$$\frac{dm}{d\alpha_i} = 0 = \lambda_m I \cos\alpha_i + (L_d - L_q) I^2 \cos 2\alpha_i$$

$$= \lambda_m \cos\alpha_i + (L_d - L_q) I \cos 2\alpha_i$$

$$= \lambda_m \cos\alpha_i + (L_d - L_q) I (2\cos^2\alpha_i - 1)$$

$$\underbrace{2(L_d - L_q) I}_{a} \cos^2\alpha_i + \underbrace{\lambda_m}_{b} \cos\alpha_i \underbrace{- (L_d - L_q) I}_{c} = 0$$

$$\cos\alpha_i = \frac{-\lambda_m \pm \sqrt{\lambda_m^2 + 8(L_d - L_q)^2 I^2}}{4(L_d - L_q) I}$$

$> \lambda_m$

$L_d < L_q \Rightarrow \bullet < 0$

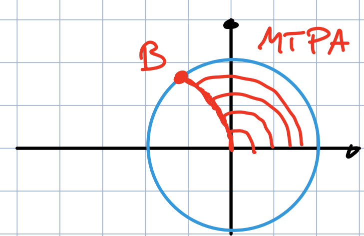
$N: > 0$, $D: < 0$ $\Big\}$ $\cos\alpha_i < 0$

II° quadrante $\quad \alpha_i > \frac{\pi}{2} \quad \cos\alpha_i < 0$

$$\cos\alpha_i = \frac{-\lambda_m + \sqrt{\lambda_m^2 + 8(L_d - L_q)^2 I^2}}{4(L_d - L_q) I}$$

Siccome $\quad I_d = I\cos\alpha_i = \dfrac{-\lambda_m + \sqrt{\lambda_m^2 + 8(L_d - L_q)^2 I^2}}{4(L_d - L_q)}$

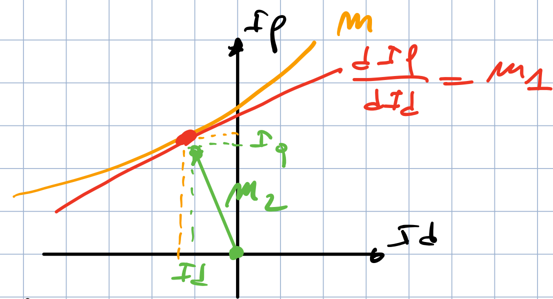
$$I_q = \sqrt{I^2 - I_d^2}$$



α_i cambia al variare dell'ampiezza $|\bar{i}| = I$

(2) $\quad m = \frac{3}{2} p \left[\lambda_m I_q + (L_d - L_q) I_d I_q \right] = \frac{3}{2} p I_q \left[\lambda_m + (L_d - L_q) I_d \right]$

$$\boxed{I_q} = \frac{2}{3} \frac{m}{p} \frac{1}{\lambda_m + (L_d - L_q) I_d}$$



$$\frac{dI_q}{dI_d} = \frac{2}{3} \frac{m}{p} \frac{d}{dI_d} \left[\frac{1}{\lambda_m + (L_d - L_q) I_d} \right]$$

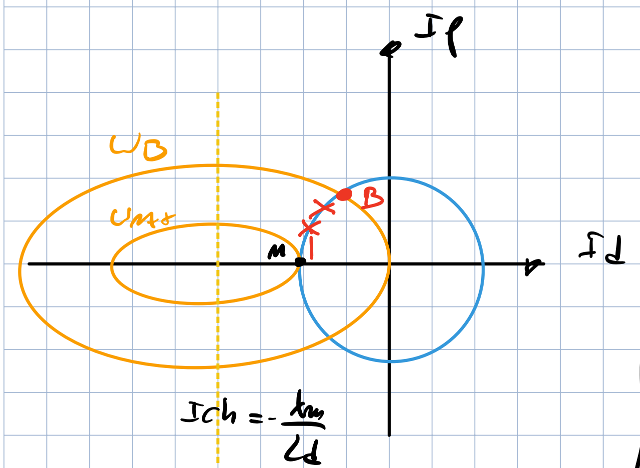
$$= \frac{2}{3} \frac{m}{p} \frac{(L_q - L_d)}{[\lambda_m + (L_d - L_q) I_d]^2} = \frac{I_q (L_q - L_d)}{\lambda_m + (L_d - L_q) I_d} = m_1$$

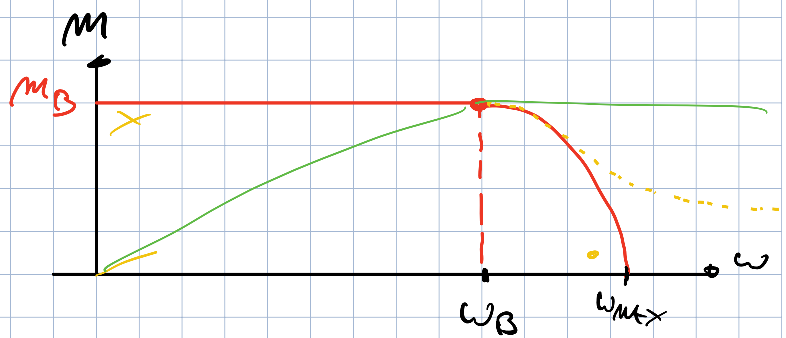
$$m_2 = \frac{I_q}{I_d} \qquad m_1 \cdot m_2 = -1 \qquad m_1 = -\frac{1}{m_2}$$

$$\frac{I_q (L_q - L_d)}{\lambda_m + (L_d - L_q) I_d} = -\frac{I_d}{I_q} \qquad I_q^2 = \frac{I_d [\lambda_m + (L_d - L_q) I_d]}{\cancel{L_q - L_d}\ L_d - L_q}$$

$$I_q = \pm \sqrt{\frac{I_d [\lambda_m + (L_d - L_q) I_d]}{\cancel{L_q - L_d}\ (L_d - L_q)}}$$

Se $\lambda_m = 0 \qquad I_q = \pm I_d$





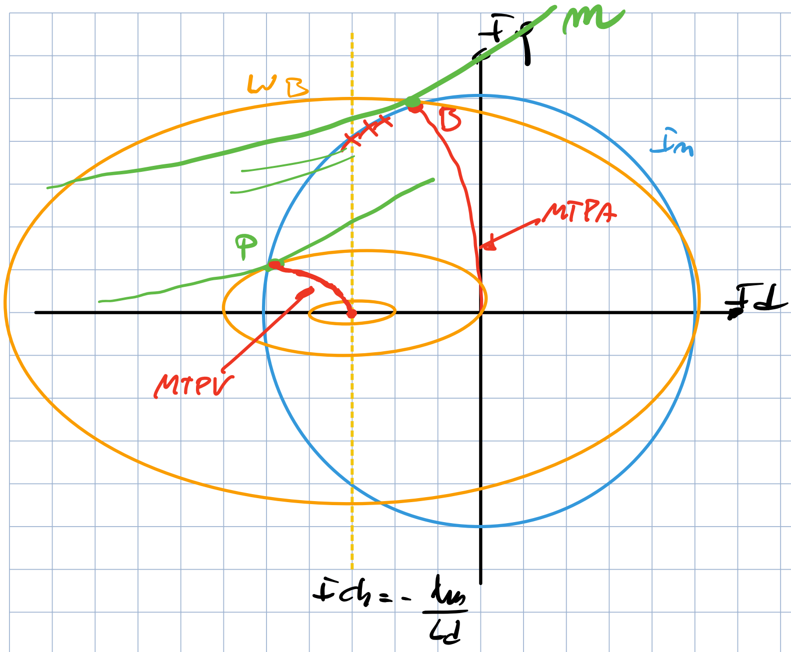
$$\begin{cases} V_d = R \cancel{I_d} - \omega_{max} \lambda_q \\ V_q = R \cancel{I_q} + \omega_{max} [\lambda_m + L_d I_d] \end{cases} \qquad \begin{cases} V_d = -\omega_{max} L_q I_q \\ V_q = \omega_{max} [\lambda_m + L_d I_d] \end{cases}$$

$$V_m^2 = V_d^2 + V_q^2 = \omega_{max}^2 \left[\cancel{(L_q I_q)^2} + [\lambda_m + L_d I_d]^2 \right]$$

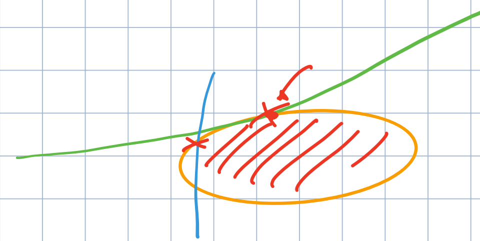
In M : $\left. \begin{array}{l} I_d = -I_m \\ I_q = 0 \end{array} \right\} \Rightarrow V_m^2 = \omega_{max}^2 [\lambda_m + L_d I_d]^2$

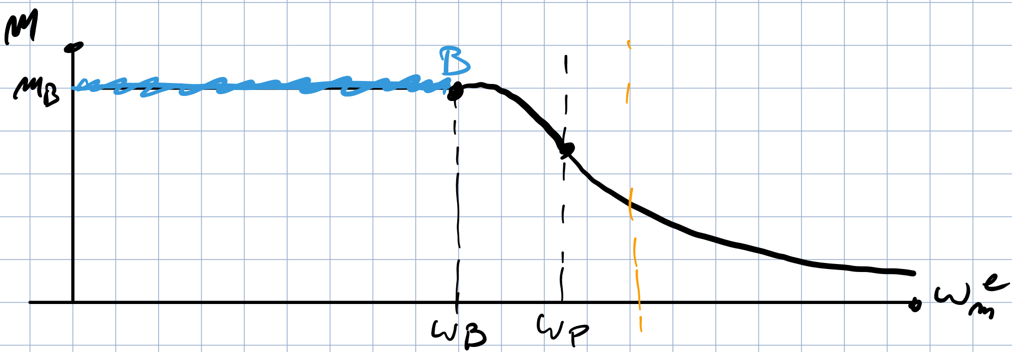
$$\omega_{max} = \frac{V_m}{\lambda_m - L_d I_m}$$

Se $|I_{ch}| = \left| -\frac{\lambda_m}{L_d} \right| > I_m$ $\qquad$ Centro ellissi "esterno" al limite di corrente



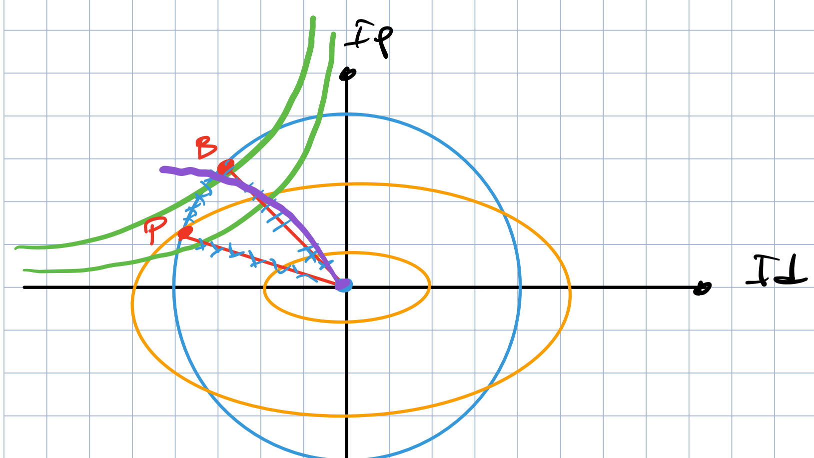
Nel punto P IPERBOLE di coppia e le ellissi DI TENSIONE sono tangenti





Si può calcolare il luogo MTPV $I_q = \frac{L_d}{L_q}\sqrt{\frac{-(I_d+\frac{\lambda_m}{L_d})\left[\lambda_m+(L_d-L_q)I_d\right]}{L_d-L_q}}$

Se $\lambda_m = 0$ (Rel)



MTPA $\alpha_i = 135^\circ$ $(45^\circ + 90^\circ)$

MTPV $I_q = \frac{L_d}{L_q} I_d$

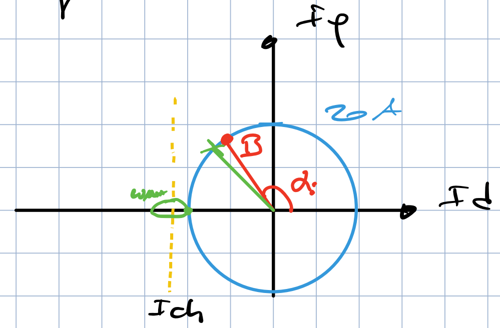
$I_q = -\frac{I_d}{\xi}$ $\quad \xi = \frac{L_q}{L_d} > 1$

Esempio Un motore IPM ha i seguenti dati:

$2p = 4$
$\lambda_m = 0,4\ Vs$
$L_d = 16\ mH$
$L_q = 20\ mH$

$R \cong 0$

$V_n = 210V$
$I_n = 20A$



$I_{ch} = -\frac{0,4}{0,016} = -25A$

a) Calcolare la massima coppia che il motore può sviluppare

$$\cos\alpha_i|_B = \frac{-\lambda_m + \sqrt{\lambda_m^2 + 8(L_d - L_q)^2 I_n^2}}{4(L_d - L_q) I_n} = \frac{-0,4 + \sqrt{0,4^2 + 8[(16-20)\cdot 10^{-3}]^2 \cdot 20^2}}{4(16-20)\cdot 10^{-3}\cdot 20}$$

$$= -0,186 \quad \rightarrow \quad \alpha_i|_B = 100,72°$$

$$\begin{cases} I_d|_B = 20\cdot\cos\alpha_i|_B = -3,72\ A \\ I_q|_B = 20\cdot \operatorname{sen}\alpha_i|_B = 19,65\ A \end{cases}$$

$$m = \frac{3}{2}p\left[\lambda_m I_q + (L_d - L_q) I_d I_q\right] = \frac{3}{2}\cdot 2\left[0,4\ 19,65 + \frac{(16-20)}{1000}(-3,72)\,19,65\right]$$

$$= 24,45\ Nm$$

b) Calcolare la velocità base

$$V_d^2 + V_q^2 = V_n^2$$

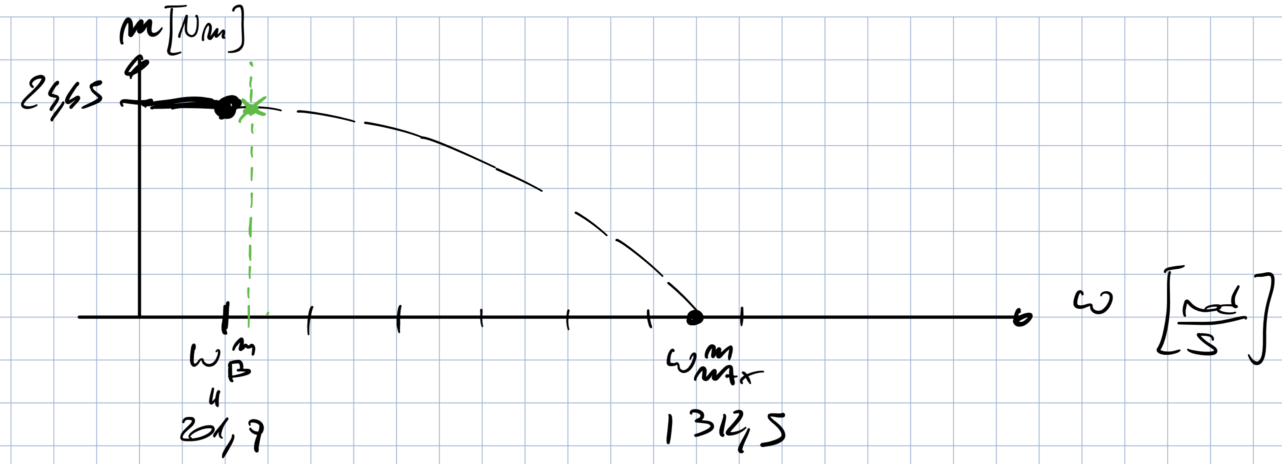
$I_d = I_{dB}$
$I_q = I_{qB}$
$\omega_m^e = \omega_B$

$$\omega_B^2 (L_q I_{qB})^2 + \omega_B^2(\lambda_m + L_d I_{dB})^2 = V_n^2$$

$$\omega_B = \frac{V_n}{\sqrt{(L_q I_{qB})^2 + (\lambda_m + L_d I_{dB})^2}} = \frac{210}{\sqrt{\left(\frac{20}{1000}19,65\right)^2 + \left(0,4 + \frac{16}{1000}(-3,72)\right)^2}}$$

$$= 403,8\ \frac{rad}{s}$$

$$\omega_B^m = \frac{\omega_B}{p} = 201,9\ \frac{rad}{s} = 1929\ rpm$$



c) La massima velocità a cui può spingersi il motore

$V_d^2 + V_q^2 = V_m$

$\omega_m^e = \omega_{MAX}$
$I_q = 0$
$I_d = I_m$

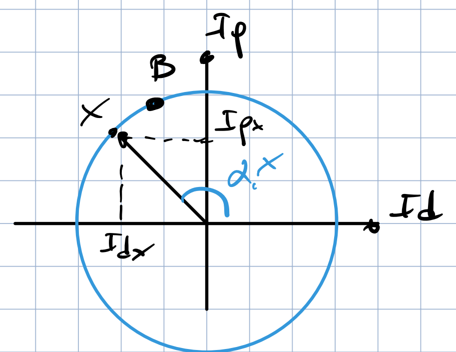
$$\omega_{MAX} = \frac{V_m}{\lambda_m - L_d I_m} = \frac{210}{0{,}4 - \frac{16}{1000}\,20} = 2625\ \frac{rad}{s}$$

$$\omega_{MAX}^m = 1312{,}5\ \frac{rad}{s} = 12\,540\ rpm$$

d) Calcolare la coppia massima disponibile alla velocità di 2200 rpm

$$\omega_m = 2200\ rpm = 230\ \frac{rad}{s}$$

$$\omega_m^e = 230 \cdot 2 = 460\ \frac{rad}{s}$$



Nel punto "x" posso scrivere che:

$V_d^2 + V_q^2 = V_m^2$

$\omega_m^e = 460\ \frac{rad}{s}$
$I_d = I_{dx}$
$I_q = I_{qx}$

$$\omega_m^{e\,2}(L_q I_{qx})^2 + \omega_m^{e\,2}(\lambda_m + L_d I_{dx})^2 = V_m^2$$

$$(L_q I_{qx})^2 + (\lambda_m + L_d I_{dx})^2 = \left(\frac{V_m}{\omega_m^e}\right)^2$$

$$\left(L_q I_m \operatorname{sen}\alpha_i^*\right)^2 + \left(\lambda_m + L_d I_m \cos\alpha_i^*\right)^2 = \left(\frac{U_m}{\omega_m^e}\right)^2$$

$$\left(0{,}4 \operatorname{sen}\alpha_i^*\right)^2 + \left(0{,}4 + 0{,}32 \cos\alpha_i^*\right)^2 = 0{,}2084$$

$$0{,}4^2(1-\underbrace{\cos^2 2\alpha_i^*}_{1-\cos^2\alpha_i^*}) + 0{,}4^2 + 0{,}32^2\cos^2\alpha_i^* + 0{,}256\cos\alpha_i^* = 0{,}2084$$

$$\cos\alpha_i^* = \begin{cases} -0{,}4 \\ \cancel{4{,}85} \end{cases}$$

$$I_{dx} = I_m \cos\alpha_i^* = -8\,A$$

$$I_{qx} = \sqrt{20^2 - 8^2} = 18{,}33\,A$$

La coppia vale $m_x = \frac{3}{2} p \left[\lambda_m I_{qx} + (L_d - L_q) I_{dx} I_{qx}\right]$

$$= \frac{3}{2} \cdot 2 \left[0{,}4 \cdot 18{,}33 + \frac{16-20}{1000}(-8)\,18{,}33\right] = 23{,}76\ Nm$$