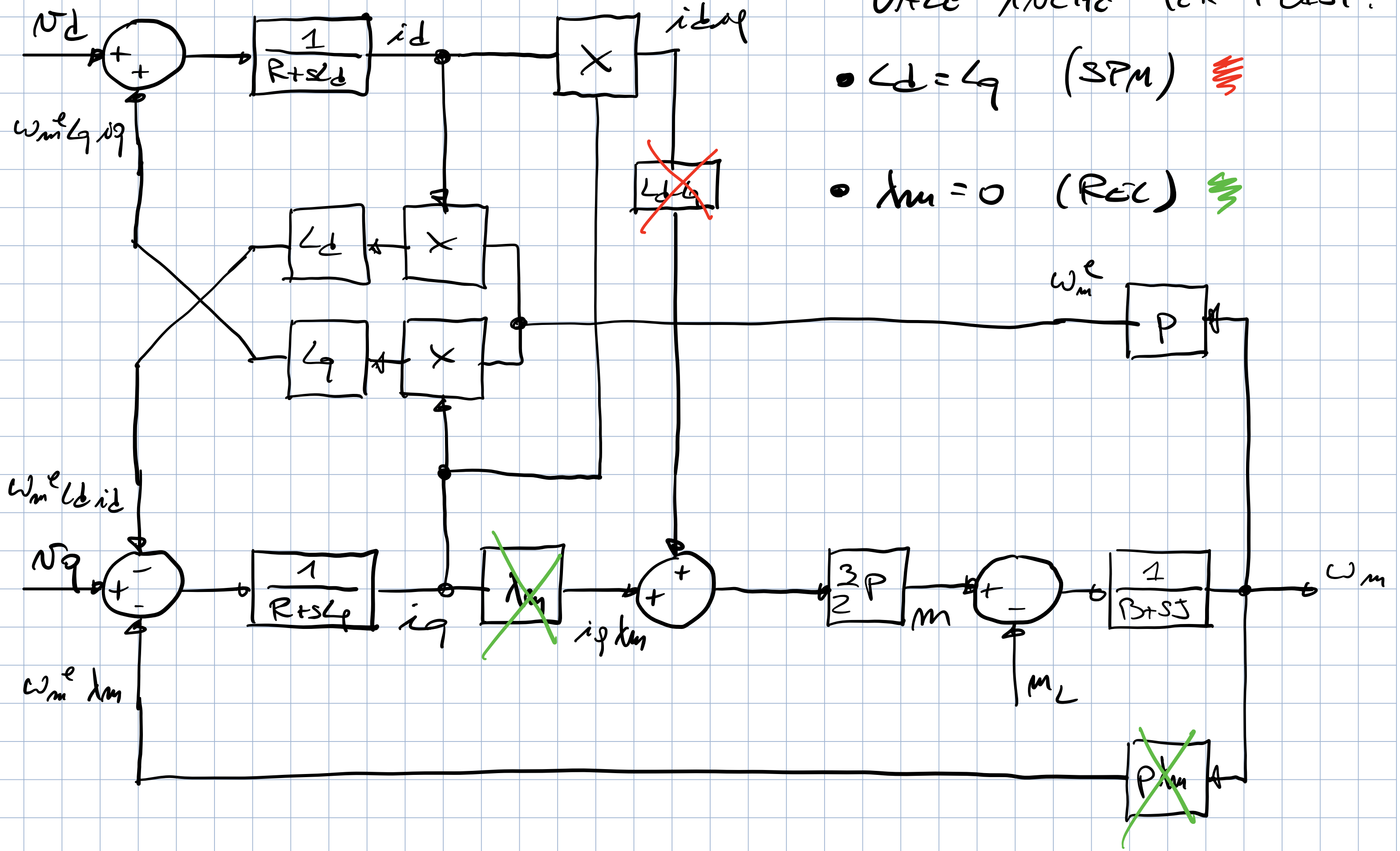
$$
\begin{cases}
v_d = R i_d + L_d \dfrac{di_d}{dt} - \omega_m^e L_q i_q \\[2ex]
v_q = R i_q + L_q \dfrac{di_q}{dt} + \omega_m^e \left( \lambda_m + L_d i_d \right)
\end{cases}
$$

$$
m = \frac{3}{2} p \left[ \lambda_m i_q + (L_d - L_q) i_d i_q \right]
$$




VALE ANCHE PER I CASI:

- $L_d = L_q$ (SPM)
- $\lambda_m = 0$ (REL)

ESERCIZIO

Dato un motore con:

$2p = 4$

$R = 2\,\Omega$

$L_d = 10\,mH$

$L_q = 50\,mH$

$\lambda_m = 0{,}6\,Vs$

$\omega_m = 150\,\frac{rad}{s}$

$I_d = -8{,}5\,A$

$I_q = 12\,A$

(tutti valori di picco)

? Tensione di alimentazione $\bar{v} = v_d + j v_q$

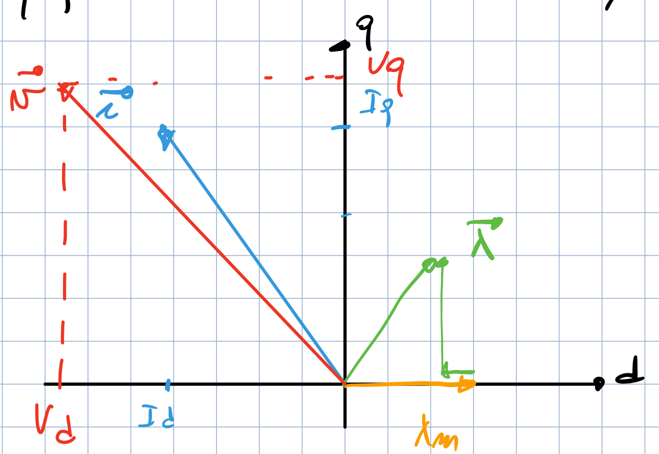
? Bilancio delle potenze ($\eta$)

$$\begin{cases} V_d = R I_d - \omega_m^e L_q I_q \\ V_q = R I_q + \omega_m^e (\lambda_m + L_d I_d) \end{cases}$$

$$\omega_m^e = p\,\omega_m = 2 \cdot 150 = 300 \ \frac{rad}{s}$$

$$\begin{cases} \lambda_d = \lambda_m + L_d I_d = 0,6 + 0,01 \cdot (-8,5) = 0,515 \ Vs \\ \lambda_q = L_q I_q = 0,04 \cdot 12 = 0,48 \ Vs \end{cases}$$

$$\left.\begin{cases} V_d = 2 \cdot (-8,5) - 300 \cdot 0,48 = -161 \ V \\ V_q = 2 \cdot 12 + 300 \cdot 0,515 = 178,5 \ V \end{cases}\right\} \quad V = \sqrt{V_d^2 + V_q^2} = 240,4 \ V$$




$$\arg(\vec{v}) = \operatorname{atan} \frac{V_q}{V_d} = 132^\circ$$

$$\vec{v} = 240,4 \ e^{j132^\circ}$$

$$m = \frac{3}{2} p \left[ \lambda_m I_q + (L_d - L_q) I_d I_q \right]$$

$$= \frac{3}{2} \cdot 2 \left[ 0,6 \cdot 12 + (10 - 40) \cdot 10^{-3} (-8,5) \cdot 12 \right] = 30,8 \ Nm$$

$$P_{out} = m\,\omega_m = 30,8 \cdot 150 = 4617 \ W$$

$$P_J = \frac{3}{2} R I^2 = \frac{3}{2} \cdot 2 \cdot (8,5^2 + 12^2) = 648,8 \ W$$

$$P_{in} = P_{out} + P_J = 4617 + 648,8 = 5265 \ W$$

$$= \frac{3}{2} \left[ V_d I_d + V_q I_q \right] = \frac{3}{2} \left[ (-161)(-8,5) + (178,5)(12) \right] = 5265 \ W$$

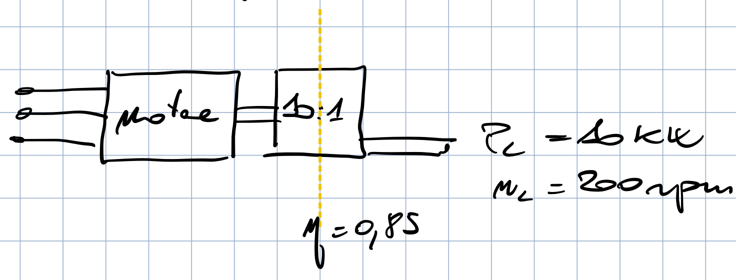
$$\eta = \frac{P_{out}}{P_{in}} = \frac{4617}{5265} = 0,877 \qquad \underline{87,7\,\%}$$

# Esercizio (motore a riluttanza)

$2p = 6$
$\lambda_m = 0$
$L_d = 50\ mH$
$L_q = 10\ mH$

$R \cong 0$

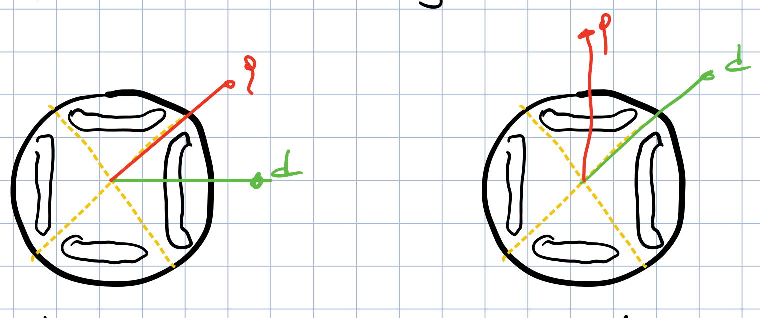
Comanda un carico $P_L = 10\ kW$ mediante un riduttore di giri 1:10 che ha rendimento $\eta = 0{,}85$
$n_L = 200\ rpm$




? $\vec{i}, \vec{v}, f$

$L_d > L_q$

$$m = \frac{3}{2} p \left[ \cancel{\lambda_m I_q} + (L_d - L_q) I_d I_q \right] \qquad \lambda_m = 0$$



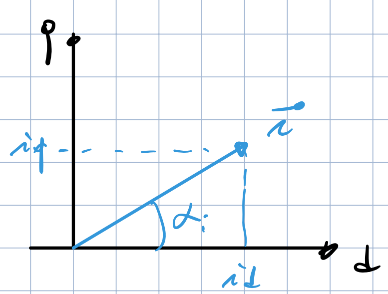

$L_d < L_q$ $\qquad$ $L_d > L_q$

$$P_m = \frac{P_L}{0{,}85} = 11765\ W$$

$n_m = 200 \cdot 10 = 2000\ rpm$ $\qquad$ $\omega_m = 209{,}3\ \frac{rad}{s}$ $\qquad$ $\omega_m^e = p\omega_m = 3 \cdot 209{,}3 = 628\ \frac{rad}{s} = 2\pi f$

$f = 100\ Hz$

$$m = \frac{3}{2} p (L_d - L_q) I_d I_q = \frac{P_m}{\omega_m} = \frac{11765}{209{,}3} = 55{,}78\ Nm$$




$I_d = I \cos\alpha_i$ $\qquad$ $I = |\vec{i}|$
$I_q = I \sin\alpha_i$

$$m = \frac{3}{2} p (L_d - L_q) I^2 \cos\alpha_i \operatorname{sen}\alpha_i$$

$$= \frac{3}{2} p (L_d - L_q) I^2 \frac{\operatorname{sen} 2\alpha_i}{2}$$

$m$ max quando $\operatorname{sen} 2\alpha_i = 1$

$2\alpha_i = \pi/2$

$\alpha_i = \pi/4$

$$\alpha_i = \frac{\pi}{4} \Rightarrow I_d = I_q$$

$$m = \frac{3}{2} p (L_d - L_q) \frac{I^2}{2} = 55{,}78 \Rightarrow I = \sqrt{\frac{4m}{3p(L_d - L_q)}} = 24{,}9 \text{ A}$$

$$\vec{i} = 24{,}9 \, e^{j\frac{\pi}{4}} \qquad I_d = I_q = 17{,}6 \text{ A}$$

$$\begin{cases} \lambda_d = L_d I_d = 0{,}05 \cdot 17{,}6 = 0{,}88 \text{ Vs} \\ \lambda_q = L_q I_q = 0{,}01 \cdot 17{,}6 = 0{,}176 \text{ Vs} \end{cases}$$

$$\left.\begin{cases} V_d = \cancel{R I_d} - \omega_m^e \lambda_q = -628 \cdot 0{,}176 = -110{,}5 \text{ V} \\ V_q = \cancel{R I_q} + \omega_m^e \lambda_d = 628 \cdot 0{,}88 = 550 \text{ V} \end{cases}\right\} \vec{v} = 561 \, e^{j 101{,}4°}$$

Regioni limite di funzionamento motore anisotropo

$$|\vec{i}| \leq I_n \qquad |\vec{v}| \leq V_n$$

$$\boxed{I_d^2 + I_q^2 \leq I_n^2}$$

CERCHIO LIMITE DI CORRENTE

$$\begin{cases} V_d = \cancel{R I_d} - \omega_m^e L_q I_q \\ V_q = \cancel{R I_q} + \omega_m^e (\lambda_m + L_d I_d) \end{cases}$$

$R = 0$

$$m = \frac{3}{2} p \left[ \lambda_m I_q + (L_d - L_q) I_d I_q \right]$$

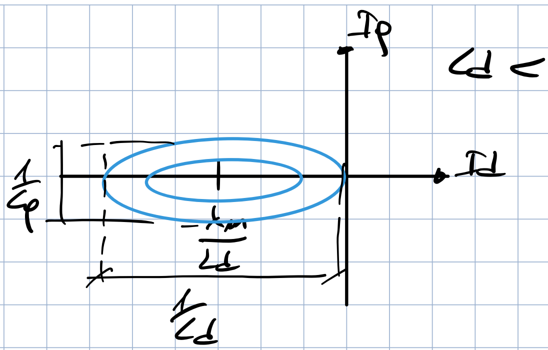
$$V_d^2 + V_q^2 \leq V_n^2 \qquad (\omega_m^e L_q I_q)^2 + \left[\omega_m^e (\lambda_m + L_d I_d)\right]^2 \leq V_n^2$$

$$(L_q I_q)^2 + (\lambda_m + L_d I_d)^2 \leq \left(\frac{V_n}{\omega_m^e}\right)^2$$

ELLISSE LIMITE DI TENSIONE

$$\begin{cases} V_d = 0 \\ V_q = 0 \end{cases} \qquad \begin{cases} I_q = 0 \\ \lambda_m + L_d I_d = 0 \end{cases} \qquad \begin{cases} I_q = 0 \\ I_d = -\dfrac{\lambda_m}{L_d} = I_{ch} \end{cases}$$

$L_d < L_q$




Note : se RēL ($L_d > L_q$)

$$\xi = \frac{L_d}{L_q}$$

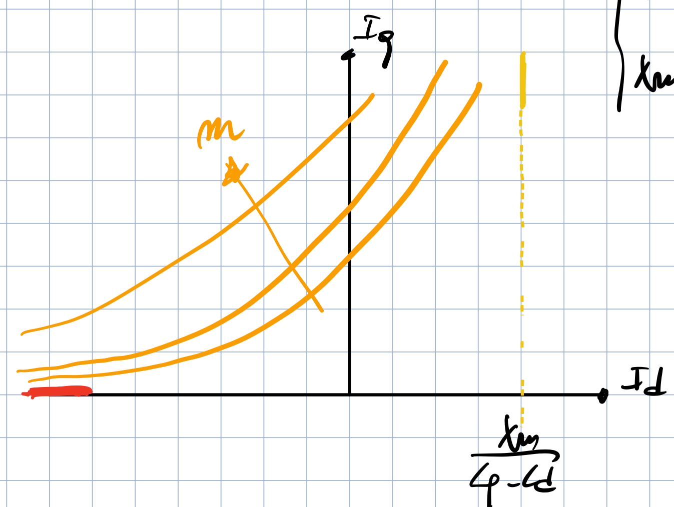
$$\text{SALIENZA} = \frac{\frac{1}{L_d}}{\frac{1}{L_q}} = \frac{L_q}{L_d} = \xi$$

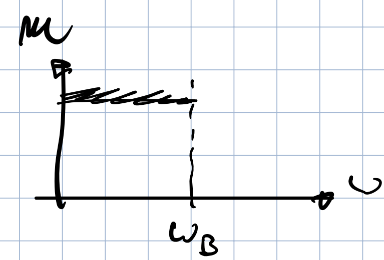
$$m = \frac{3}{2} p I_q \left[ \lambda_m + (L_d - L_q) I_d \right]$$

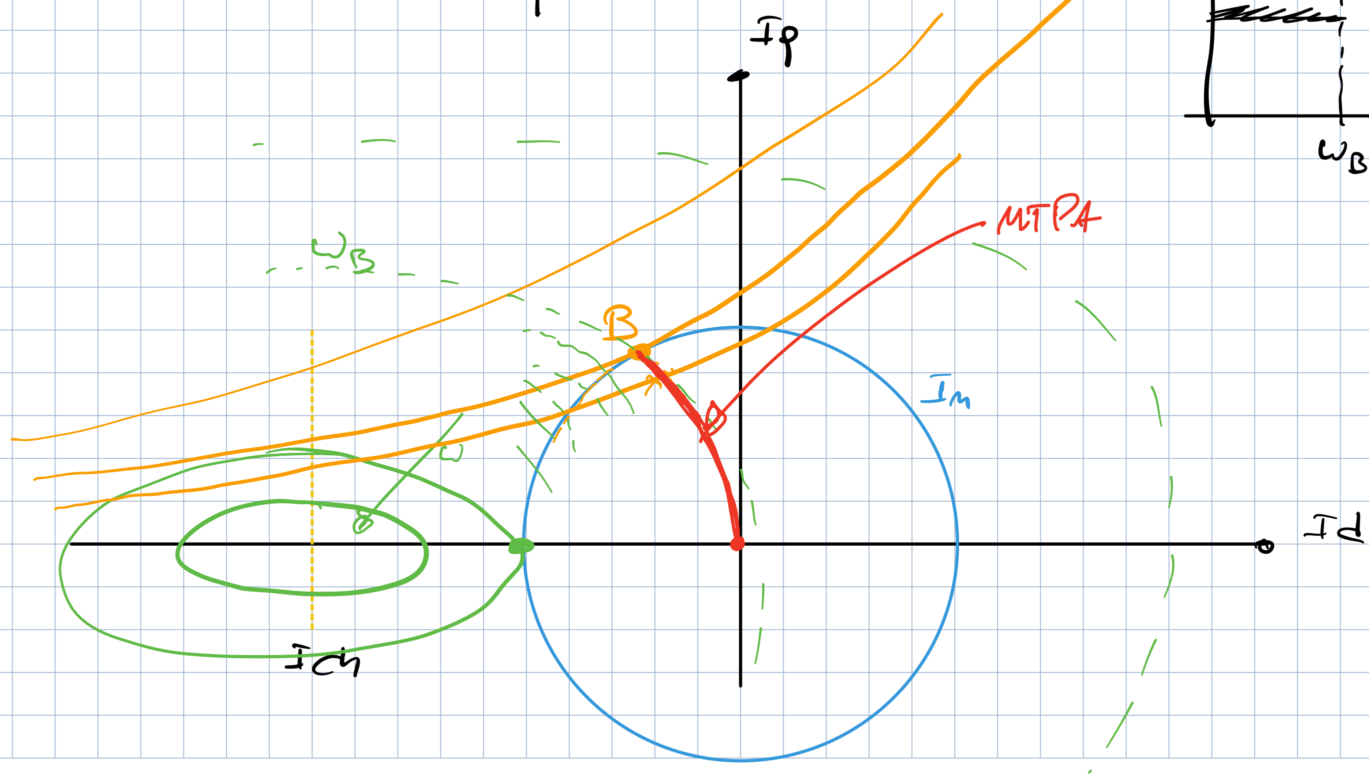
$$I_q = \frac{2}{3} \frac{m}{p} \frac{1}{\left[ \lambda_m + (L_d - L_q) I_d \right]}$$

IPERBOLI (CURVE ISOCOPPIA)

$$\begin{cases} I_q = 0 \\ \lambda_m + (L_d - L_q) I_d = 0 \end{cases} \qquad \begin{cases} I_q = 0 \\ I_d = \dfrac{\lambda_m}{(L_q - L_d)} \end{cases}$$

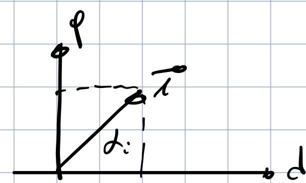


TRAIETTORIA MTPA

$$m = \frac{3}{2} p \left[ \Lambda_m I_q + (L_d - L_q) I_d I_q \right]$$




$$I_d = I \cos\alpha_i$$
$$I_q = I \sin\alpha_i$$

$$= \frac{3}{2} p \left[ \Lambda_m I \sin\alpha_i + (L_d - L_q) I^2 \sin\alpha_i \cos\alpha_i \right]$$

$$= \frac{3}{2} p \left[ \Lambda_m I \sin\alpha_i + (L_d - L_q) I^2 \frac{\sin 2\alpha_i}{2} \right]$$

$$\frac{dm}{d\alpha_i} = 0 = \Lambda_m I \cos\alpha_i + (L_d - L_q) I^2 \cos 2\alpha_i$$

$$= \Lambda_m \cos\alpha_i + (L_d - L_q) I \cos 2\alpha_i$$

$$= \Lambda_m \cos\alpha_i + (L_d - L_q) I (2\cos^2\alpha_i - 1)$$

$$\underbrace{2(L_d - L_q) I}_{a} \cos^2\alpha_i + \underbrace{\Lambda_m}_{b} \cos\alpha_i \underbrace{- (L_d - L_q) I}_{c} = 0$$

$$\cos\alpha_i = \underline{\hspace{6cm}}$$