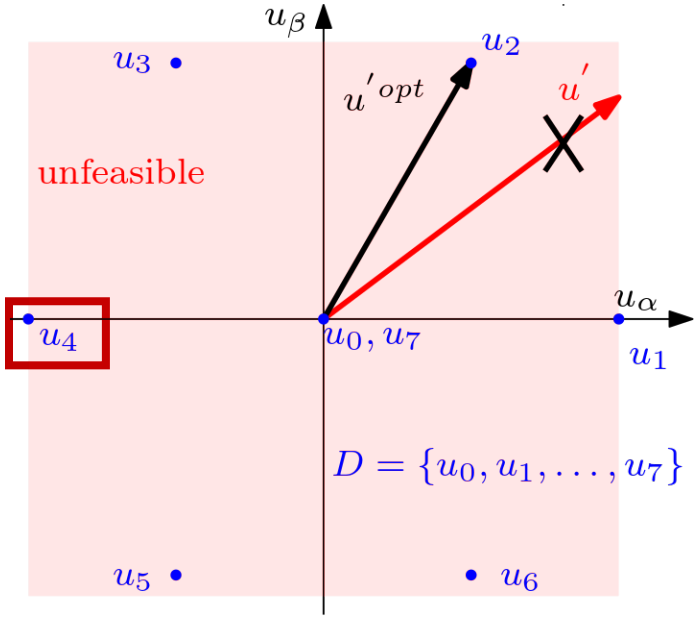
8 cost function evaluations to find the minimum:

$$\min_{u_{dq}} J = \left(i_{dq}^{ref} - i_{dq}(k+1)\right)^2 =$$

$$= \left(i_{dq}^{ref} - Ai_{dq}(k) - Bu_{dq}(k) + w(k)\right)^2$$

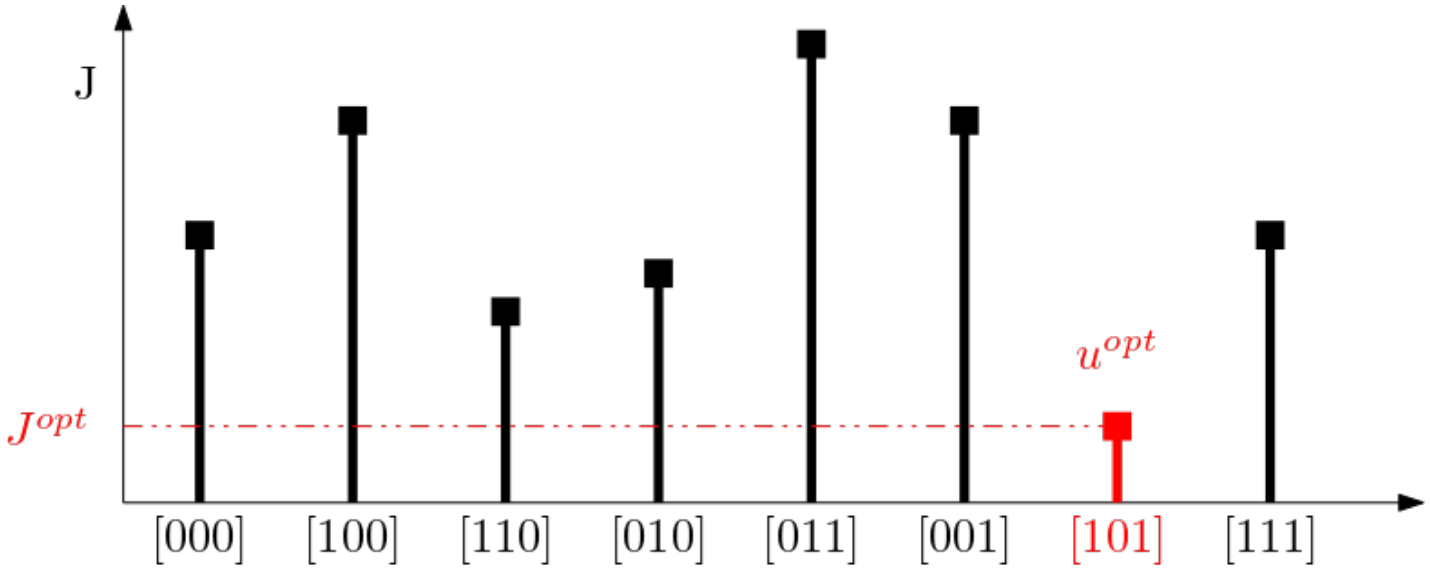
$u_{dq} = T_{Park}T_{Clarke}s$, where $s = [s_a, s_b, s_c]^T$

and $s_i = \{0,1\}$



Design FS MPC (II) – Enumeration Solver

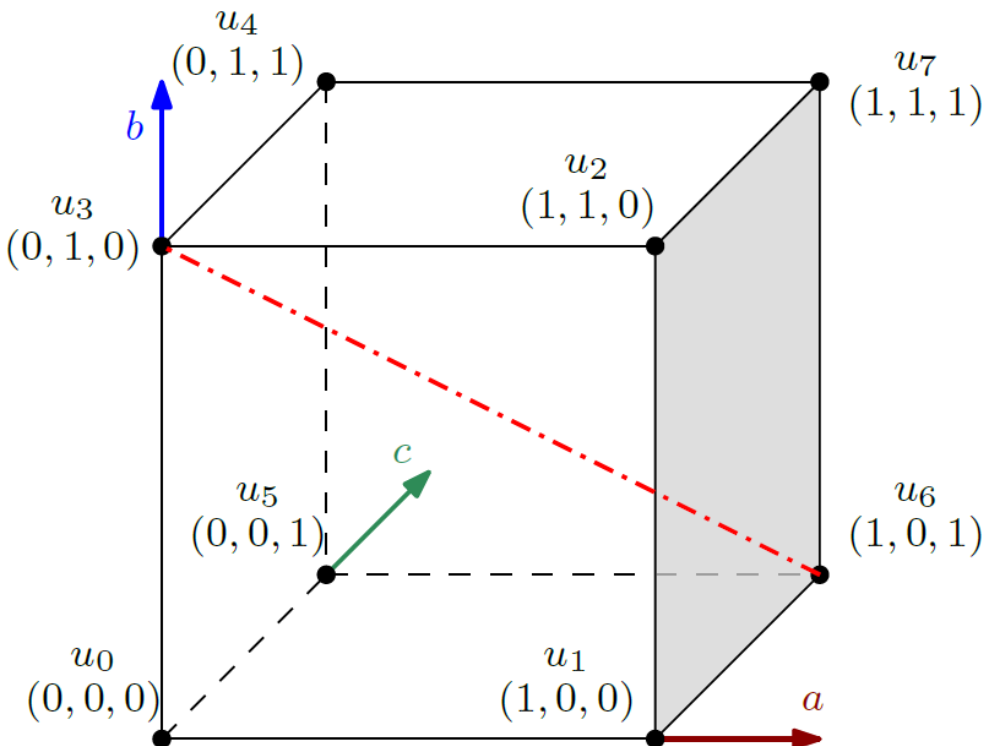
8 cost function evaluations to find the minimum



Q: How to decide between [000] and [111]?

Design FS MPC (III) – Enumeration Solver

With R≠0 a penalty on the switching can be included



Case study: Finite Set MPC - Performances

Performances FS MPC (I)

FS MPC allows

✓Low switching frequency

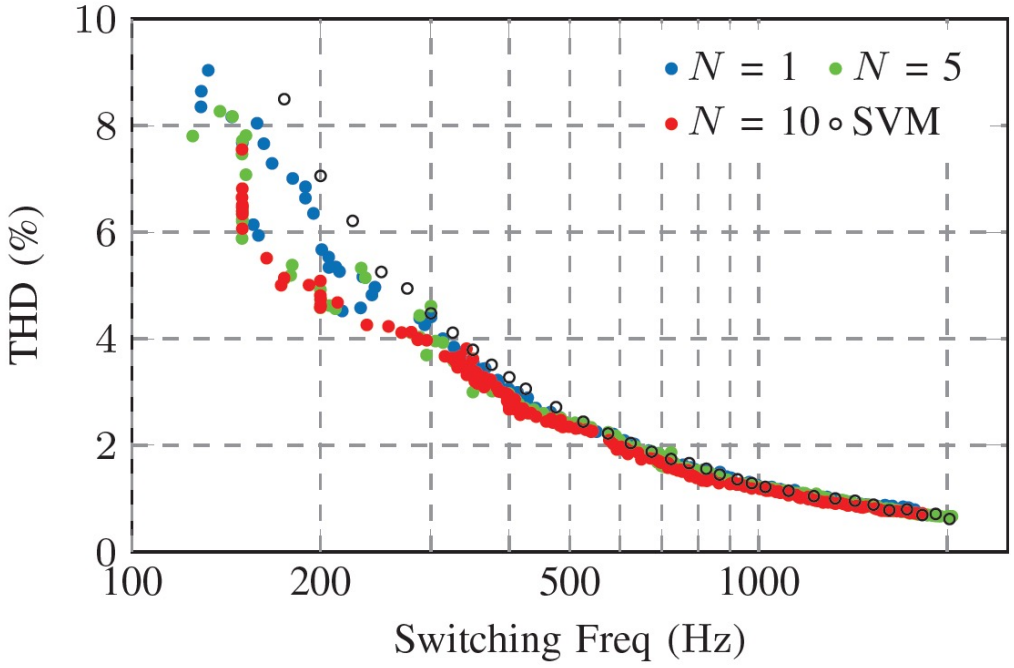
- Possibly high current ripple

✓Fast dynamics

- High computation cost

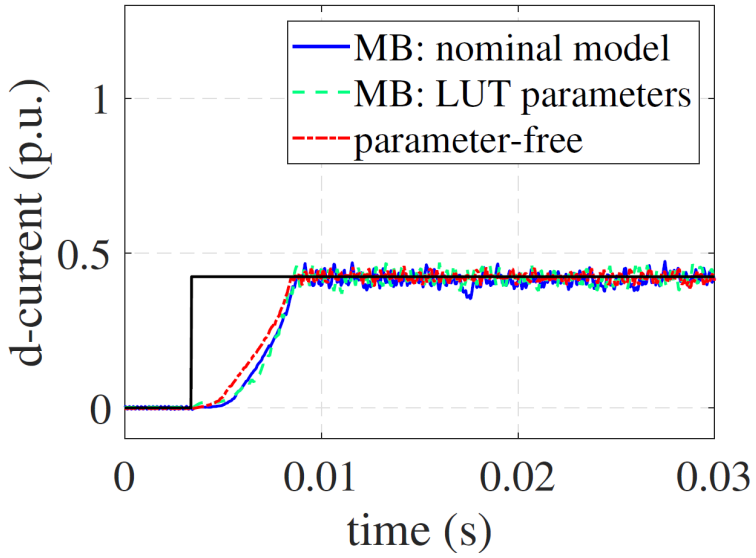
Performances FS MPC (II)

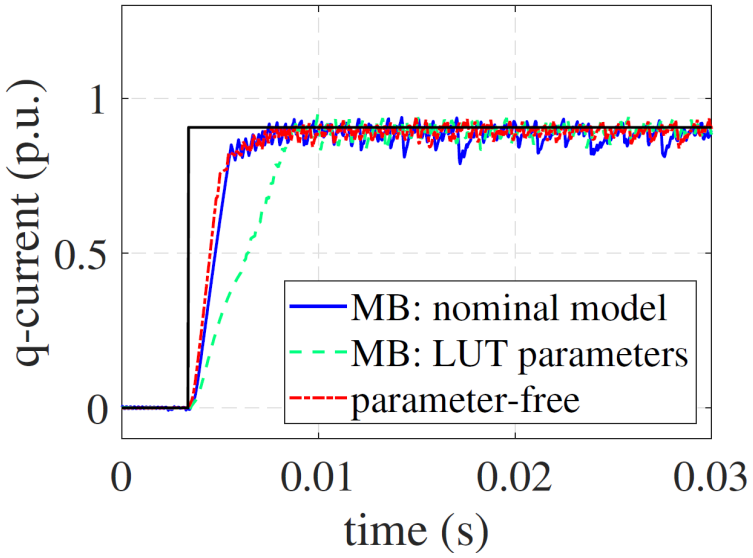
Advantages at low switching frequencies when compared to PWM



Performances FS MPC (III)

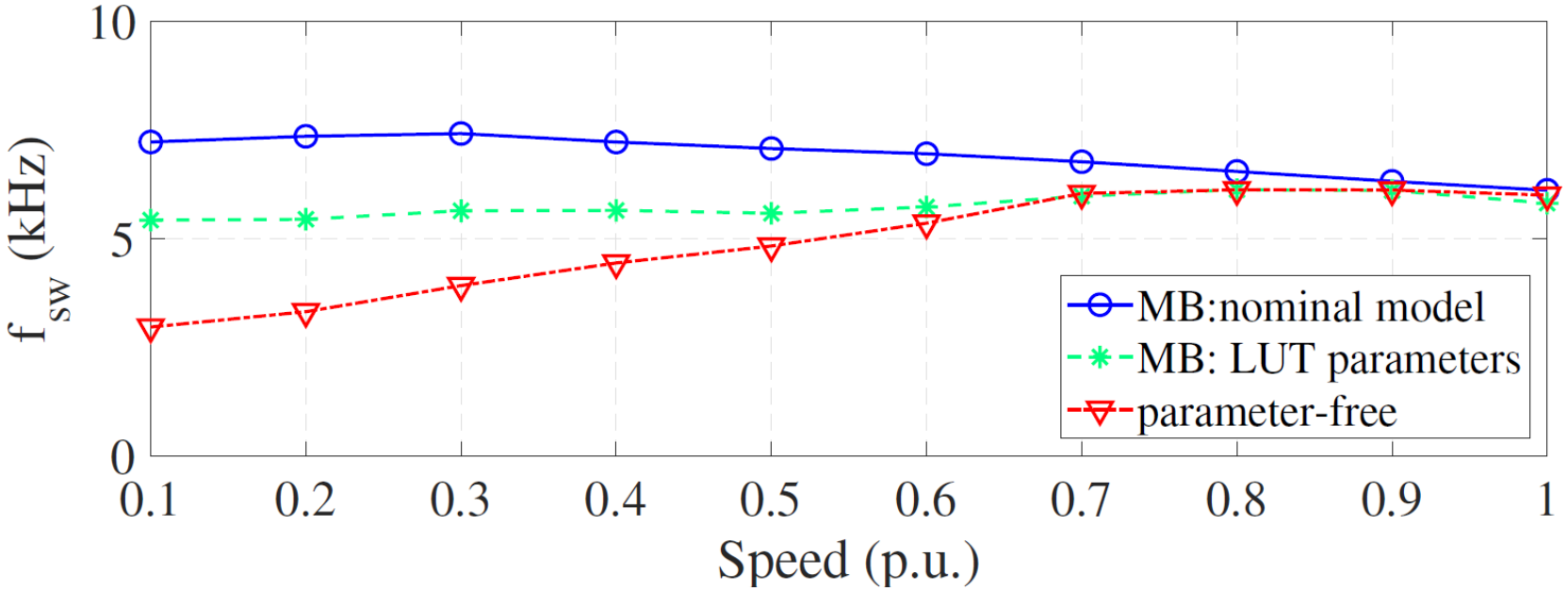
High performances during transients but influenced by the model parameters





Performances FS MPC (IV)

Converter resulting switching frequency also depends on the accuracy of the model



Thank you!

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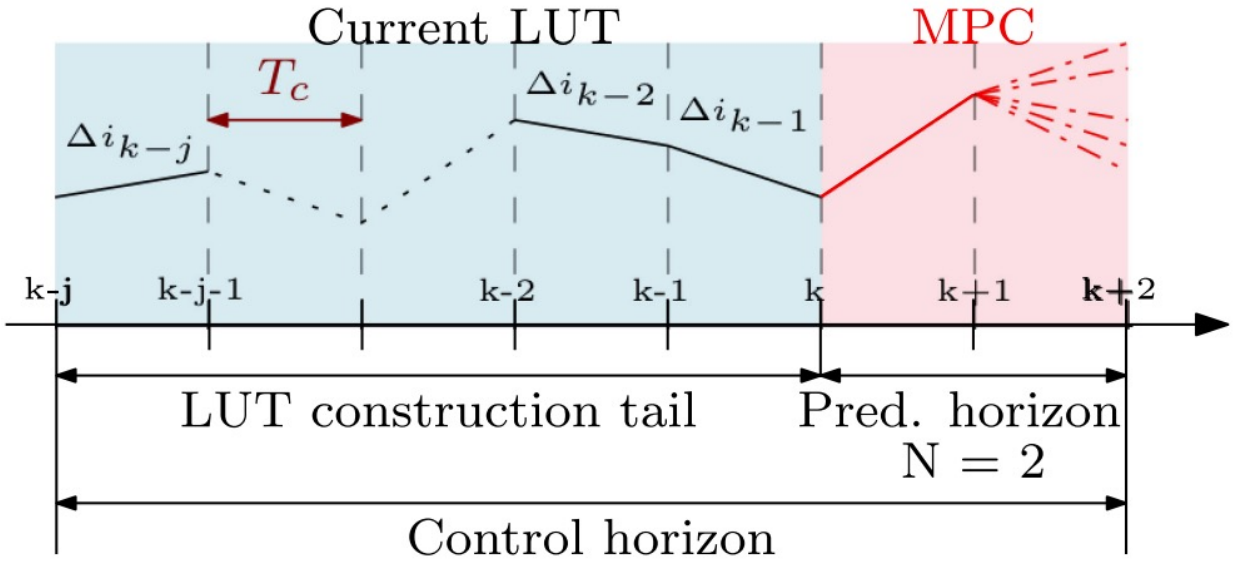
Design FS MPC (II) – Delay Compensation

Open loop prediction to correct the digital delay compensation:

$$i_{dq}(k+1) = Ai_{dq}(k) + Bu_{dq}(k) + w(k)$$



Control Problem (V) – Equality constraints

Predicted currents coherent with the prediction model

$$\min_{U}\left(\boldsymbol{X}^{ref} - \boldsymbol{X}(k)\right)^{T} Q\left(\boldsymbol{X}^{ref} - \boldsymbol{X}(k)\right) + \boldsymbol{U}(k)^{T} R \boldsymbol{U}(k)$$

$$\boldsymbol{X}(k) = \boldsymbol{A}\, x(k) + \boldsymbol{B}\, U(k)$$

We are simply telling how to compute current predictions.