$$\begin{cases} v_a = R i_a + \dfrac{d\lambda_a}{dt} \\ v_b = R i_b + \dfrac{d\lambda_b}{dt} \\ v_c = R i_c + \dfrac{d\lambda_c}{dt} \end{cases} \qquad \begin{cases} \lambda_a = \lambda_m \cos \vartheta_m^e + L i_a \\ \lambda_b = \lambda_m \cos\left(\vartheta_m^e - \frac{2}{3}\pi\right) + L i_b \\ \lambda_c = \lambda_m \cos\left(\vartheta_m^e - \frac{4}{3}\pi\right) + L i_c \end{cases} \qquad \vartheta_m^e = \omega_m^e t$$

$$\begin{cases} \dfrac{d\lambda_{ma}}{dt} = -\omega_m^e \lambda_m \operatorname{sen} \vartheta_m^e = \omega_m^e \lambda_m \cos\left(\vartheta_m^e + \frac{\pi}{2}\right) = e_a \\ \dfrac{d\lambda_{mb}}{dt} = -\omega_m^e \lambda_m \operatorname{sen}\left(\vartheta_m^e - \frac{2}{3}\pi\right) = \omega_m^e \lambda_m \cos\left(\vartheta_m^e - \frac{2}{3}\pi + \frac{\pi}{2}\right) = e_b \\ \dfrac{d\lambda_{mc}}{dt} = -\omega_m^e \lambda_m \operatorname{sen}\left(\vartheta_m^e - \frac{4}{3}\pi\right) = \omega_m^e \lambda_m \cos\left(\vartheta_m^e - \frac{4}{3}\pi + \frac{\pi}{2}\right) = e_c \end{cases}$$

3 FORZE CONTRO ELETTROMOTRICI $\omega_m^e = \text{cost} \iff$ 3 SINUSOIDI

BRUSHLESS SINUSOIDALE ω_m^e PULSAZIONE SINCRONA CON IL ROTORE

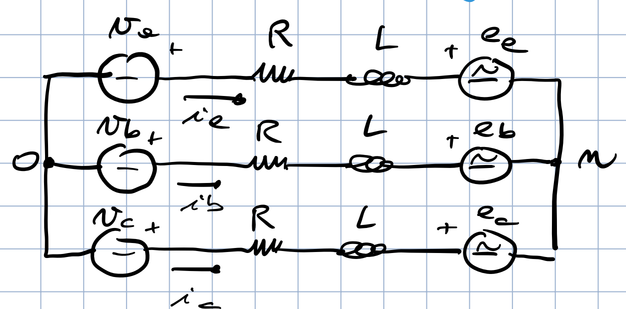
Ampiezze: $E_m = \omega_m^e \lambda_m$

$$E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{\omega_m^e \lambda_m}{\sqrt{2}}$$

$$V_{rms} = \sqrt{3} \cdot E_{rms} = \sqrt{\frac{3}{2}}\, \lambda_m \omega_m^e = K_E\, \omega_m^e$$

$$\left[V \cdot s \cdot \frac{1}{s}\right] = [V]$$

$$\begin{cases} v_a = R i_a + L \dfrac{di_a}{dt} + e_a \\ v_b = R i_b + L \dfrac{di_b}{dt} + e_b \\ v_c = R i_c + L \dfrac{di_c}{dt} + e_c \end{cases}$$

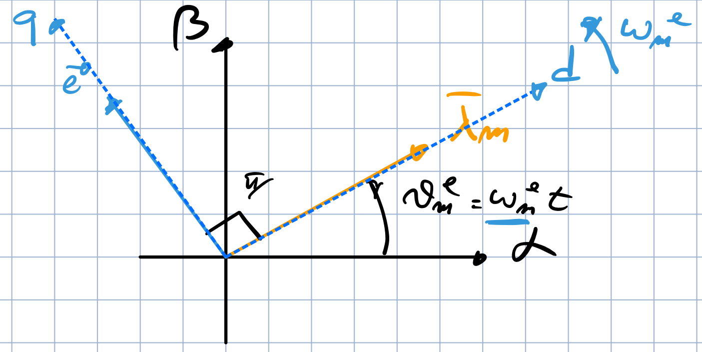


$$\vec{v} = R\vec{i} + L\frac{d\vec{i}}{dt} + \vec{e} \quad \text{in } \alpha\beta \quad \text{(STAZIONARIO)}$$

In particolare posso definire:

$$\vec{\lambda}_m = \lambda_m e^{j\vartheta_m^e} \qquad \vec{e} = \frac{d\vec{\lambda}_m}{dt} = \frac{d}{dt}\lambda_m e^{j\omega_m^e t} = j\omega_m^e \lambda_m e^{j\omega_m^e t}$$

$$= j\omega_m^e \vec{\lambda}_m$$



$$\bar v = R\bar i + L\frac{d\bar i}{dt} + \bar e$$
$$= R\bar i + L\frac{d\bar i}{dt} + j\omega\bar\lambda_m$$

$$v_\alpha + j v_\beta = R(i_\alpha + j i_\beta) + L\left(\frac{di_\alpha}{dt} + j\frac{di_\beta}{dt}\right) + j\omega_m^e\left(\lambda_{m\alpha} + j\lambda_{m\beta}\right)$$

$$\begin{cases} v_\alpha = R i_\alpha + L\dfrac{di_\alpha}{dt} - \omega_m^e\lambda_{m\beta} \\[2ex] v_\beta = R i_\beta + L\dfrac{di_\beta}{dt} + \omega_m^e\lambda_{m\alpha} \end{cases}$$

$$\bar g^R = \bar g^s e^{-j\vartheta_m^e}$$

R: Rotante dq

S: STAZIONARIO αβ

$$\bar v^s = R\bar i^s + L\frac{d\bar i^s}{dt} + j\omega_m^e\bar\lambda_m^s$$

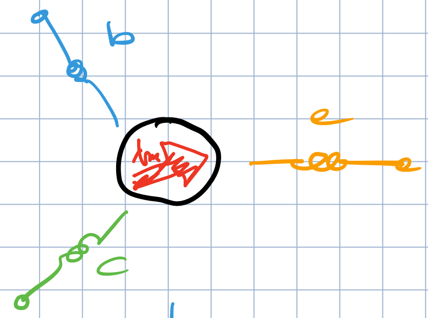
$$\bar v^R e^{j\vartheta_m^e} = R\bar i^R e^{j\vartheta_m^e} + L\frac{d}{dt}\left(\bar i^R e^{j\vartheta_m^e}\right) + j\omega_m^e\bar\lambda_m^R e^{j\vartheta_m^e}$$

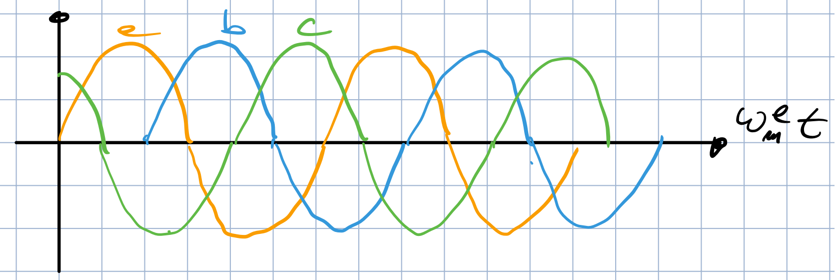
$$\bar v^R \cancel{e^{j\vartheta_m^e}} = R\bar i^R \cancel{e^{j\vartheta_m^e}} + L\frac{d\bar i^R}{dt}\cancel{e^{j\vartheta_m^e}} + L\bar i^R j\omega_m^e \cancel{e^{j\vartheta_m^e}} + j\omega_m^e\bar\lambda_m^R \cancel{e^{j\vartheta_m^e}}$$

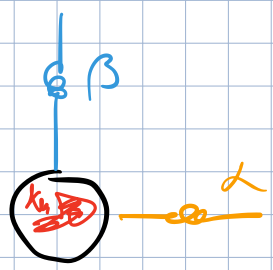
$$\bar v^R = R\bar i^R + L\frac{d\bar i^R}{dt} + j\omega_m^e L\bar i^R + j\omega_m^e\bar\lambda_m^R$$
$$= R\bar i^R + L\frac{d\bar i^R}{dt} + j\omega_m^e\underbrace{\left[L\bar i^R + \bar\lambda_m^R\right]}_{\bar\lambda^R = \bar\lambda_m^R + L\bar i^R}$$

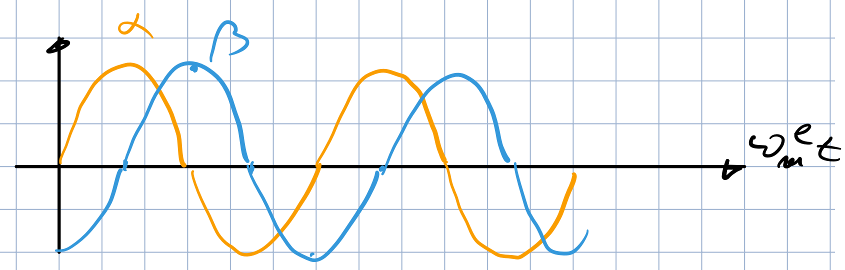
$$\begin{cases} v_d = R i_d + L\dfrac{di_d}{dt} - \omega_m^e L i_q \\[2ex] v_q = R i_q + L\dfrac{di_q}{dt} + \omega_m^e\left[\lambda_m + L i_d\right] \end{cases}$$

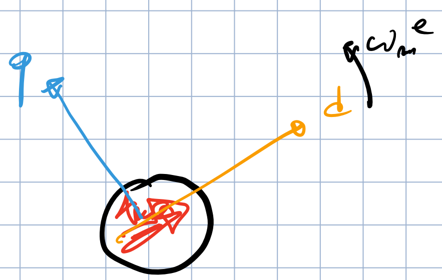
Note $\bar\lambda_m^R = \lambda_d + j0$

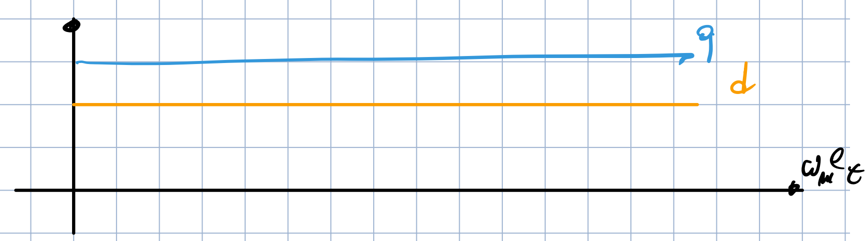












BILANCIO ENERGETICO

$$\begin{cases} v_d = R\, i_d + L \dfrac{d i_d}{dt} - \omega_m^e L\, i_q \\[2ex] v_q = R\, i_q + L \dfrac{d i_q}{dt} + \omega_m^e \left[\lambda_m + L\, i_d \right] \end{cases}$$

$$\frac{3}{2}\left[v_d i_d + v_q v_q \right] = P_{in}$$

$$\begin{cases} v_\alpha = R\, i_\alpha + L \dfrac{d i_\alpha}{dt} - \omega_m^e \lambda_{m\beta} \\[2ex] v_\beta = R\, i_\beta + L \dfrac{d i_\beta}{dt} + \omega_m^e \lambda_{m\alpha} \end{cases}$$

$$\frac{3}{2}\left[v_\alpha i_\alpha + v_\beta i_\beta \right] = P_{in}$$

Inizio da α-β

$$\frac{3}{2} v_\alpha i_\alpha = \frac{3}{2} i_\alpha \left[R i_\alpha + L \frac{d i_\alpha}{dt} - \omega_m^e \lambda_{m\beta} \right]$$

$$\frac{3}{2} v_\beta v_\beta = \frac{3}{2} i_\beta \left[R i_\beta + L \frac{d i_\beta}{dt} + \omega_m^e \lambda_{m\alpha} \right]$$

(+)

$$\underbrace{\frac{3}{2} R i_\alpha^2 + \frac{3}{2} R i_\beta^2}_{P_{Joule}} + \underbrace{\frac{3}{2} L i_\alpha \frac{di_\alpha}{dt} + \frac{3}{2} L i_\beta \frac{di_\beta}{dt}}_{\frac{3}{2}\left\{\frac{d}{dt}\left[\frac{1}{2} L i_\alpha^2 + \frac{1}{2} L i_\beta^2\right]\right\}} + \underbrace{\frac{3}{2}\omega_m^e\left[\lambda_{m\alpha} i_\beta - \lambda_{m\beta} i_\alpha\right]}_{P_{mecc.}} = P_{in}$$

[redacted]

Variazione Energia magnetostatica

$$P_m = m \cdot \omega_m = \frac{3}{2}\overbrace{\omega_m^e}^{\omega_m \cdot p}\left[\lambda_{m\alpha} i_\beta - \lambda_{m\beta} i_\alpha\right]$$

$$m = \frac{3}{2} p \left(\underline{\lambda_{m\alpha} i_\beta} - \underline{\lambda_{m\beta} i_\alpha}\right)$$

$$\bar{\lambda}_m^s = \lambda_{m\alpha} + j\lambda_{m\beta} \qquad \bar{\lambda}_m^{s*} = \lambda_{m\alpha} - j\lambda_{m\beta}$$

$$\bar{i}^s = i_\alpha + j i_\beta$$

$$m = \frac{3}{2} p \, Im\left[\bar{i}^s \bar{\lambda}_m^{s*}\right]$$

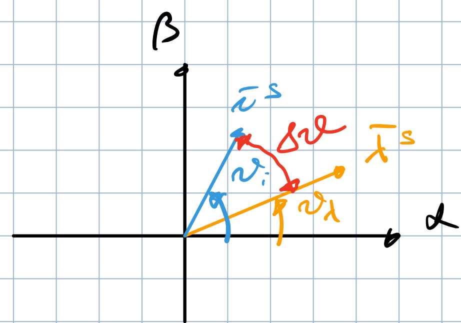
$$= \frac{3}{2} p \, Im\left[(i_\alpha + j i_\beta)(\lambda_{m\alpha} - j\lambda_{m\beta})\right]$$

$$= \frac{3}{2} p \, Im\left[|\bar{i}| e^{j\vartheta_i} \, |\bar{\lambda}_m| e^{-j\vartheta_\lambda}\right]$$

$$= \frac{3}{2} p \, |\bar{i}| \, |\bar{\lambda}_m| \, Im\left[e^{j(\vartheta_i - \vartheta_\lambda)}\right]$$

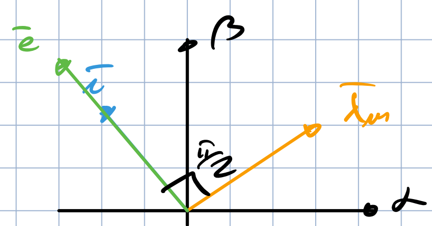
$$= \frac{3}{2} p \, |\bar{i}| \, |\bar{\lambda}_m| \, sen \Delta\vartheta$$

sfasamento fra $\bar{i}$ e $\bar{\lambda}_m$



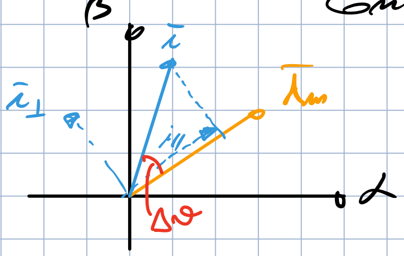
① λ_m è costante (PM) posso controllare m controllando $\bar{i}$

② m è massima se $sen \Delta\vartheta = 1$ $\quad \Delta\vartheta = \pi/2$



③ $\bar{i}$ è in fase con $\bar{i}$ quando $\Delta\vartheta = \pi/2$ ossia si ha la massima coppia

④ $m = \frac{3}{2} p |\bar{\lambda}_m| \underbrace{|\bar{i}| \operatorname{sen} \Delta\vartheta}_{\text{COMPONENTE ORTOGONALE A } \bar{\lambda}_m}$



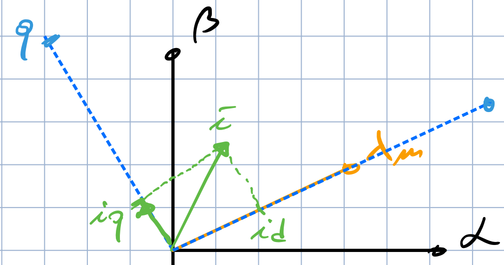
$$m = \frac{3}{2} p |\bar{\lambda}_m| \underbrace{\bar{i}_\perp}_{|\bar{i}| \operatorname{sen} \Delta\vartheta}$$

Se faccio il bilancio di potenza nel sistema dq

$$\underbrace{\frac{3}{2}\left(v_d i_d + v_q i_q\right)}_{P_{in}} = \underbrace{\frac{3}{2}\left(R i_d^2 + R i_q^2\right)}_{P_{Joule}} + \underbrace{\frac{3}{2} L \left(i_d \frac{di_d}{dt} + i_q \frac{di_q}{dt}\right)}_{\Delta W_m} + \underbrace{\frac{3}{2}\omega_m^e \left(\left[L i_d + \lambda_m\right] i_q - L i_q i_d\right)}_{P_m}$$

$$P_m = m \cdot \omega = \frac{3}{2}\omega_m^e \left[\cancel{L i_d i_q} + \lambda_m i_q - \cancel{L i_q i_d}\right] = \frac{3}{2} p \omega \lambda_m i_q$$

$$m = \frac{3}{2} p \lambda_m i_q$$



SINTESI

Riferimento dq ROTANTE:

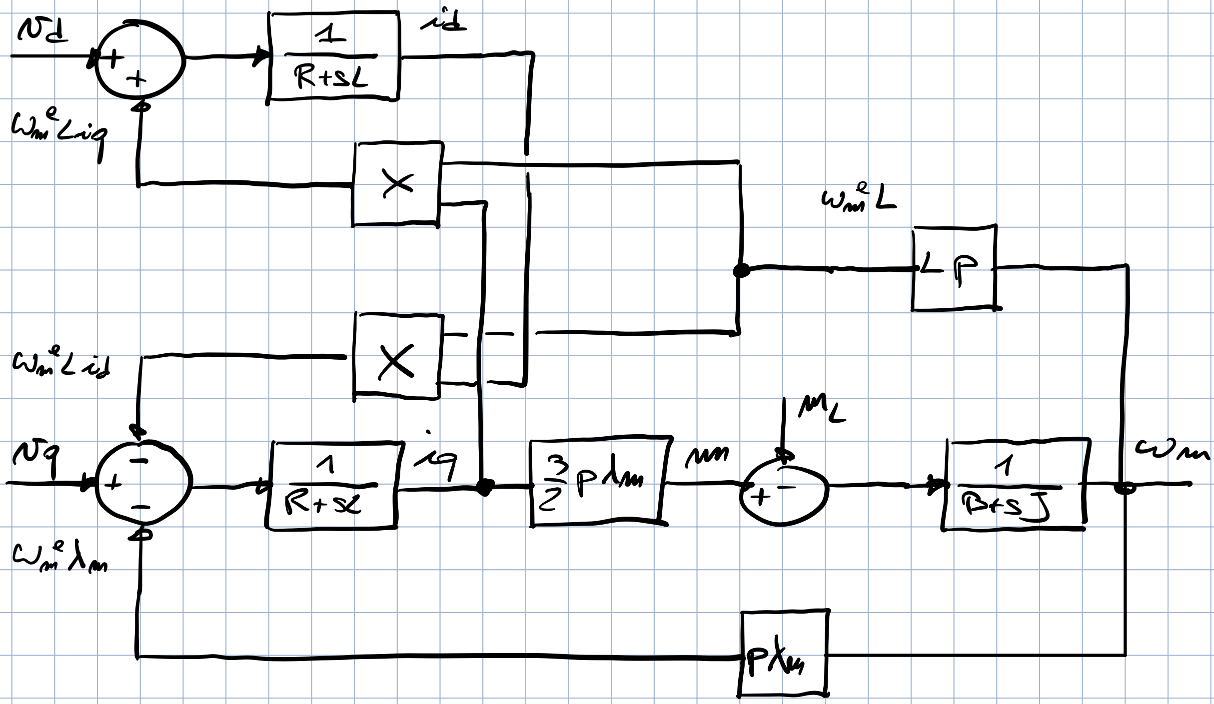
$$\bar{v}^R = R \bar{i}^R + L \frac{d\bar{i}^R}{dt} + j \omega_m^e \bar{\lambda}^R$$

$$\bar{\lambda}^R = \lambda_m + L \bar{i}_R = \lambda_d + j \lambda_q$$

$$\begin{cases} \lambda_d = \lambda_m + L i_d \\ \lambda_q = 0 + L i_q \end{cases}$$

$$\begin{cases} v_d = R i_d + L \dfrac{di_d}{dt} - \omega_m^e L i_q \\ v_q = R i_q + L \dfrac{di_q}{dt} + \omega_m^e \left(\lambda_m + L i_d\right) \end{cases}$$

$$m = \frac{3}{2} p \lambda_m i_q = m_L + B \omega_m + J \frac{d\omega_m}{dt}$$



vd
id
1
R+sL
ωm e L iq
×
ωm e L
Lp
ωm e L id
×
vq
iq
mL
mm
1
R+sL
3/2 p λm
1
B+sJ
ωm
ωm e λm
pλm