LEZIONE 17 ~ 18 — 11.11.2022

VISCOELASTICITÀ

ELASTICO — Legge di Hooke

$\sigma = E\varepsilon$

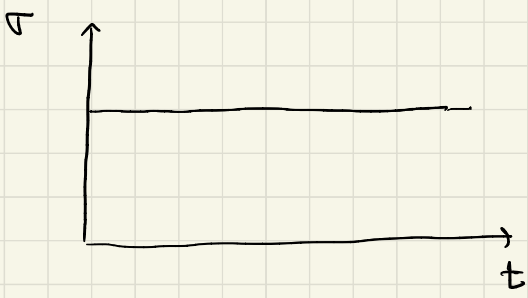
↳ Modulo di Young [GPa]

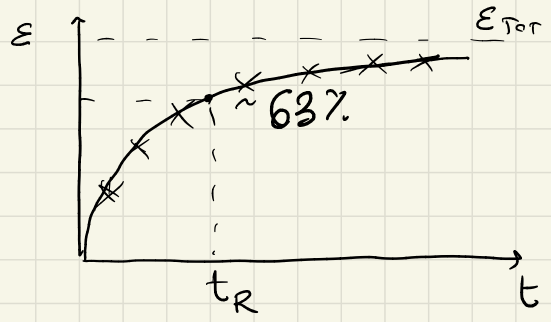
VISCOSO — Legge di Newton

$\sigma = \eta \dfrac{d\varepsilon}{dt}$

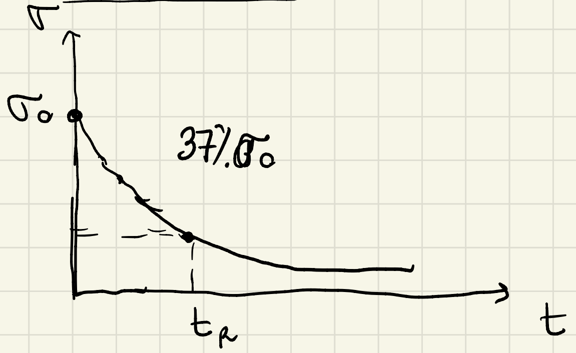
↳ VISCOSITÀ [Pa·s]

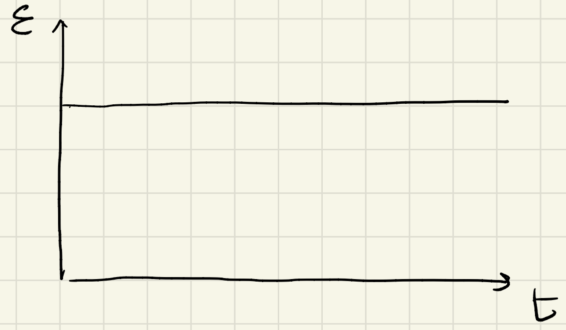
CREEP



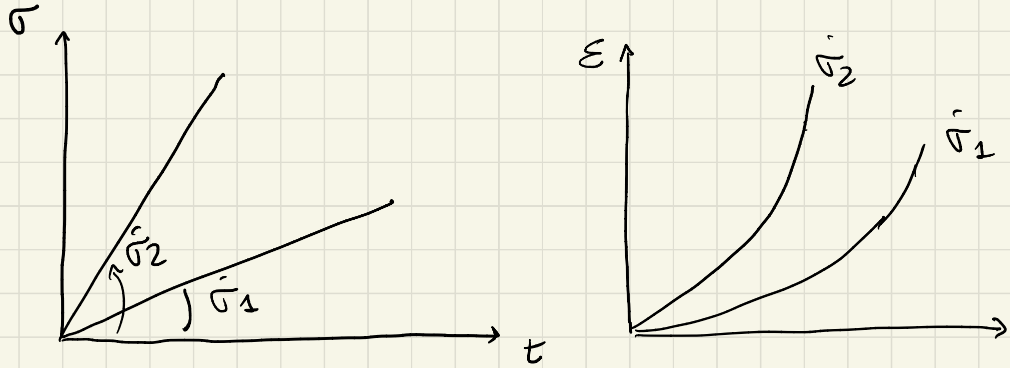


RILASSAMENTO

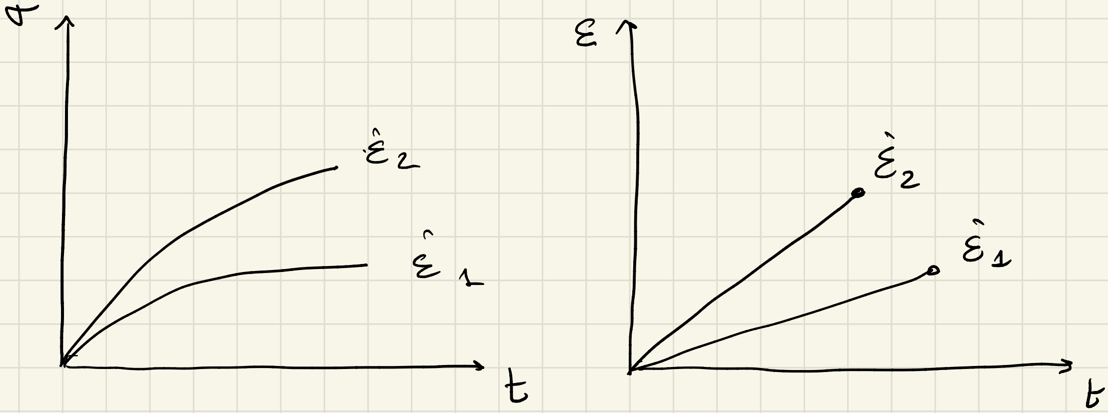




STRESS RATE = cost



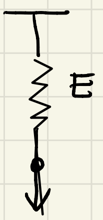
DEFORMATION RATE = cost



MODELLI MECCANICI x DESCRIVERE il COMPORTAMENTO VISCOELASTICO

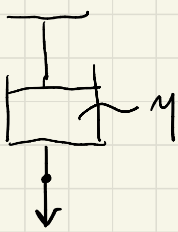
MODELLI ad 1 ELEMENTO

ELASTICO



$\sigma = E\varepsilon$

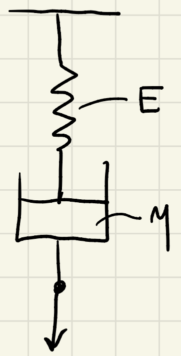
VISCOSO



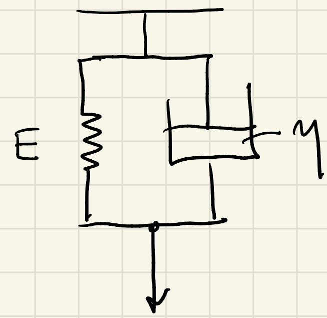
$\sigma = \eta \frac{d\varepsilon}{dt}$

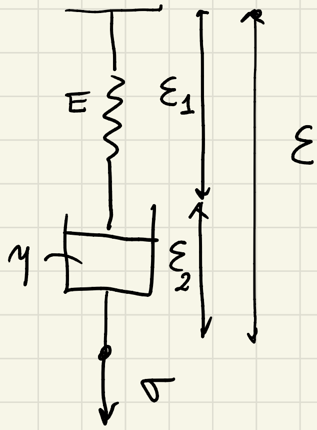
MODELLI a 2 ELEMENTI

MAXWELL



KELVIN-VOIGT





$$\varepsilon = \varepsilon_1 + \varepsilon_2$$

$$\sigma = \sigma_1 = \sigma_2 \leftarrow$$

$$\sigma_1 = E\varepsilon_1 \qquad \frac{d\varepsilon_1}{dt} = \frac{1}{E}\frac{d\sigma}{dt}$$

$$\sigma_2 = \eta \frac{d\varepsilon_2}{dt} \qquad \frac{d\varepsilon_2}{dt} = \frac{\sigma_2}{\eta}$$

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon_1}{dt} + \frac{d\varepsilon_2}{dt}$$

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta}$$

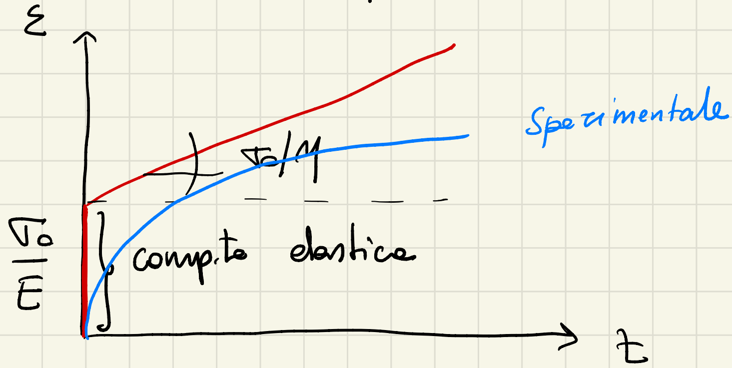
CREEP $\boxed{\sigma = \sigma_0}$ $\frac{d\sigma}{dt} = 0$ ε_t $t=0 \quad \varepsilon = \varepsilon_0$

$$\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta} \qquad \int_0 d\varepsilon = \int_0^{\infty} \frac{\sigma}{\eta} dt$$

$$\varepsilon_0 = \frac{\sigma_0}{E}$$

$$\boxed{\varepsilon(t) = \frac{\sigma_0}{E} + \frac{\sigma_0}{\eta} t}$$

$$y = q + mx$$



IL MODELLO di MAXWELL <u>NON</u> SI ADATTA AL CREEP

RILASSAMENTO delle TENSIONI

$\varepsilon = \varepsilon_0 = \text{cost}$; $\sigma = \sigma(t)$

$$\varepsilon = \varepsilon_0 \Rightarrow \frac{d\varepsilon}{dt} = 0$$

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta} \Rightarrow \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta} = 0$$

$$\frac{d\sigma}{\sigma} = -\frac{E}{\eta} t \qquad \begin{array}{l} t = 0 \\ \sigma = \sigma_0 \end{array}$$

$$\sigma = \sigma_0 \exp\left(-\frac{E}{\eta} t\right)$$

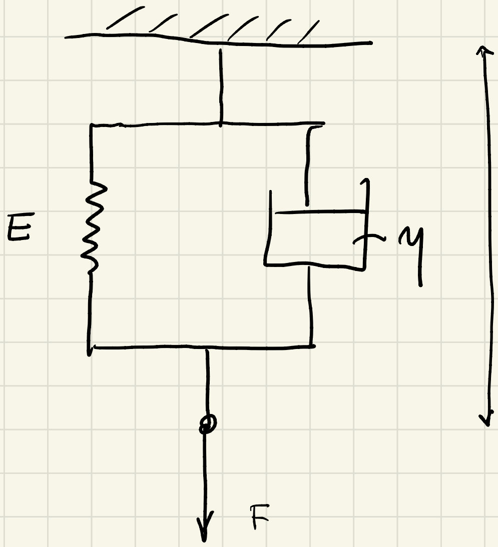
Definisco t_R TEMPO di RILASSAMENTO

$$t_R = \frac{\eta}{E} \qquad \frac{[\cancel{Pa} \cdot s]}{[\cancel{Pa}]} = [s]$$

$$\boxed{\sigma = \sigma_0 \, e^{-t/t_R}}$$

BUON FITTING
del COMP.TO a
RILASSAMENTO

MODELLO di VOIGT



$$\varepsilon = \varepsilon_1 = \varepsilon_2$$

$$\sigma = \sigma_1 + \sigma_2 \qquad (*)$$

$$\left.\begin{aligned} \sigma_1 &= \varepsilon E \\ \sigma_2 &= \eta \frac{d\varepsilon}{dt} \end{aligned}\right\} \qquad \boxed{\sigma = \varepsilon E + \eta \frac{d\varepsilon}{dt}}$$

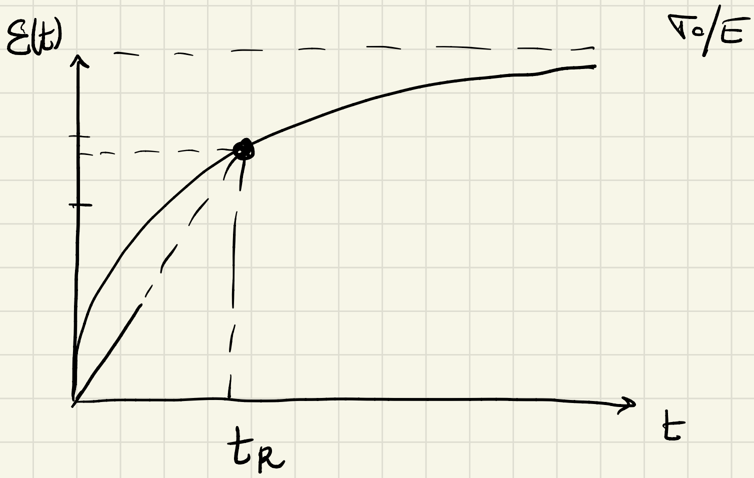
$$\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta} - \varepsilon \frac{E}{\eta}$$

CREEP $\quad \sigma = \sigma_0$

$$\frac{d\varepsilon}{dt} + \varepsilon \frac{E}{\eta} = \frac{\sigma_0}{\eta} \qquad \varepsilon = \frac{\sigma_0}{E}\left[1 - \exp\left(-\frac{E}{\eta} t\right)\right]$$

t_R = TEMPO di RITARDO

$$t_R = \frac{\eta}{E} \qquad \boxed{\varepsilon(t) = \frac{\sigma_0}{E}\left[1 - e^{-\frac{t}{t_R}}\right]}$$



SIGNIFICATO di t_R

CREEP → DESCRITTO da VOIGT

se $t = t_R$

$$\varepsilon(t_R) = \frac{\sigma_0}{E}\left(1 - e^{-t_R/t_R}\right)$$

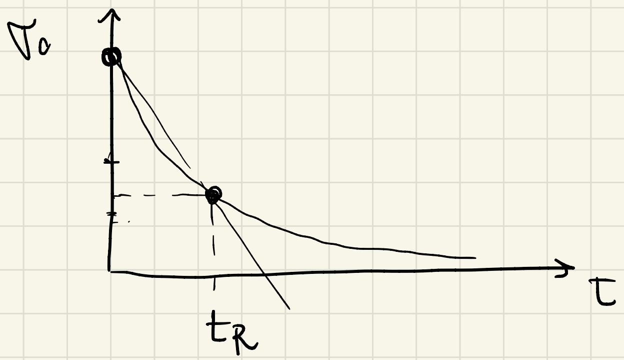
$$\overset{!}{=} \frac{\sigma_0}{E}\left(1 - \frac{1}{e}\right)$$

$$\varepsilon(t_R) = \frac{\sigma_0}{E}\left(1 - 0.37\right) = 0.63\,\frac{\sigma_0}{E}$$

$t = t_R$ TEMPO di RILASSAMENTO

$$\sigma(t_R) = \sigma_0\, e^{-1} \qquad \left[\sigma(t) = \sigma_0\, e^{-t/t_R}\right]$$

$$= \sigma_0 \frac{1}{e} \simeq 0.37\,\sigma_0$$

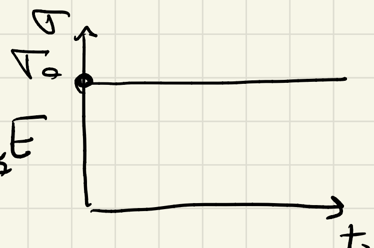


<u>CASO 2</u> RILASSAMENTO delle TENSIONI con VOIGT

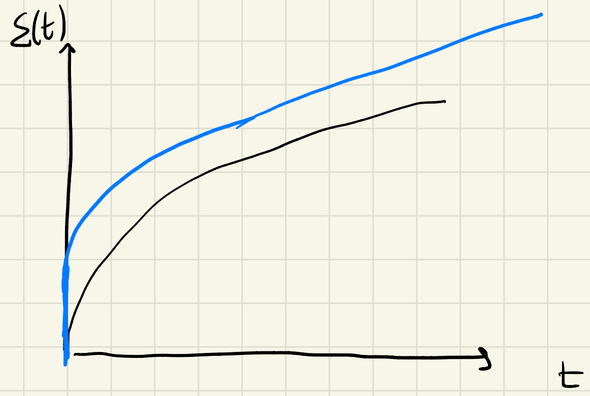
$$\sigma = \varepsilon E + \eta \frac{d\varepsilon}{dt}$$

se $\varepsilon = \varepsilon_0 = cost$

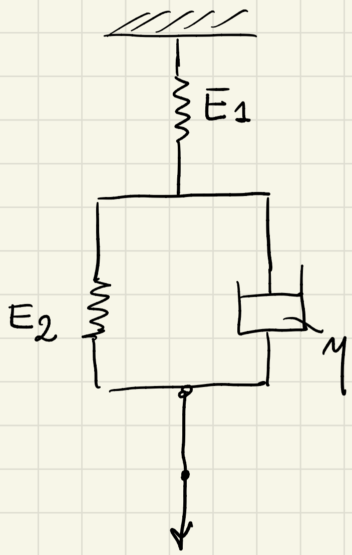
$\sigma = \varepsilon_0 E$

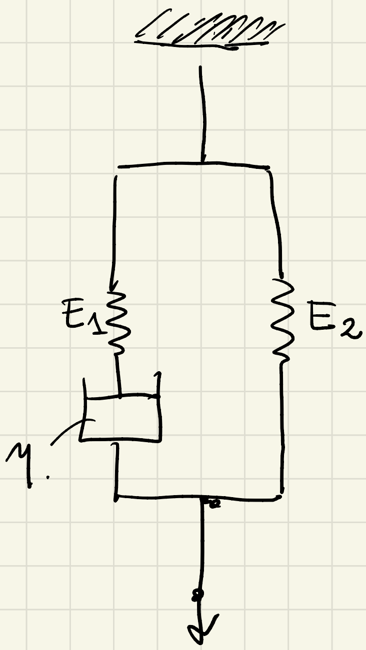


NON INTERPETRA il RILASSAMENTO delle TENSIONI



MODELLI a 3 ELEMENTI o MODELLI di ZENER

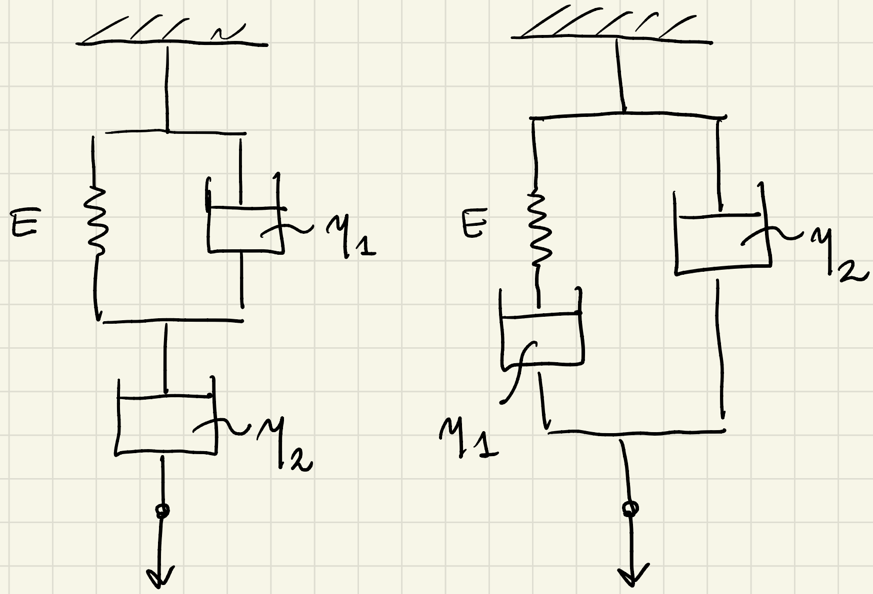




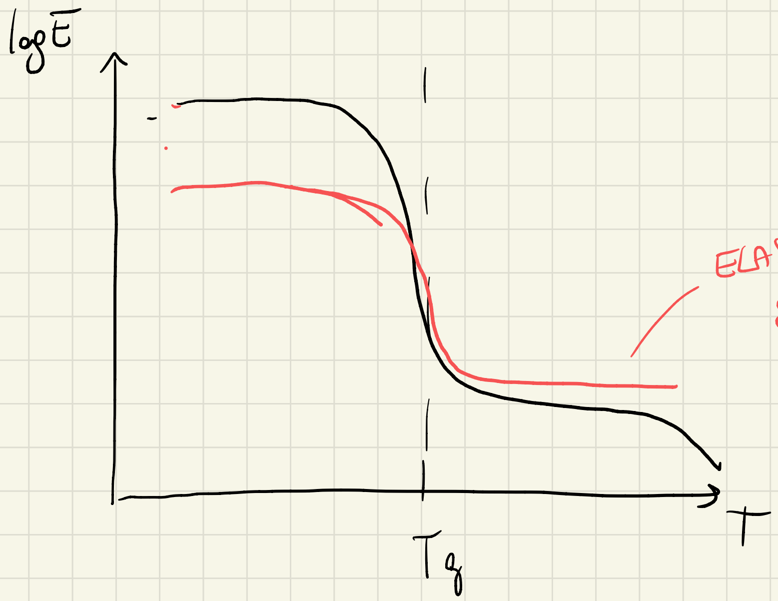
COMPORTAMENTO GOMMOSO

ELASTOMERI

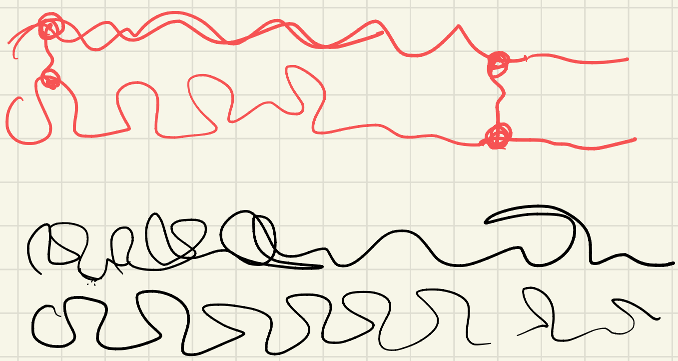
MODELLI A 3 ELEMENTI   ANTI-ZENER



COMPORTAMENTO VISCOELASTICO
dei POLIMERI TERMOPLASTICI

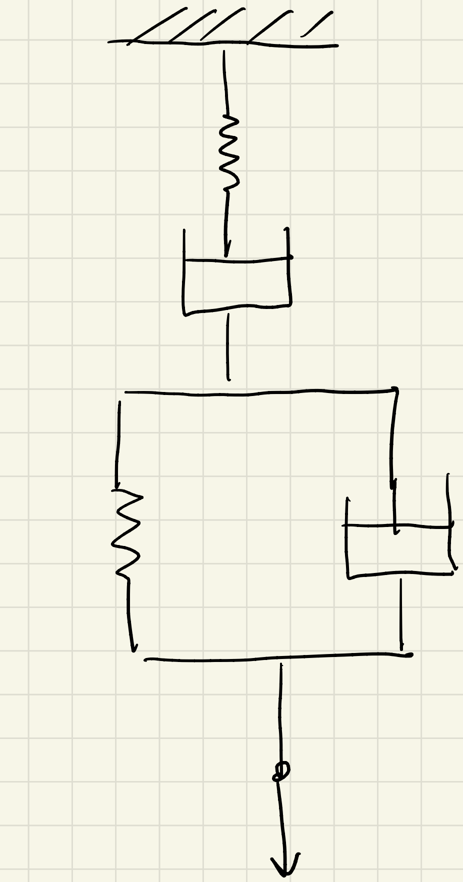


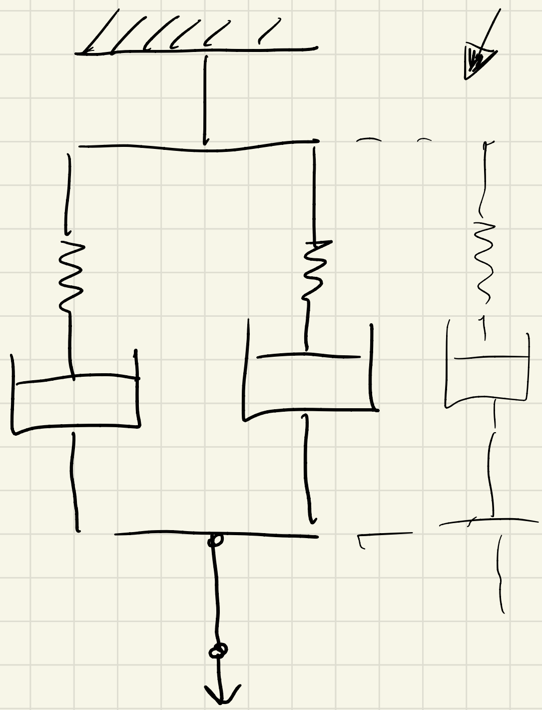
ELASTOMERI
STRUTTURA PARZIALMENTE RETICOLATA

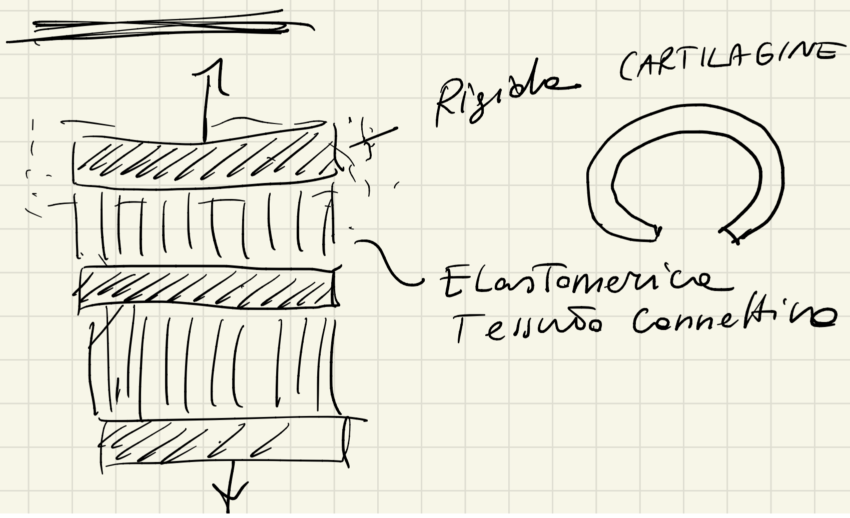


si rompono i punti di aggancio fisico

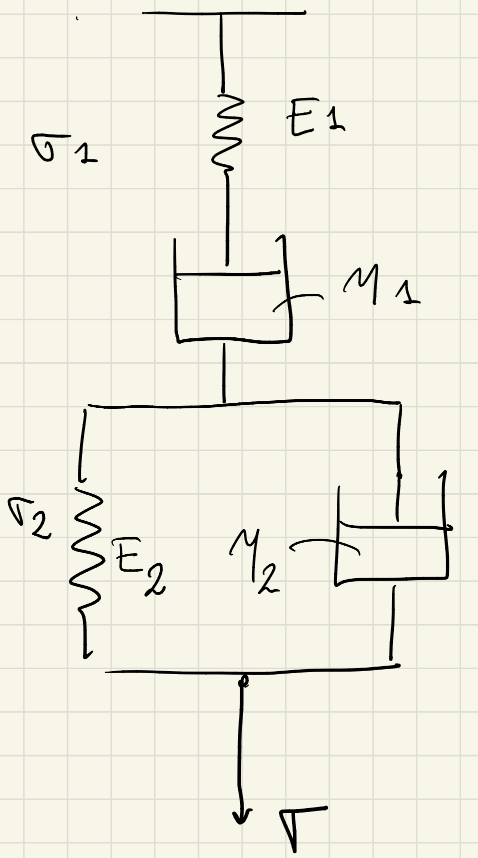
MODELLI a 4 ELEMENTI (BURGER)

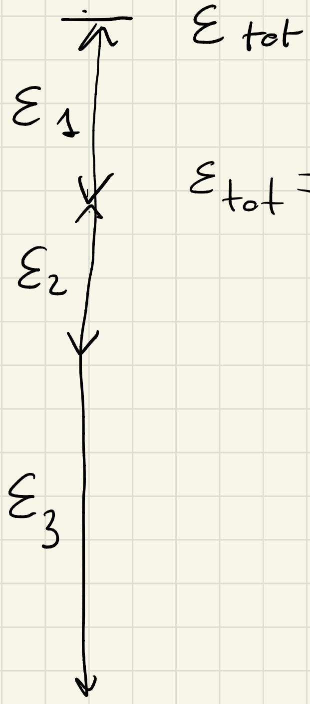






MODELLO di BURGER





$$\varepsilon_{tot} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

$$\sigma = \sigma_1 + \sigma_2$$

$$\sigma_2 = \varepsilon_3 E_2 + \eta_2 \frac{d\varepsilon_3}{dt}$$

$$\sigma_1 = \varepsilon_1 E_1 = \eta_1 \frac{d\varepsilon_2}{dt}$$

$$\varepsilon = \frac{\sigma_1}{E_1} + \frac{\sigma_1}{\eta} t + \frac{\sigma_2}{E_2}\left(1 - e^{-t/t_R}\right)$$

COMP. TO A CREEP DESCRITTO da VOIGT

$\sigma = \sigma_0$

$$\varepsilon(t) = \frac{\sigma_0}{E(t)}$$

$\sigma = E(t)\,\varepsilon$

$D(t)$ MODULO di CREEP

$C(t)$ $[GPa^{-1}]$

$J(t)$

$$\sigma(t) = E(t)\,\varepsilon$$

↳ MODULO di RILASSAMENTO

$$\boxed{J(t) \neq \frac{1}{E(t)}}$$

ATTENZIONE!!!

$$\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-t/t_R}\right)$$

MODELLO di VOIGT

$$\varepsilon(t) = \sigma_0 D(t) \longrightarrow D(t) = \frac{1}{E}\left(1 - e^{-t/t_{RIT}}\right)$$

$$D'(t) = 2 - e^{-0.5t}$$

ESEMPIO

ESERCIZIO 1

KELVIN-VOIGT

$\sigma_0 = 10\ MPa$

$D_0 = 2\ GPa^{-1}$

$t_{RIT} = 5760h = 2.07 \times 10^7\ s$

CALCOLARE $\varepsilon(t = 4\ mesi)$

KELVIN-VOIGT

$$\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-t/t_R}\right)$$

$$D(t) = \underset{\substack{\shortparallel \\ D_0}}{\frac{1}{E}}\left(1 - e^{-t/t_R}\right)$$

$$D(t) = D_0\left(1 - e^{-t/t_{RIT}}\right)$$

$$\varepsilon(4mesi) = \sigma_0 D_0\left(1 - e^{-t/t_{RIT}}\right)$$

$$t = 4mesi = 4 \times 30 \times 24 \times 3600 = 1.04 \times 10^7\ [s]$$

$$\varepsilon(4\,\text{mesi}) = 10 \times 10^{6} \times 2 \times 10^{-9}\left(1 - e^{-\frac{1}{2}}\right)$$

$$\frac{t}{t_{RIT}} = \frac{1.04 \times 10^{7}}{2.07 \times 10^{7}} \simeq \frac{1}{2}$$

$$= 20 \times 10^{-3}\left(1 - \frac{1}{\sqrt{e}}\right)$$

$$\overset{!}{=} 7{,}8 \times 10^{-3}$$

$\boxed{\text{ESERCIZIO 2}}$ MODELLO di MAXWELL

$\varepsilon_0 = \text{cost}$

$\sigma_0 = 7{,}6\ \text{MPa}$

$t_1 = 40_{gg}$ @ $20°C$ $\sigma_1 = \underline{4{,}8\ \text{MPa}}$

- CALCOLARE t_R
- σ_2, $t_2 = 60$ gg

$$\sigma(t) = \sigma_0\, e^{-t/t_R}$$

$$\sigma(t_1) = \sigma_0\, e^{-t_1/t_R}$$

$$4.8 \times 10^6 = 7.6 \times 10^6 \, e^{-\frac{3.46 \times 10^6}{t_R}}$$

$t_1 = 40_{gg} = 3.46 \times 10^6$ [s]

$$\frac{4.8}{7.6} = e^{-\frac{3.46 \times 10^6}{t_R}} \Rightarrow t_R = 7.5 \times 10^6 \text{ [s]}$$

$t_2 = 60_{gg}$

$t_2 = 60 \times 24 \times 3600 = 5.18 \times 10^6$ s

$$\sigma(t_2) = 7.6 \times 10^6 \, e^{-\frac{5.18 \times 10^6}{7.5 \times 10^6}} = 3.8 \text{ MPa}$$

$\boxed{\text{ESERCIZIO 3}}$ CONFRONTO MODELLO a 2 ELEMENTI

$t = 0$ $\sigma_0 = 10$ MPa per 200 giorni $= t_1$

$\varepsilon(t_1)$

$E = 3$ GPa

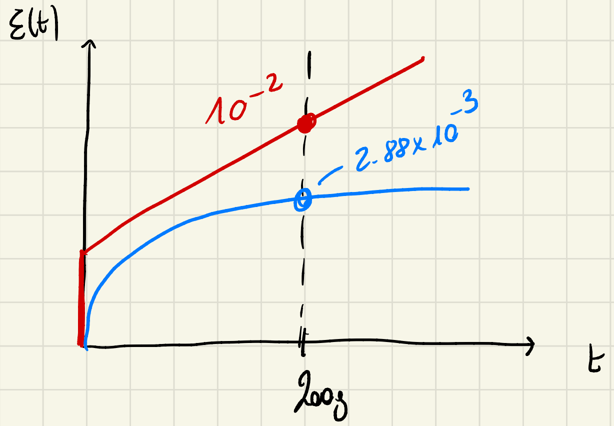
$\eta = 300$ GPa · gg [Pa·s]

MAXWELL

$$\varepsilon(t) = \frac{\sigma_0}{\eta} t + \frac{\sigma_0}{E}$$

KELVIN – VOIGT

$$\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-t/t_R}\right)$$



$$\varepsilon(200g) = \frac{10\times10^{6}}{300\times10^{9}}\cdot 200 + \frac{10\times10^{6}}{3\times10^{9}}$$

$$= 10\times10^{-3}\cdot\frac{2}{3} + \frac{10}{3}\times10^{-3}$$

$$= \frac{10}{3}\times10^{-3}(2+1)$$

$$= 10^{-2}$$

$$\varepsilon(200g) = \frac{10\times10^{6}}{3\times10^{9}}\left(1-e^{-200/100}\right)$$

$$t_{RIT} = \frac{\eta}{E} = \frac{300\ \text{GPa}\cdot gg}{3\ \text{GPa}} = 100\,gg$$

$$\varepsilon(200g) = 3.33\times10^{-3}\left(1-e^{-2}\right)$$

$$\overset{!}{=}\ 2.88\times10^{-3}$$

PRINCIPIO di EQUIVALENZA t-T

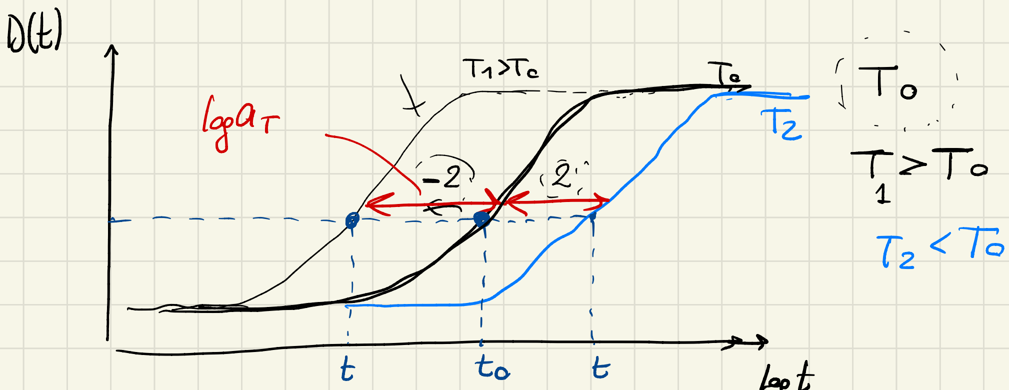
a_T = FATTORE di SPOSTAMENTO

$$a_{T_0}^{T} = \frac{t}{t_0}$$

$$\log a_T = \frac{-C_1 (T - T_0)}{C_2 + (T - T_0)}$$

$C_1 = 17.4$

$C_2 = 51.6$ [°C]



$$\log a_T = \log \frac{t}{t_0}$$

ESEMPIO

$$\boxed{D_{T_0}(t) = 2 - e^{-0.1 t_0}} \qquad D_T(t)$$

$T < T_0 \qquad \log a_{T_0}^{T} = 2 \qquad a_{T_0}^{T} = 100$

$T > T_0 \qquad \log a_{T_0}^{T} = -2 \qquad a_{T_0}^{T} = 0.01$

$$a_T = \frac{t}{t_0} \Rightarrow t = t_0 a_{T_0}^{T} \Rightarrow t_0 = \frac{t}{a_{T_0}^{T}}$$

$T < T_0$

$$D(t) = 2 - e^{-0.1\, t / a^{T}_{T_0}}$$

$$D(t) = 2 - e^{-0.1 \frac{t}{100}} = 2 - e^{-0.001 t}$$

$T > T_0$

$$D(t) = 2 - e^{-0.1 \frac{t}{0.01}} = 2 - e^{-10 t}$$