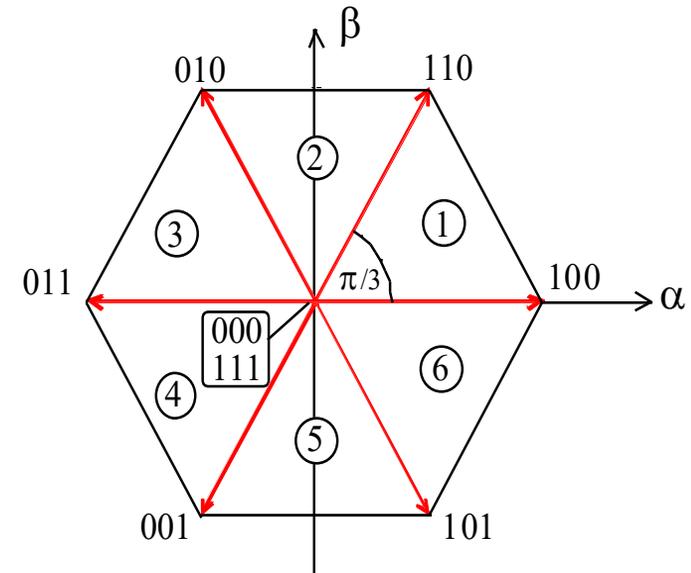
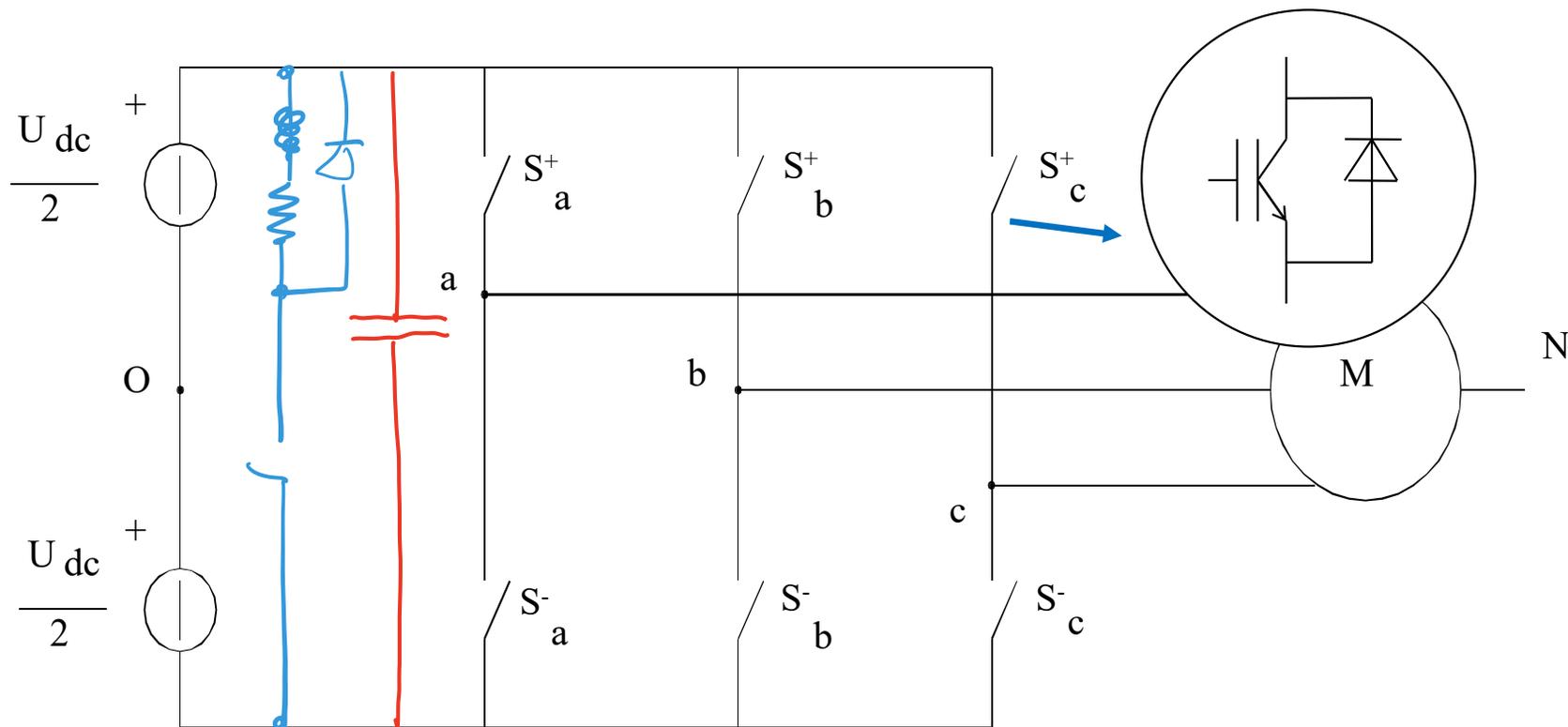


Invertitore trifase di tensione

Luigi Alberti
Silverio Bolognani

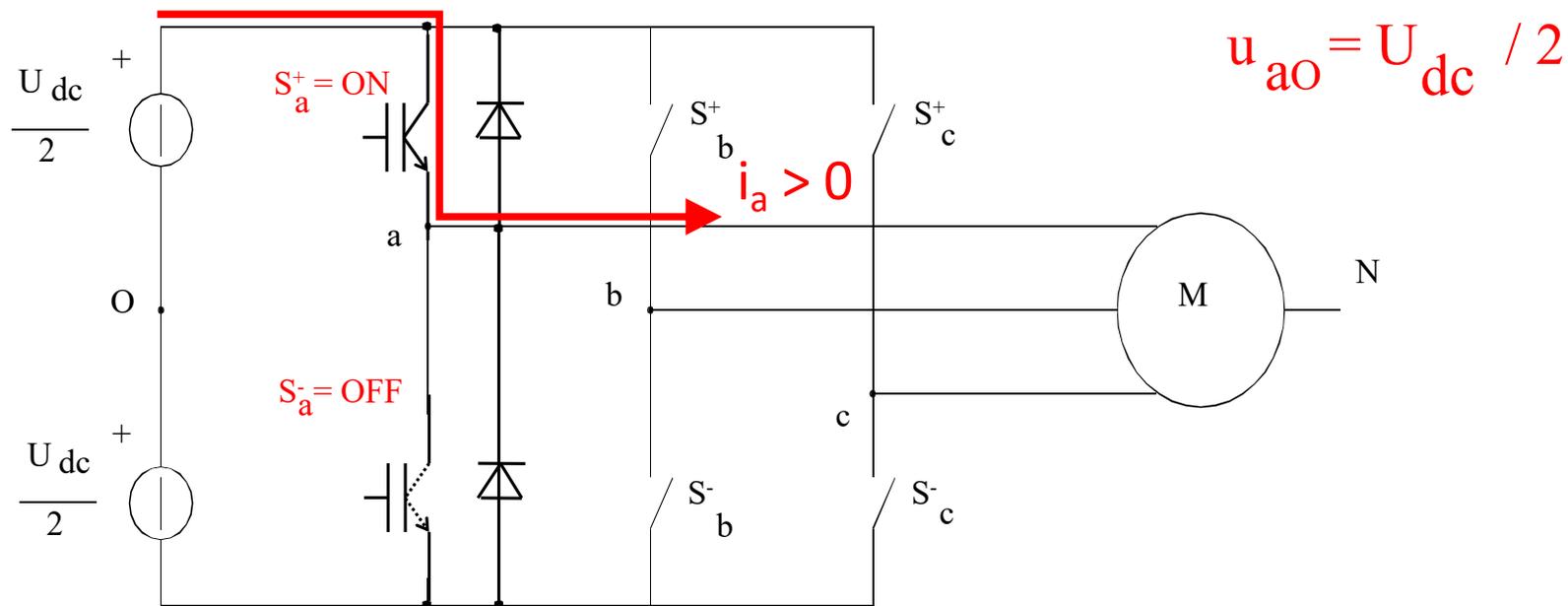
Lezioni di Azionamenti Elettrici AA 2021-2022





Rappresentazione schematica di un invertitore di tensione trifase

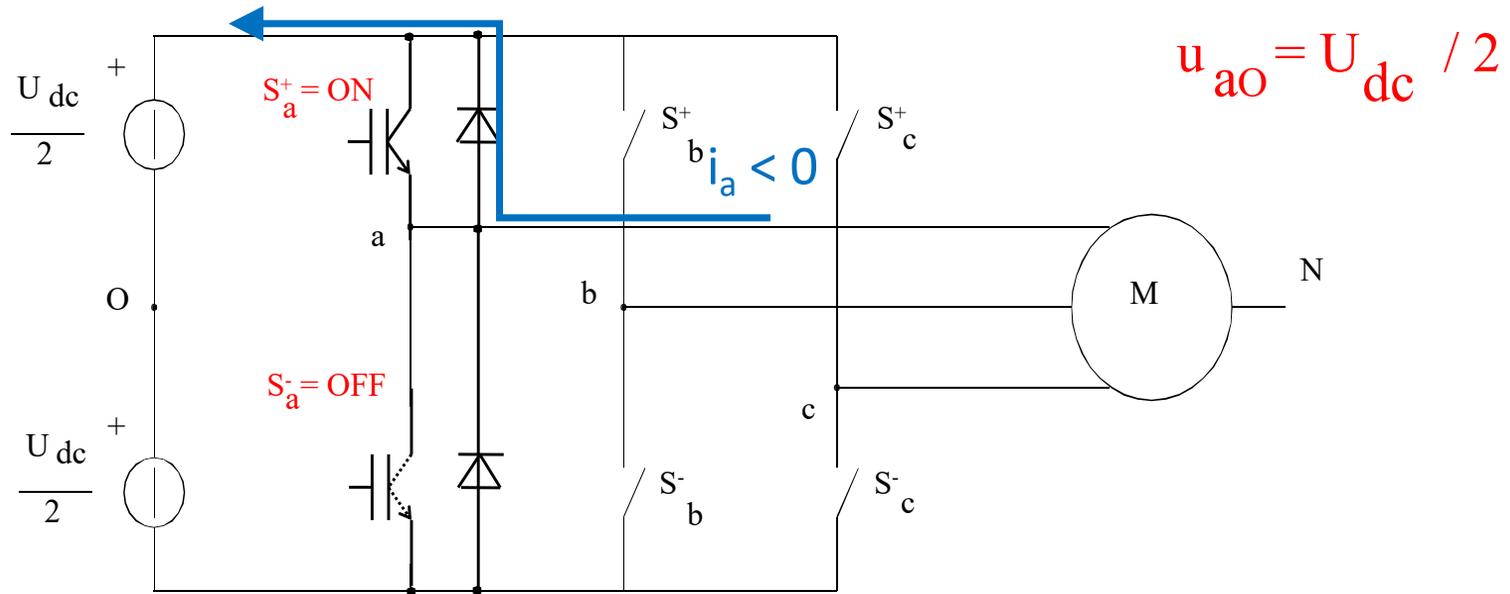
Percorso della corrente di fase «a» positiva



$$u_{aO} = U_{dc} / 2$$

Con $S_a^+ = \text{ON}$ si ottiene $u_{aO} > 0$

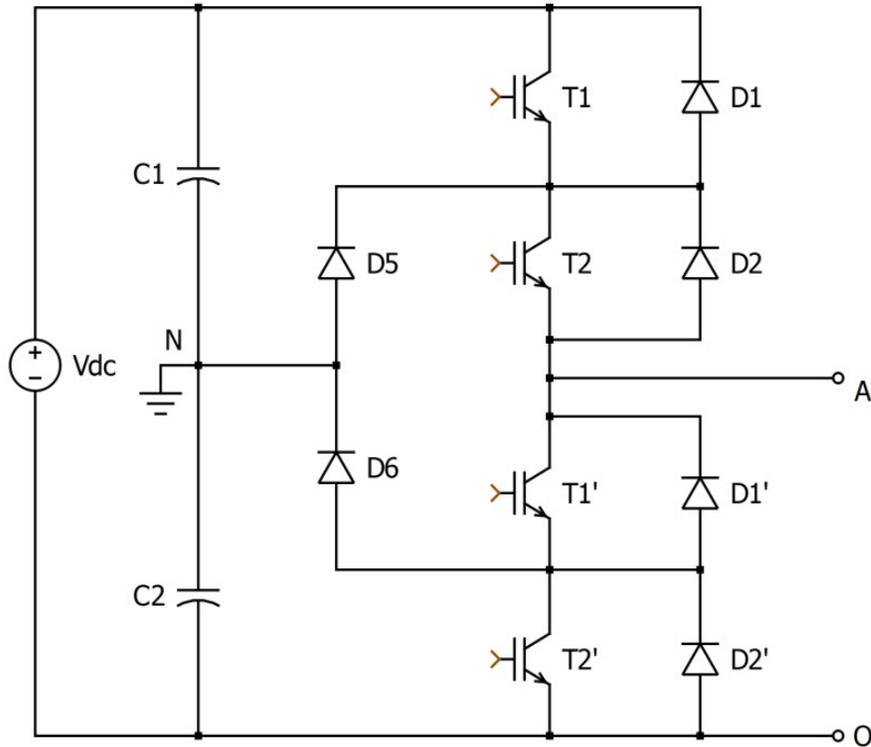
Percorso della corrente di fase «a» negativa



Con $S_a^+ = \text{ON}$ si ottiene ANCORA $u_{aO} > 0$

Il comando degli interruttori statici fissa la polarità della tensione di uscita

Configurazione (1 gamba) di un inverter a tre livelli



T1	T2	T1'	T2'	V_{AN}	V_{AO}
1	1	0	0	$V_{dc}/2$	V_{dc}
0	1	1	0	0	$V_{dc}/2$
0	0	1	1	$-V_{dc}/2$	0

1
0
-1

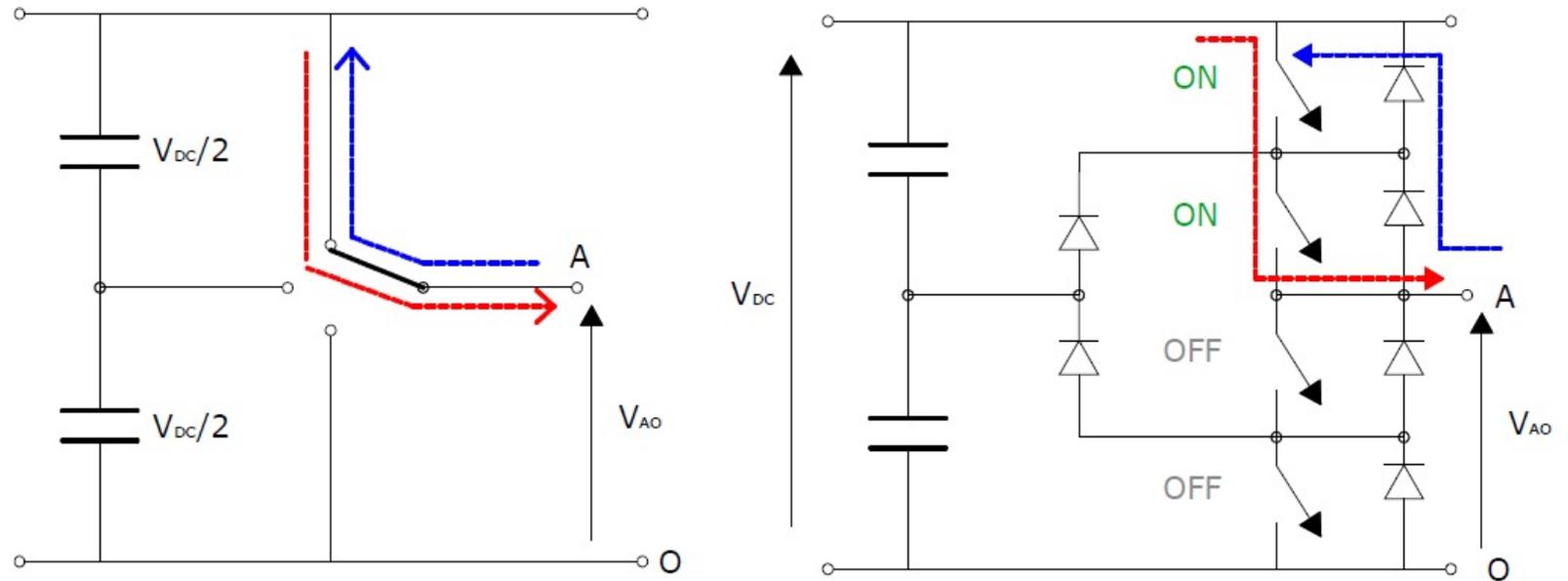
Tabella 1.1: Relazione fra gli stati degli interruttori del ramo di un inverter 3-livelli diode-clamped e le tensioni d'uscita.

NB La mezzaria dell'alimentazione è indicata con N ; il bus negativo con O .

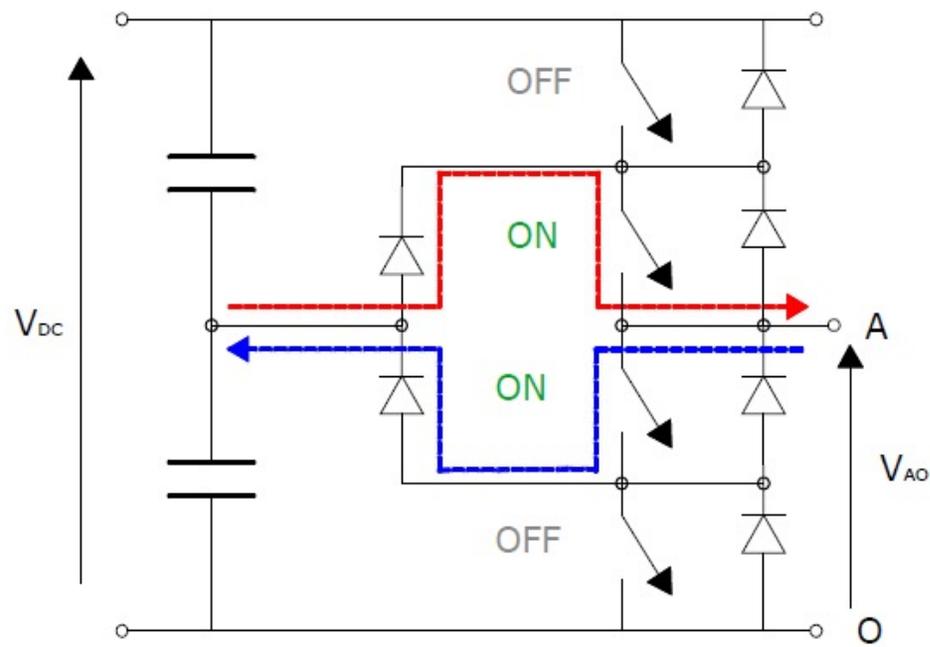
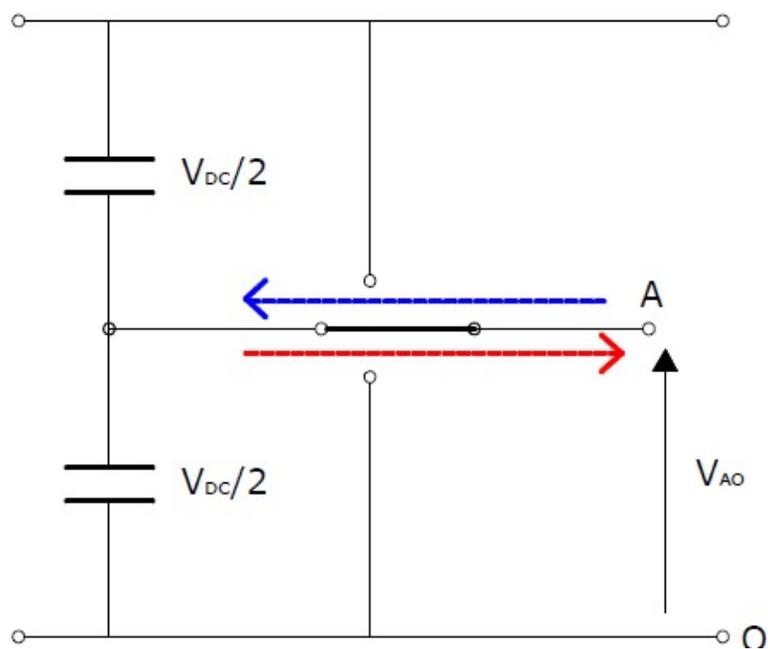
Ci sono 4 transistor e 6 diodi per ogni fase!

Ci sono $3^3 = 27$ configurazioni (24 vettori attivi (18 diversi) e 3 nulli)

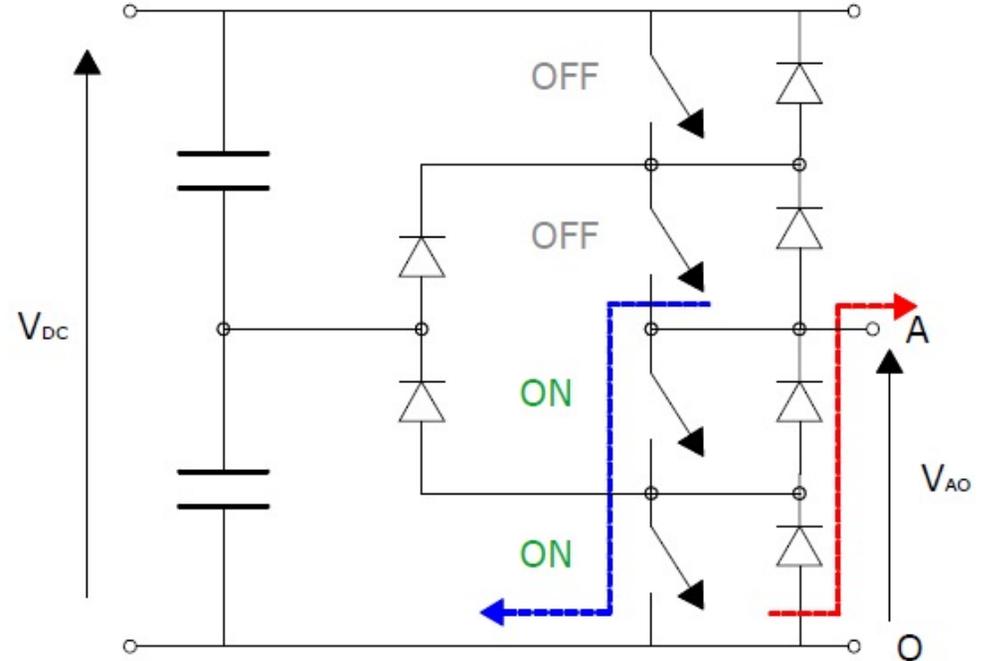
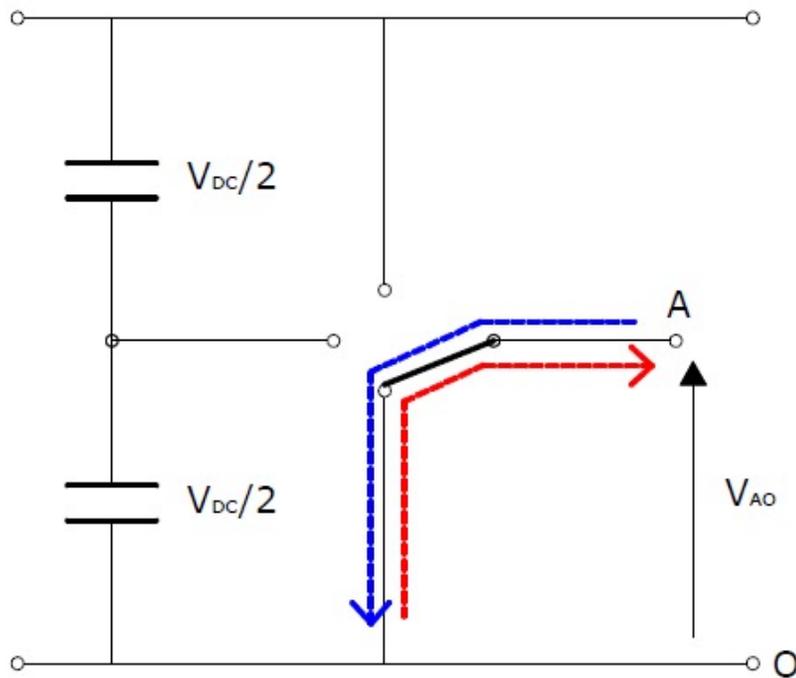
Configurazione per tensione di uscita positiva



Configurazione per tensione di uscita nulla



Configurazione per tensione di uscita negativa



Funzionamento in onda quadra: tensioni di fase dell'invertitore (a due livelli)

Componente fondamentale

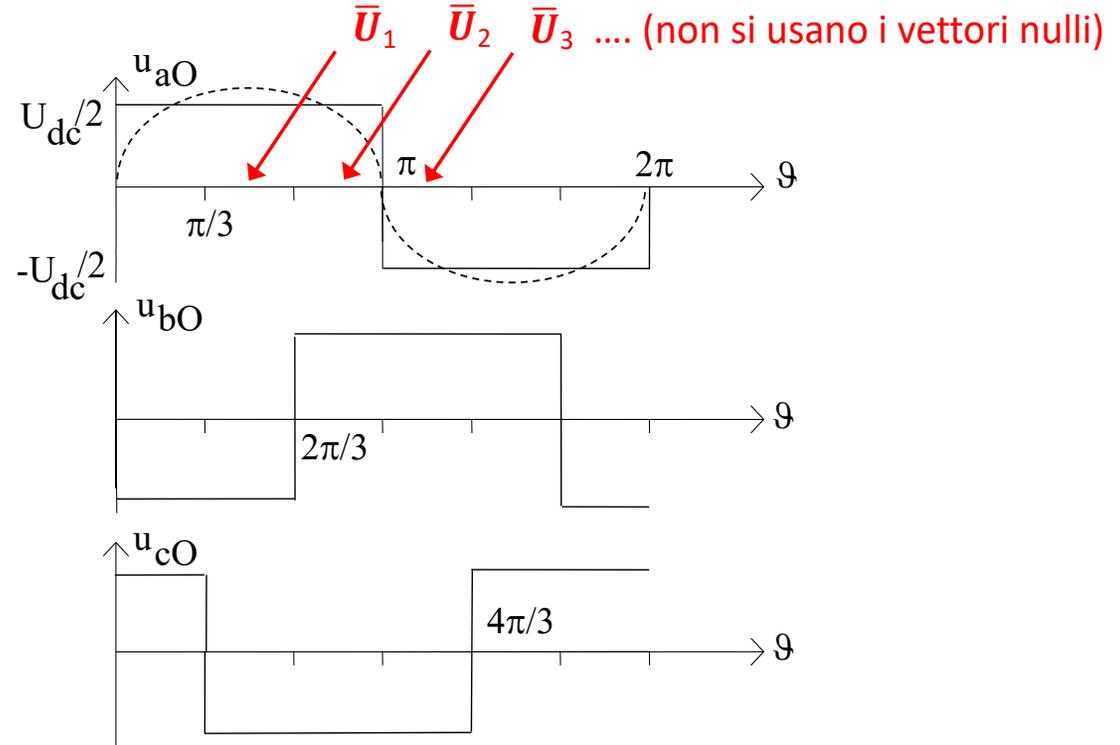
$$U_{aO,1} = \frac{4}{\pi} \frac{U_{dc}}{2} \approx 0.6366 U_{dc}$$

1.27

Componenti armoniche: $n=2k+1$

$$U_{aO,n} = \frac{4}{n\pi} \frac{U_{dc}}{2} = \frac{U_{aO,1}}{n}$$

(valori di picco)



Funzionamento in onda quadra: tensioni concatenate (quasi-quadre)

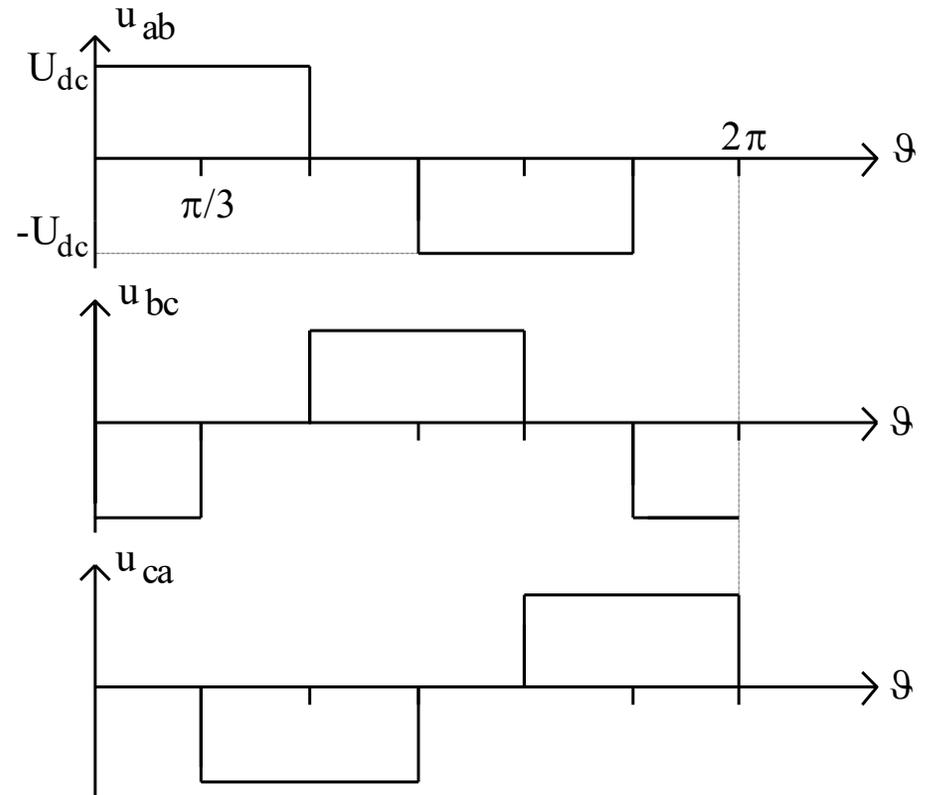
Componente fondamentale

$$U_{ab,1} = \sqrt{3} U_{a0,1}$$

Componenti armoniche: $n=6k\pm 1$

$$U_{ab,n} = \sqrt{3} U_{a0,n} = U_{ab,1} / n$$

(valori di picco)



Funzionamento in onda quadra: tensioni di fase del carico (6-gradini)

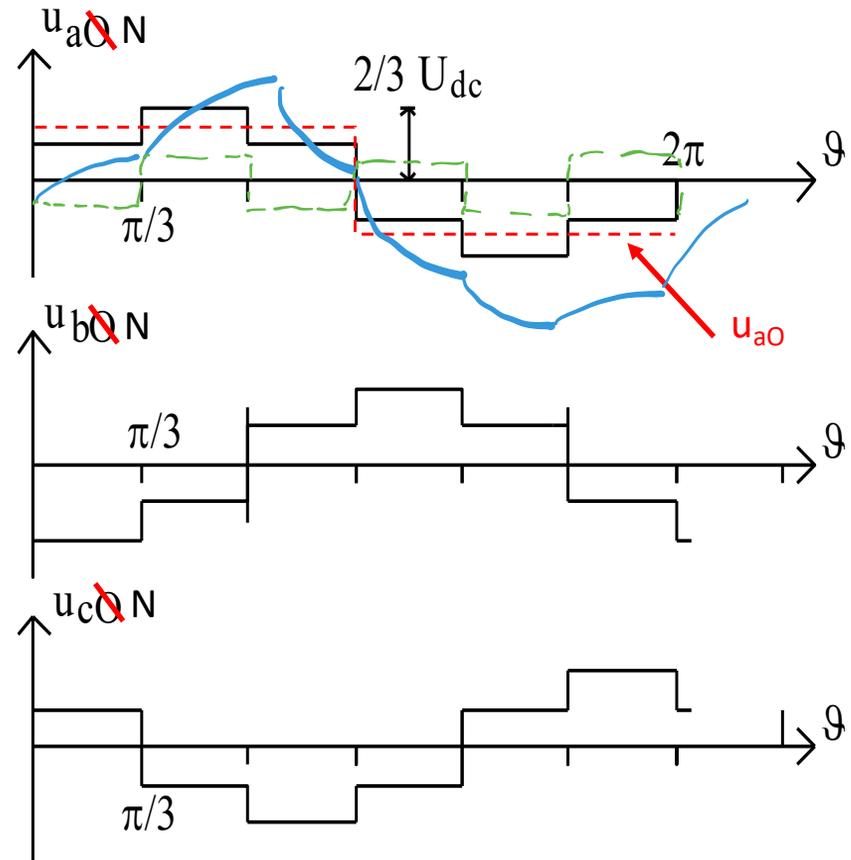
Componente fondamentale

$$U_{aN,1} = U_{aO,1} = \frac{4}{\pi} \frac{U_{dc}}{2} \approx 0.6366 U_{dc}$$

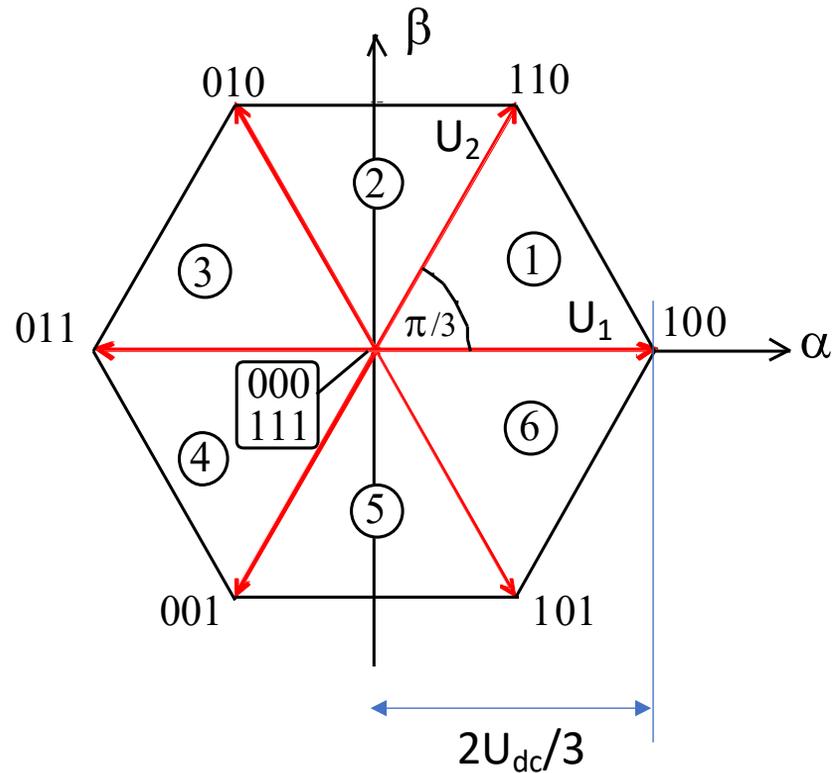
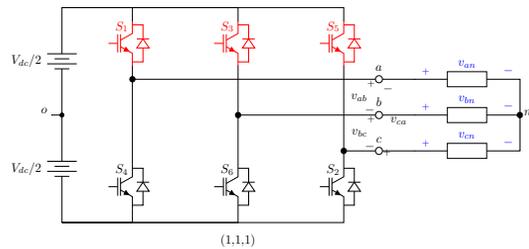
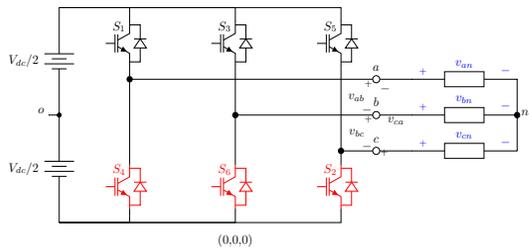
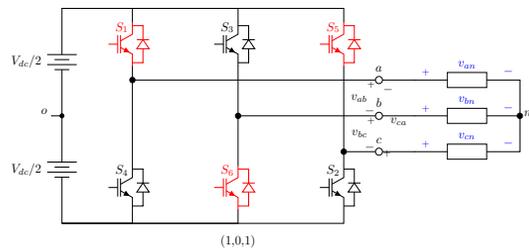
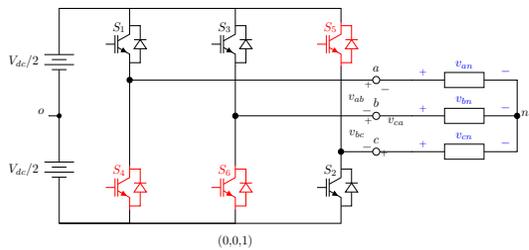
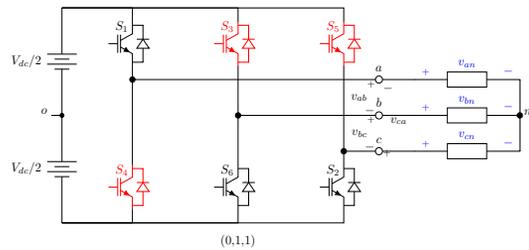
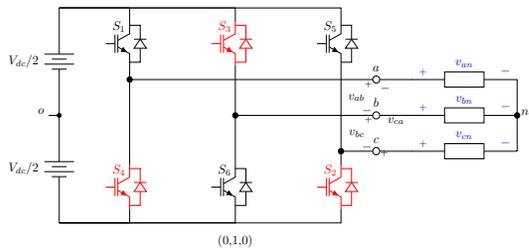
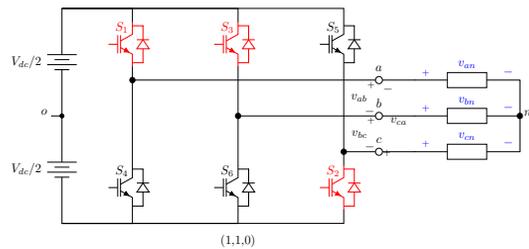
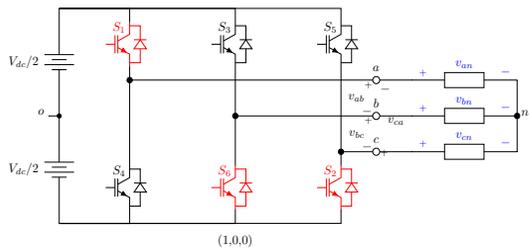
Componenti armoniche: $n=6k\pm 1$

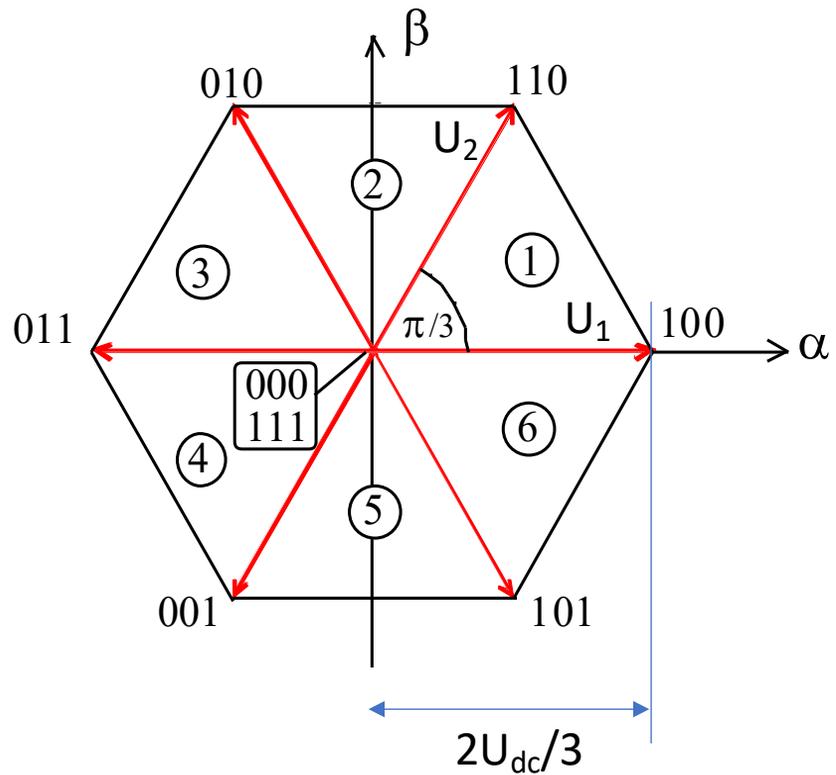
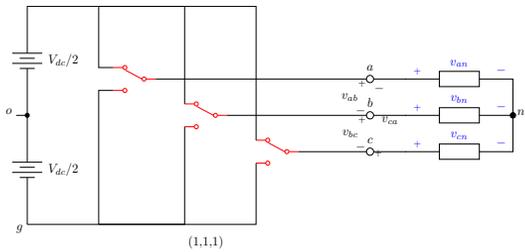
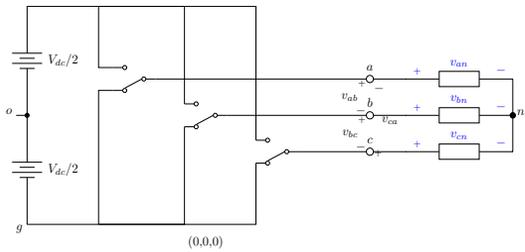
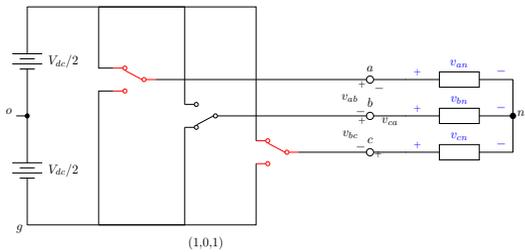
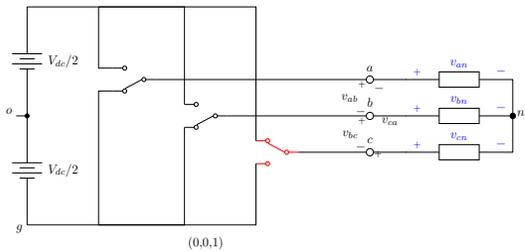
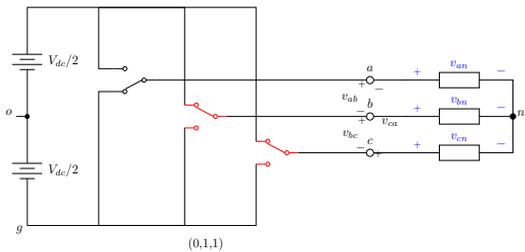
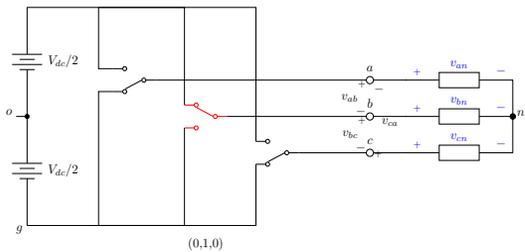
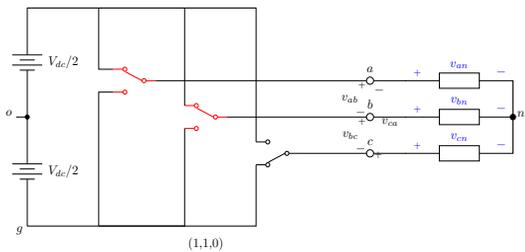
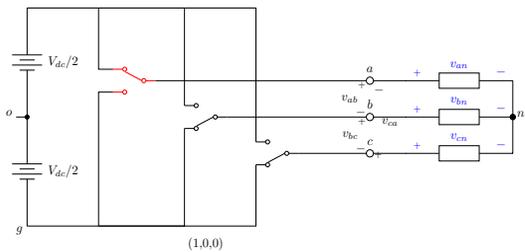
$$U_{aN,n} = U_{aO,n} = \frac{4}{n\pi} \frac{U_{dc}}{2} = \frac{U_{aO,1}}{n}$$

(valori di picco)



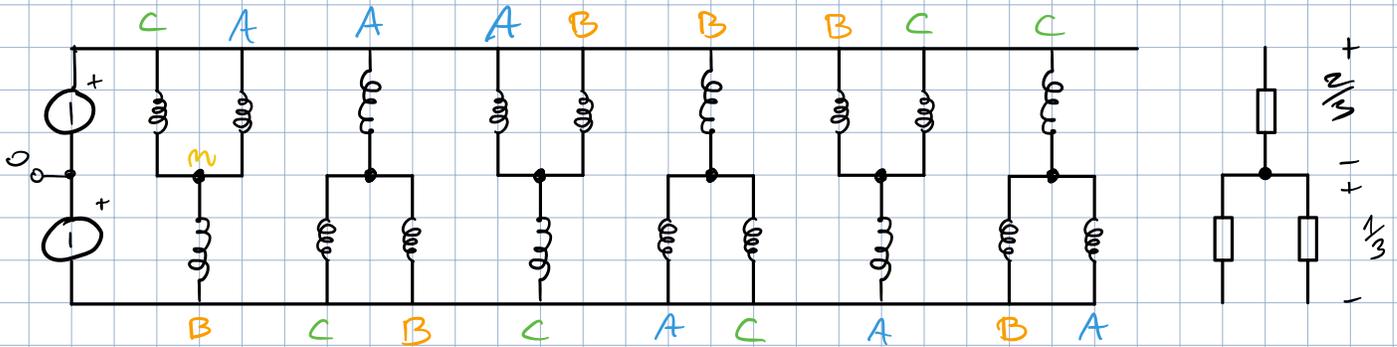
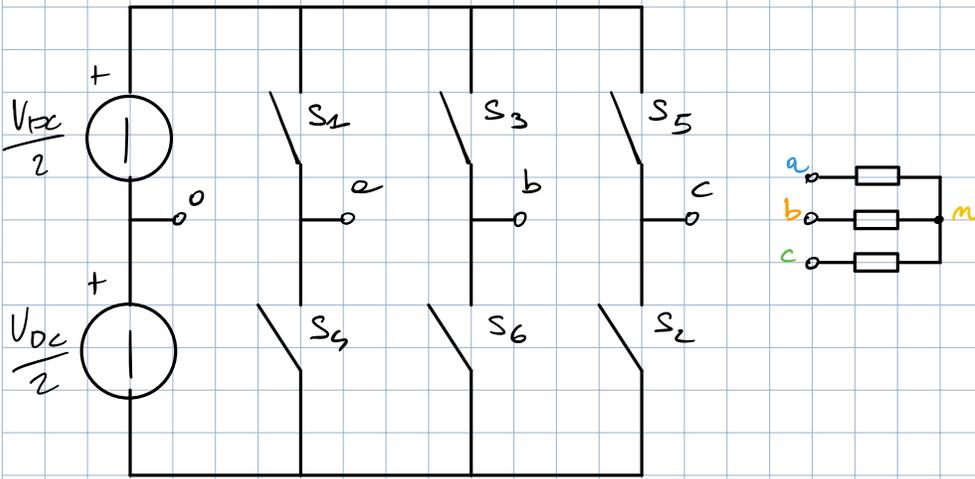
Vettori spaziali relativi agli 8 stati dell'inverter





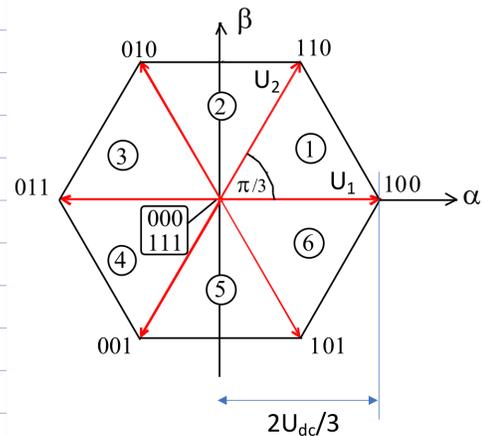
3 operating modes

- Square wave
- Sclar PWM
- SVM



101 Q561 Q612 Q123 010 Q234 011 Q345 001 Q456

\bar{u}_1 \bar{u}_2



$$\bar{u}_1 = \frac{2}{3} \left[u_a + u_b e^{j\frac{2\pi}{3}} + u_c e^{j\frac{4\pi}{3}} \right]$$

$$= \frac{2}{3} \left[\frac{2}{3} V_{dc} - \frac{1}{3} V_{dc} \left[-\frac{1}{2} + j\frac{\sqrt{3}}{2} \right] - \frac{1}{3} V_{dc} \left[-\frac{1}{2} - j\frac{\sqrt{3}}{2} \right] \right]$$

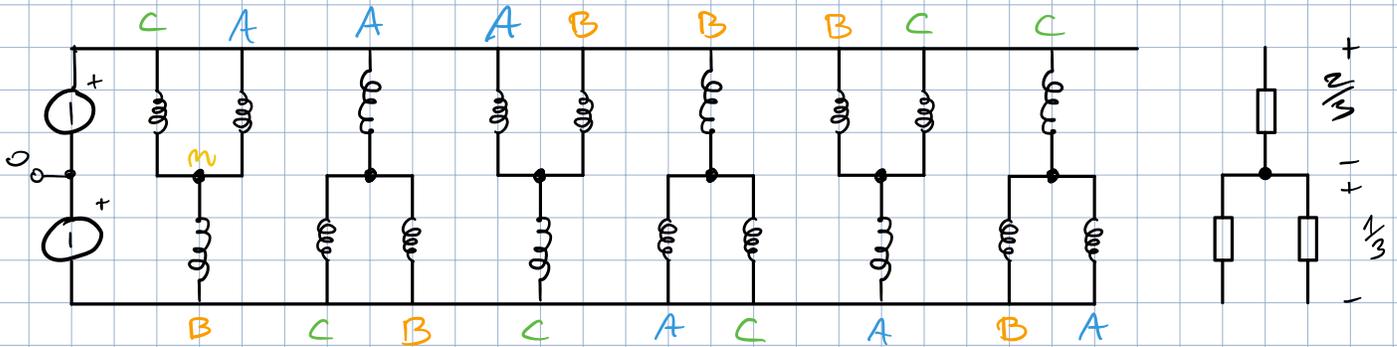
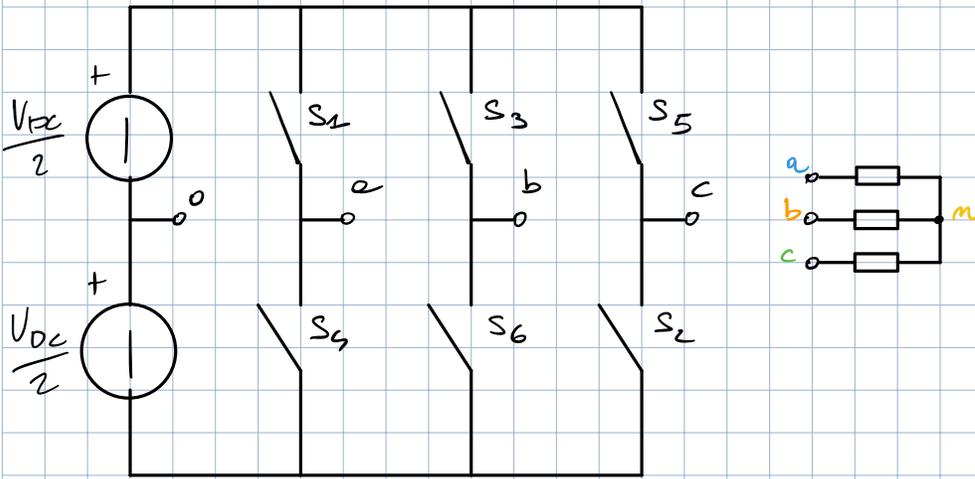
$$= \frac{2}{3} V_{dc} \left[\frac{2}{3} + \frac{1}{6} + \frac{1}{6} \right] = \frac{2}{3} V_{dc}$$

$$\bar{u}_2 = \frac{2}{3} V_{dc} \left[\frac{1}{3} + \frac{1}{3} \left[-\frac{1}{2} + j\frac{\sqrt{3}}{2} \right] - \frac{2}{3} \left[-\frac{1}{2} - j\frac{\sqrt{3}}{2} \right] \right]$$

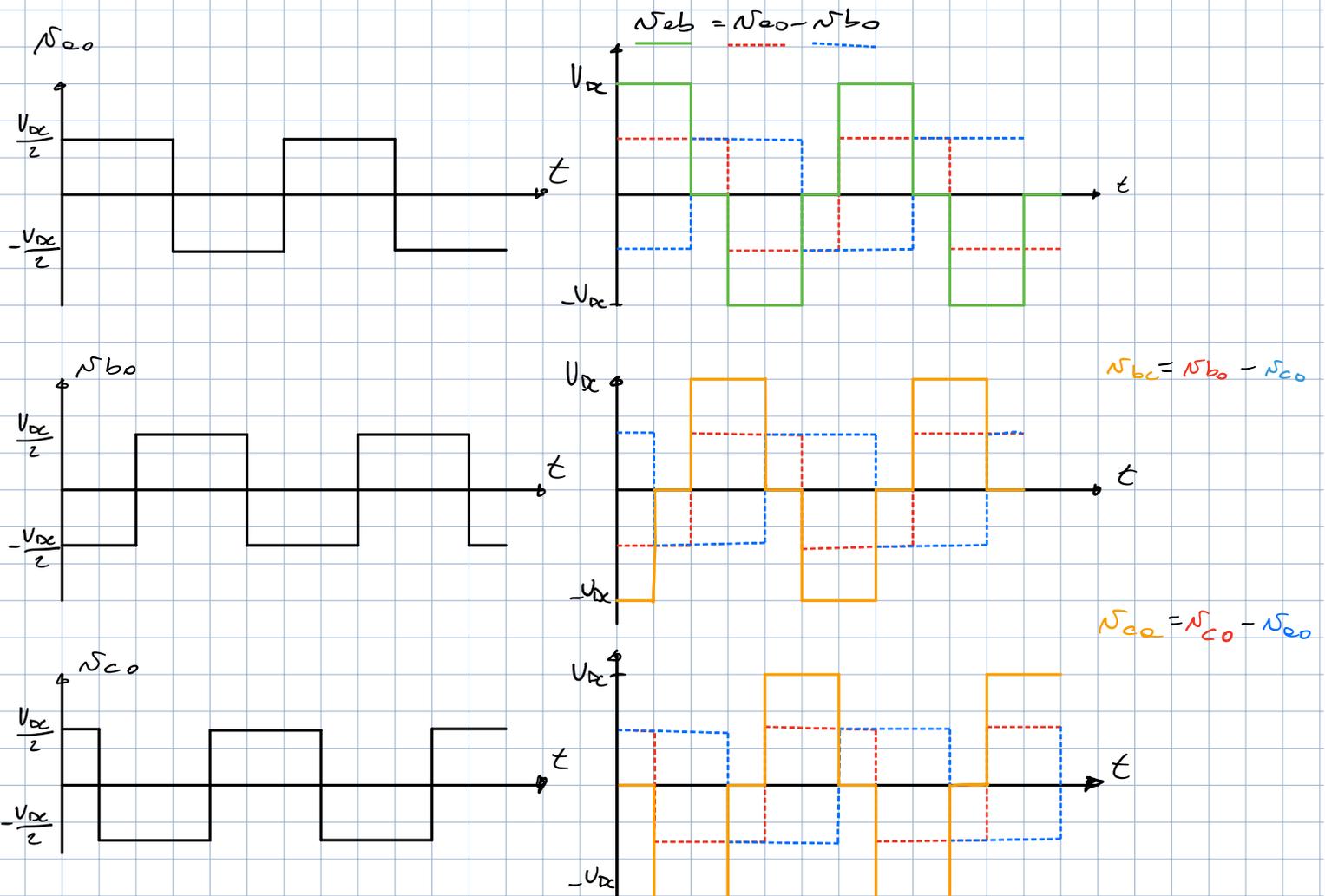
$$= \frac{2}{3} V_{dc} \left[\frac{1}{3} - \frac{1}{6} + \frac{1}{3} + j \left(\frac{\sqrt{3}}{6} + \frac{\sqrt{3}}{3} \right) \right] = \frac{2}{3} V_{dc} \left[\frac{1}{2} + j\frac{\sqrt{3}}{2} \right]$$

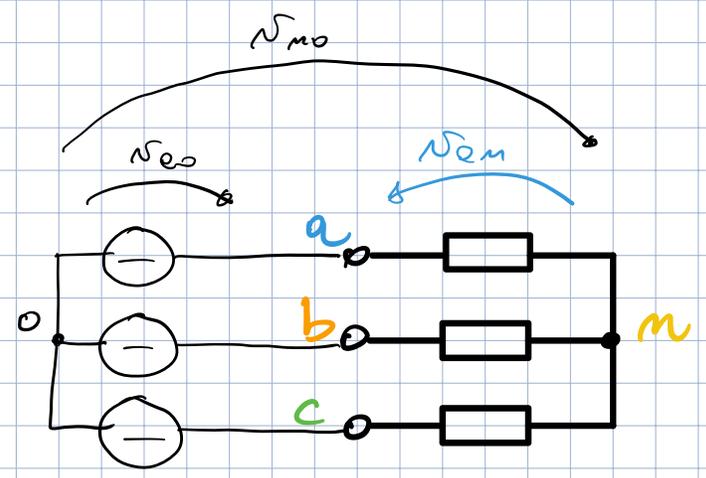
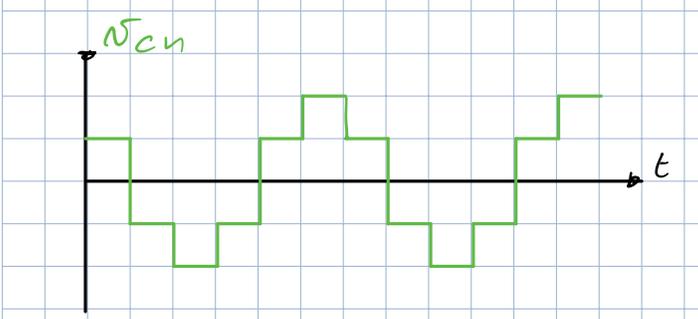
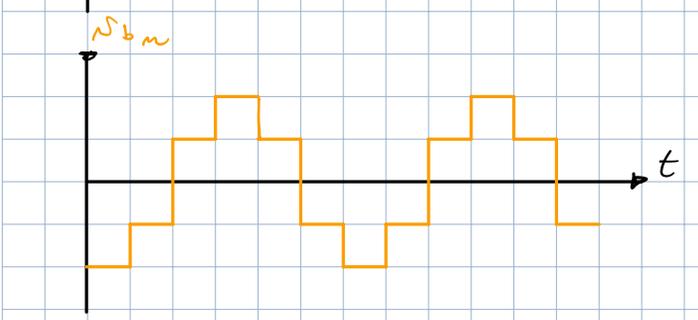
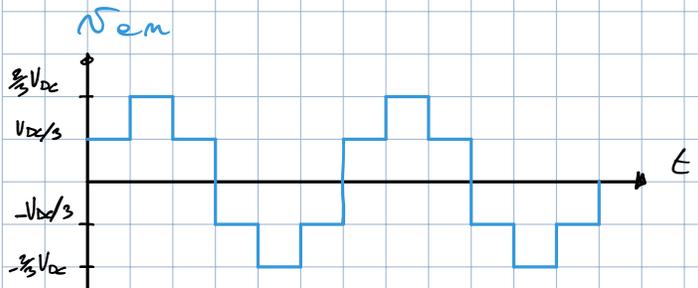
3 operating modes

- Square wave
- Sclar PWM
- SVM



101 100 110 010 011 001
 Q561 Q612 Q123 Q234 Q345 Q456

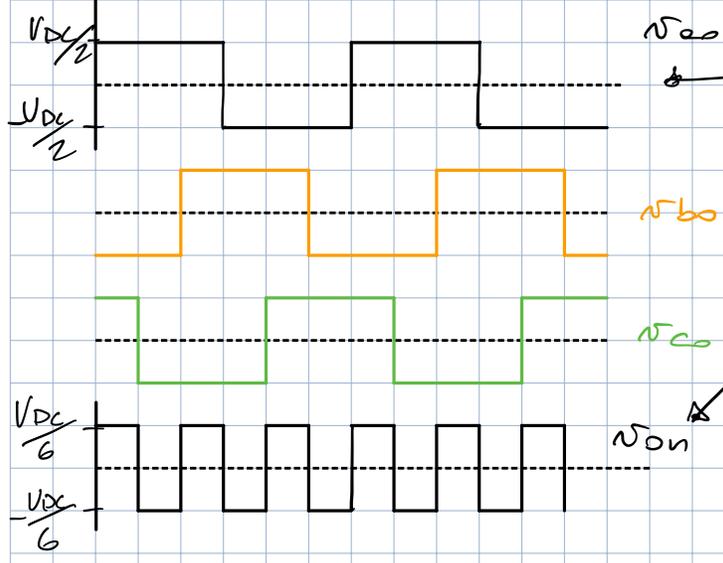




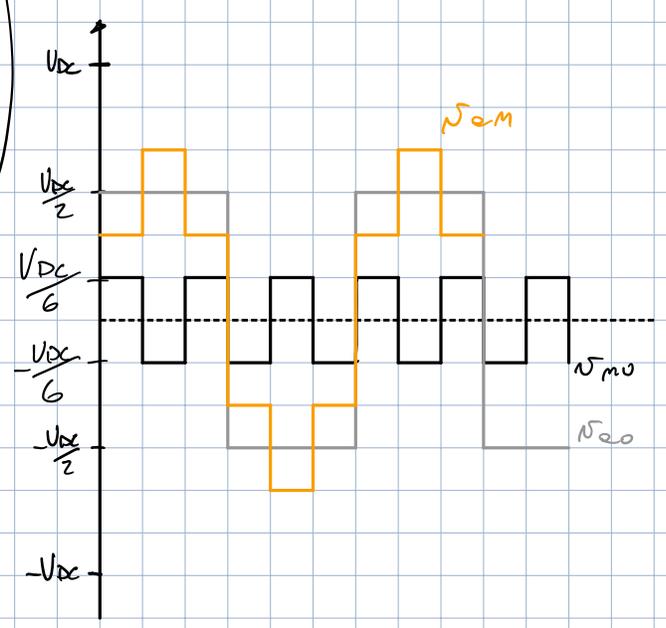
$$N_{m0} = \frac{N_{e0} + N_{b0} + N_{c0}}{3}$$

homopolar component

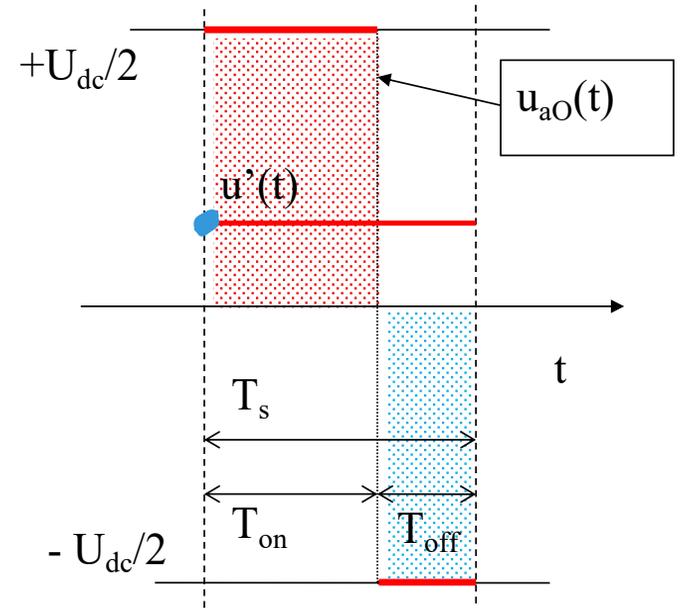
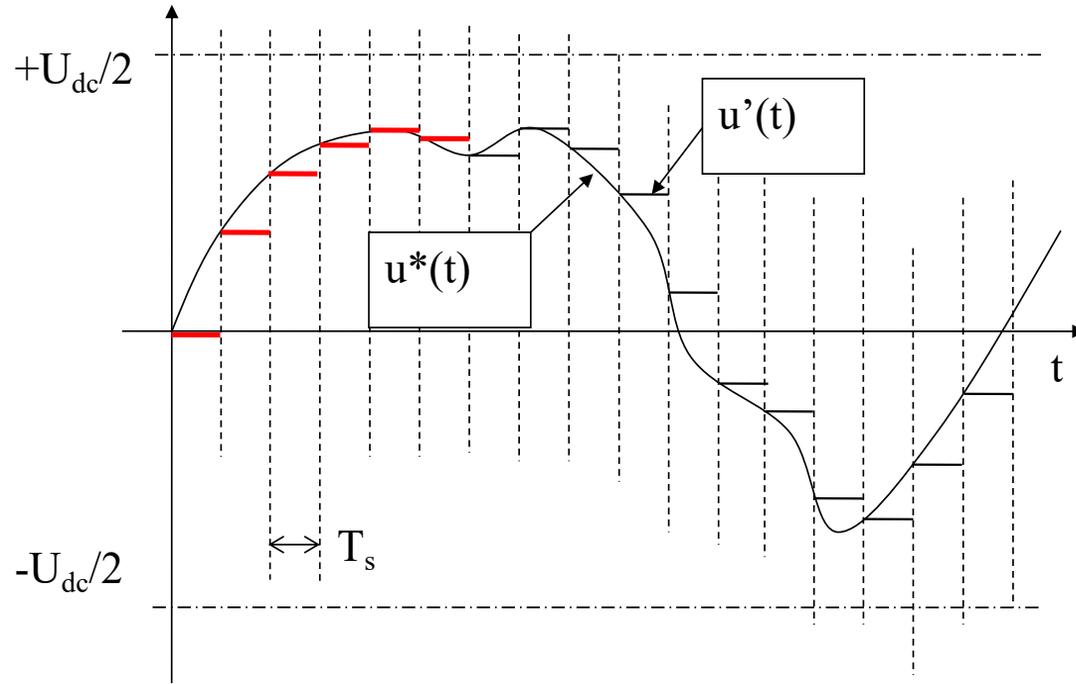
$$N_{en} = N_{e0} - N_{m0}$$



not in the same scale!

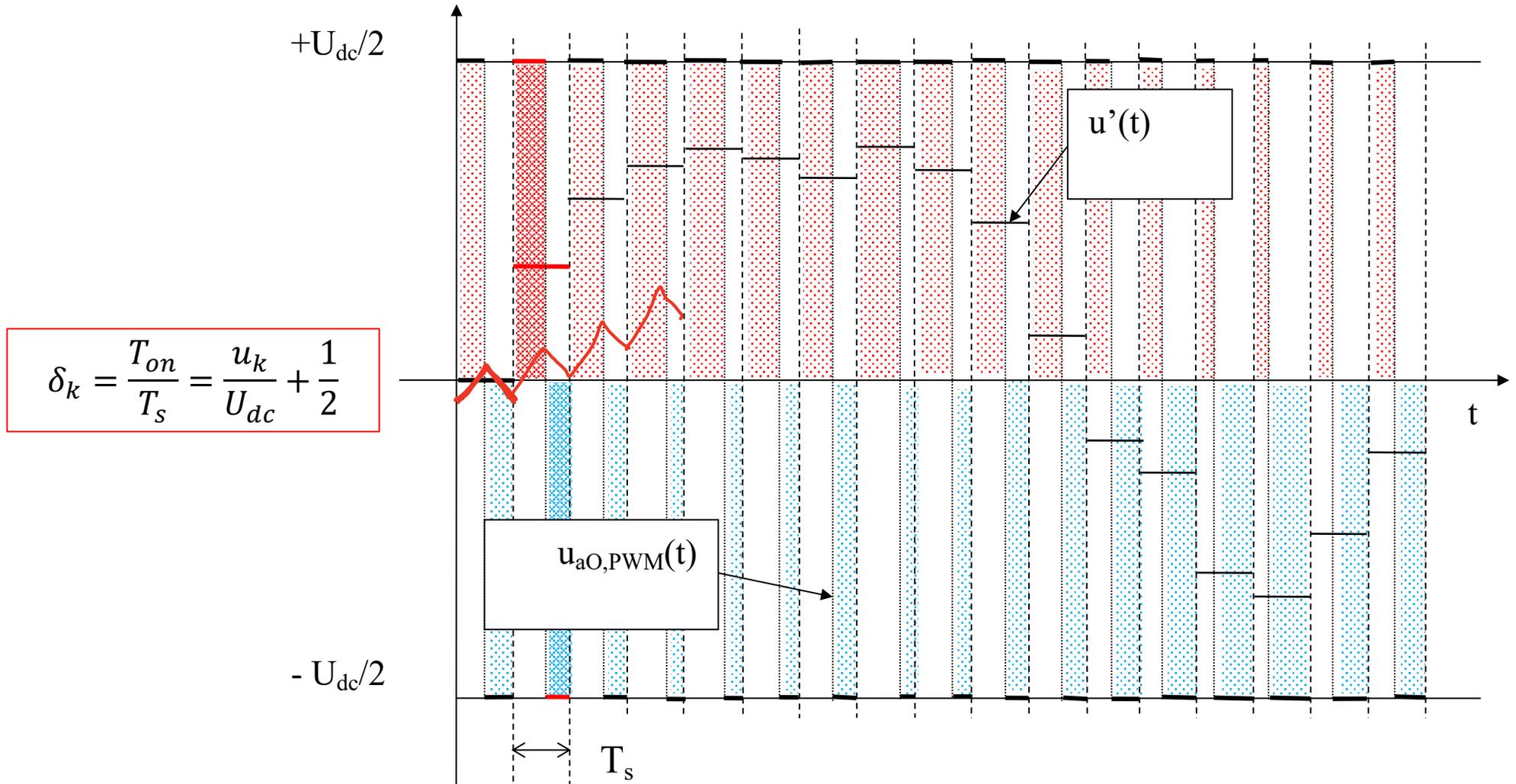


4.3 - Principio generale della PWM scalare



$$u'(t) = \frac{U_{dc} T_{on} - T_{off}}{2 T_s} = \frac{U_{dc} T_{on} - (T_s - T_{on})}{2 T_s} = \frac{U_{dc}}{2} \left(2 \frac{T_{on}}{T_s} - 1 \right) = U_{dc} \left(\delta - \frac{1}{2} \right)$$

(cont.) - Principio generale della PWM scalare



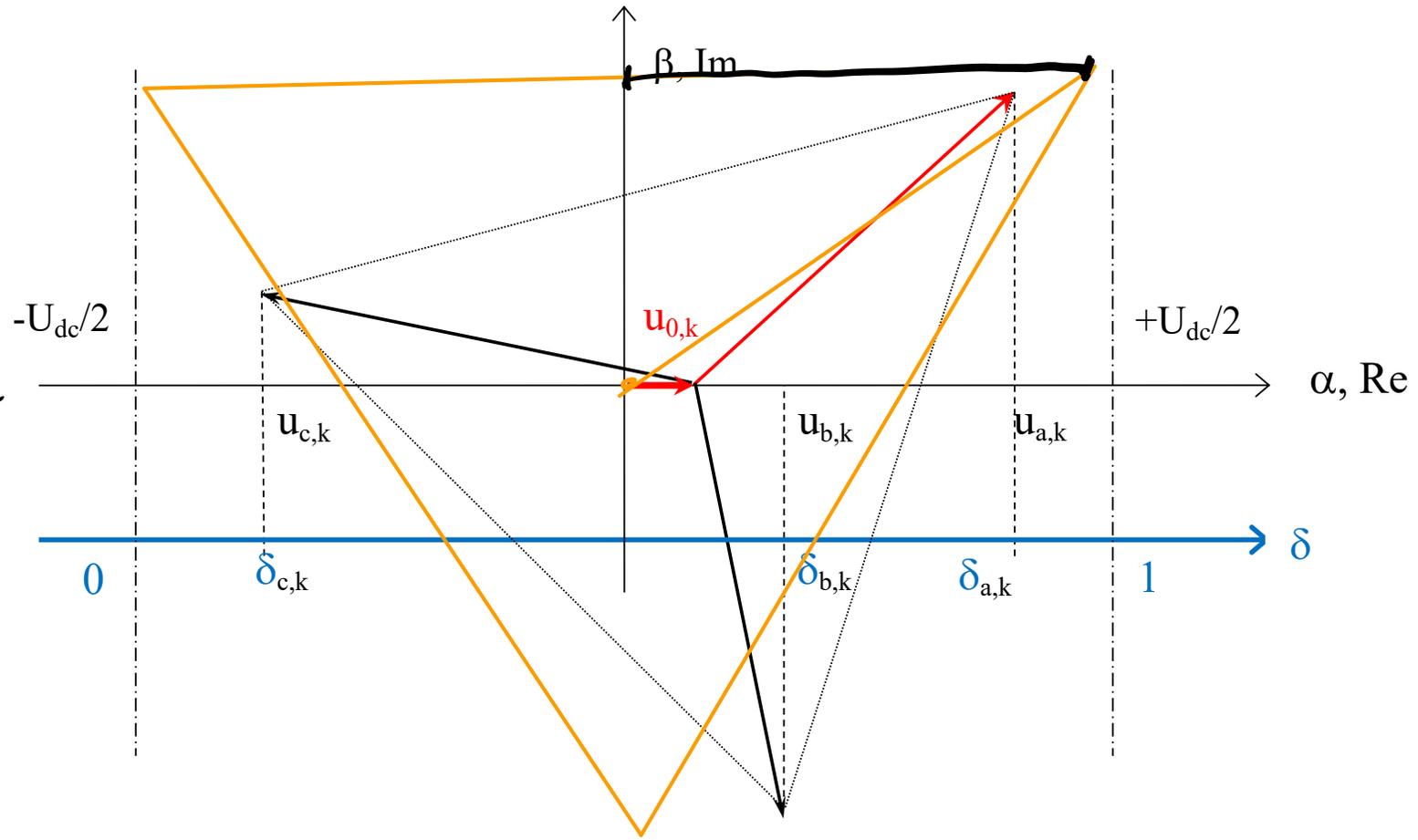
(cont.) - PWM trifase sinusoidale (con iniezione di omopolare)

$$2|\tilde{u}| \frac{\sqrt{3}}{2} = U_{DC}$$

$$|\tilde{u}| = \frac{U_{DC}}{\sqrt{3}}$$

$$= 0,57 U_{DC}$$

Tensione
con iniezione
 $\frac{\sqrt{3}|\tilde{u}|}{2}$
" U_{DC}



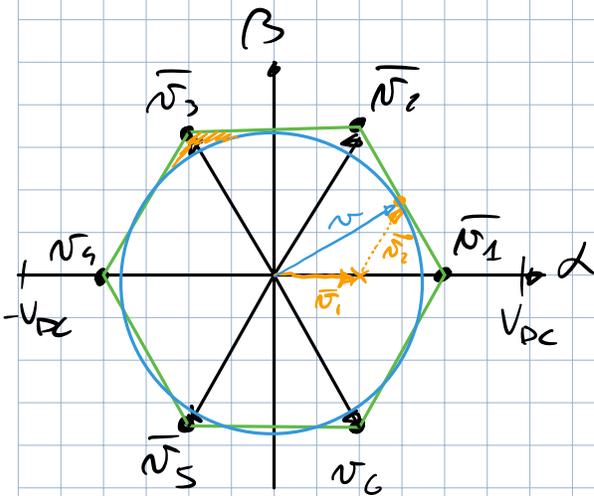
Confronto fra le diverse tecniche di PWM

Sono riportati i valori massimi (M) ed efficaci (rms) delle tensioni fondamentali (prima armonica) concatenate e di fase

Strategia di PWM	$U_{M,conc,1}$	$U_{rms,conc,1}$	$U_{M,fase,1}$	$U_{rms,fase,1}$
(senza PWM) Onda quadra	$\sqrt{3} \cdot 2/\pi U_{dc}$ (1,10 U_{dc})	$\sqrt{3} \cdot \sqrt{2}/\pi U_{dc}$ (0,78 U_{dc})	$2/\pi U_{dc}$ (0,64 U_{dc})	$\sqrt{2}/\pi U_{dc}$ (0,45 U_{dc})
PWM scalare senza omopolare <i>(da 0 fino a)</i>	$\sqrt{3}/2 U_{dc}$ (0,87 U_{dc})	$\sqrt{3}/\sqrt{2} U_{dc}$ (0,61 U_{dc})	$1/2 U_{dc}$ (0,50 U_{dc})	$1/(2\sqrt{2}) U_{dc}$ (0,35 U_{dc})
PWM scalare con omopolare (oppure PWM vettoriale) <i>(da 0 fino a)</i>	U_{dc} (1,00 U_{dc})	$1/\sqrt{2} U_{dc}$ (0,71 U_{dc})	$1/\sqrt{3} U_{dc}$ (0,58 U_{dc})	$1/(\sqrt{3}\sqrt{2}) U_{dc}$ (0,41 U_{dc})

NB: la tensione U_{dc} è ottenuta dalla rete trifase con un ponte trifase a diodi: $U_{dc} = 1.35 U_{rms,conc,rete}$. Per esempio raddrizzando la tensione trifase di rete di 400V si ottiene $U_{dc} = 540 \text{ V}$ (rapporto $U_{rms,conc,rete}/U_{dc} = 0.74$).

NOTE INVERTER SUM



$$e^{j\frac{\pi}{3}} = \cos\frac{\pi}{3} + j\sin\frac{\pi}{3}$$

$$= \frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$\vec{v}_1 = \frac{2}{3} V_{DC}$$

$$\vec{v}_2 = \frac{2}{3} V_{DC} e^{j\frac{2\pi}{3}} = \frac{2}{3} V_{DC} \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} \right)$$

$$\vec{v}_3 = \frac{2}{3} V_{DC} e^{j\frac{4\pi}{3}} = \frac{2}{3} V_{DC} \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} \right)$$

\vec{v}^* se applico \vec{v}_1 per $T_1 = \frac{T_s}{2}$ e \vec{v}_2 per $T_2 = \frac{T_s}{2}$

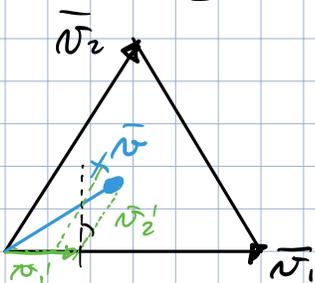
$$\vec{v}^* = \frac{\vec{v}_1 T_1 + \vec{v}_2 T_2}{T_s} = \frac{\frac{2}{3} V_{DC} \frac{T_s}{2} + \frac{2}{3} V_{DC} \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) \frac{T_s}{2}}{T_s} = \frac{V_{DC}}{3} + \frac{V_{DC}}{3} \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} \right)$$

$$= \frac{V_{DC}}{3} \left[\frac{3}{2} + j\frac{\sqrt{3}}{2} \right] = \frac{V_{DC}}{\sqrt{3}} \left[\frac{\sqrt{3}}{2} + j\frac{1}{2} \right]$$

- ① Dato \vec{v}^* che si vuole realizzare
- ② Individuo il settore da usare
- ③ proietto \vec{v}^* sui lati dell'esagono
- ④ Calcolo i tempi con cui applico i vettori di tensione

ESEMPIO:

$$\vec{v}^* = \frac{1}{2} \frac{V_{DC}}{\sqrt{3}} \left[\frac{\sqrt{3}}{2} + j\frac{1}{2} \right] = \frac{V_{DC}}{\sqrt{3}} \left[\frac{\sqrt{3}}{4} + j\frac{1}{4} \right]$$



$$\vec{v}^* = \vec{v}_1' + \vec{v}_2'$$

$$\vec{v}_1' = \frac{T_1}{T_s} \vec{v}_1$$

$$T_0 + T_1 + T_2 = T_s$$

$$\vec{v}_2' = \frac{T_2}{T_s} \vec{v}_2$$

$$I_m[\bar{v}] = \frac{V_{DC}}{\sqrt{3}} \cdot \frac{1}{4} = |\bar{v}_2| \frac{\cos \pi}{6} = \frac{T_2}{T_s} |\bar{v}_2| \frac{\sqrt{3}}{2}$$

$$T_2 = \frac{V_{DC}}{\sqrt{3}} \cdot \frac{1}{4} \frac{T_s \cdot 2}{|\bar{v}_2| \sqrt{3}} = \frac{V_{DC}}{\sqrt{3}} \frac{T_s \cdot 2}{4 \sqrt{3} \frac{2V_{DC}}{3}} = \frac{T_s}{4}$$

$$I_R[\bar{v}] = \frac{V_{DC}}{4} = |\bar{v}_1| + |\bar{v}_2| \cdot \frac{\sin \pi}{6} = \frac{T_1}{T_s} \frac{2V_{DC}}{3} + \frac{T_2}{T_s} \frac{2V_{DC}}{3} \frac{1}{2}$$

$$T_1 = \dots = \frac{T_s}{4}$$

$$T_1 = T_s/4$$

$$\bar{v}_1 \quad 100$$

$$T_2 = T_s/4$$

$$\bar{v}_2 \quad 110$$

$$T_0 = T_s - T_1 - T_2 = T_s/2$$

$$\bar{0} \quad 111 \quad 000$$

$$\begin{array}{c|c|c} & T_s & \\ \hline 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{array}$$

0	1	1	1	1	1	0	0	1	1	1	1	1	0
0	0	1	1	1	0	0	0	0	1	1	1	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0	0
$\frac{T_2}{4}$	$\frac{T_1}{2}$	$\frac{T_2}{2}$	$\frac{T_0}{2}$	$\frac{T_2}{2}$	$\frac{T_1}{2}$	$\frac{T_0}{4}$							

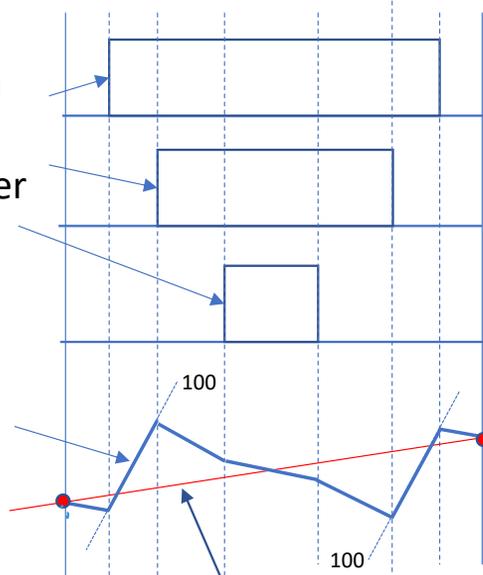
Misure delle correnti e delle tensioni di fase

Misura delle correnti

0	1	1	1	1	1	0
0	0	1	1	1	0	0
0	0	0	1	0	0	0

Tensioni di fase dell'inverter

$i(t)$



Corrente «media»

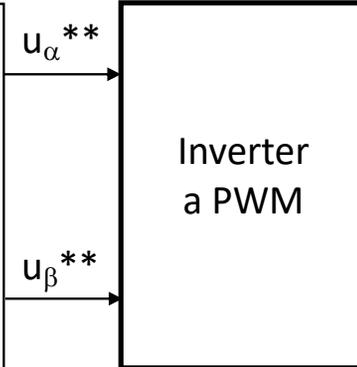
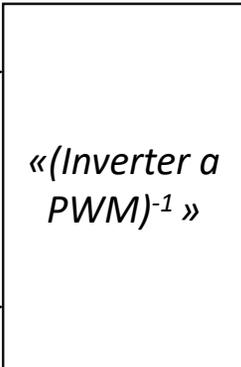
Istanti di campionamento (filtraggio sincrono)

«Misura» delle tensioni

Usati al posto delle misure di tensione

u_{α}^*

u_{β}^*



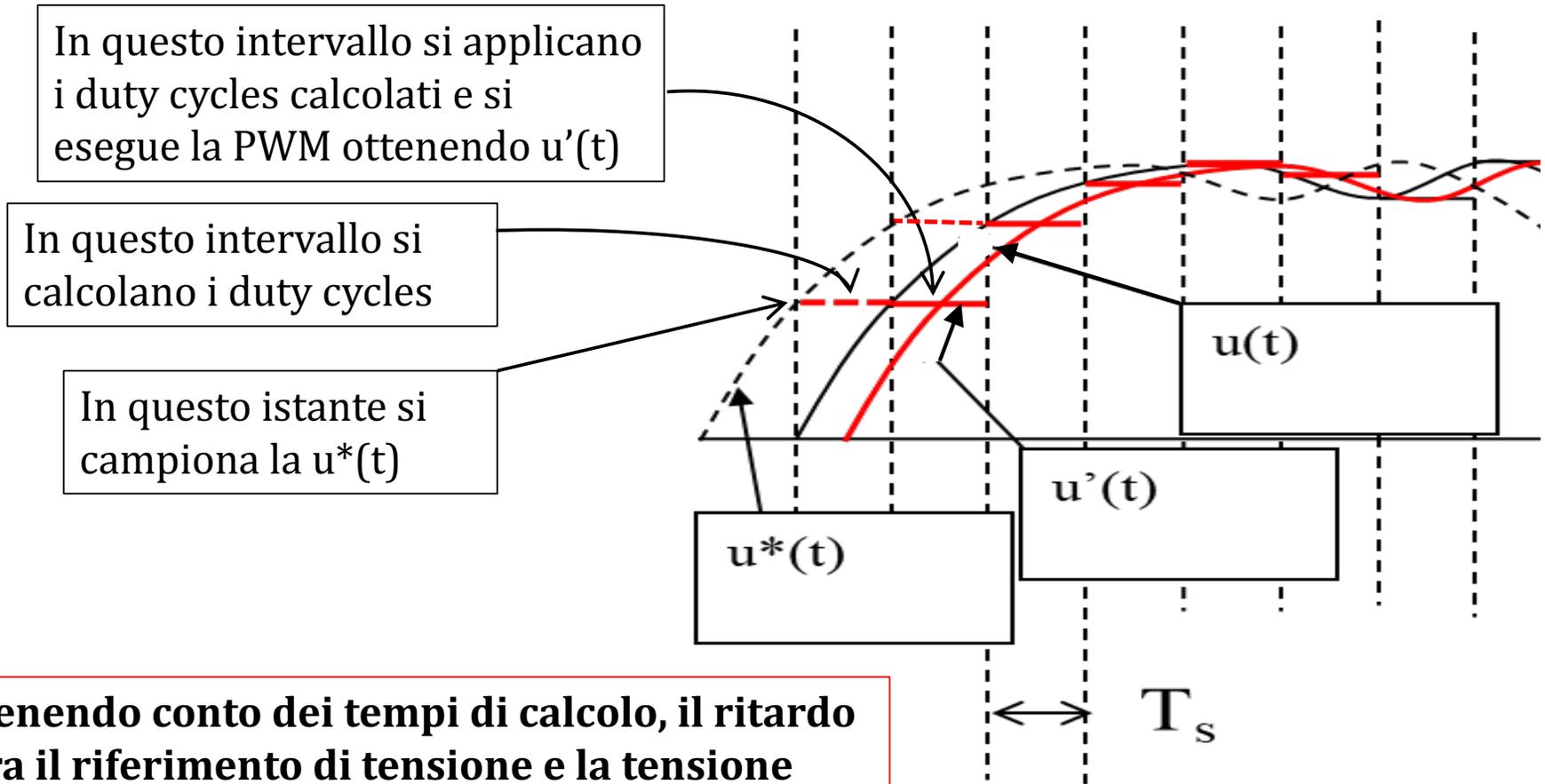
u_{α}^{**}

u_{β}^{**}

$u_{\alpha} = u_{\alpha}^*$

$u_{\beta} = u_{\beta}^*$

Funzione di trasferimento del controllo di tensione a PWM



Tenendo conto dei tempi di calcolo, il ritardo fra il riferimento di tensione e la tensione prodotta è pari a $3T_s/2 = 1.5T_s$

(cont.) - Funzione di trasferimento del controllo di tensione a PWM

Tenendo conto dei tempi di calcolo, il ritardo fra il riferimento di tensione e la tensione prodotta è pari a $3T_s/2 = 1.5T_s$ e pertanto

$$G_c(s) = \frac{U(s)}{U^*(s)} = e^{-s(1.5T_s)} \cong \frac{1}{1 + s(1.5T_s)}$$

Il polo è alla pulsazione (frequenza angolare) $\nu_c = 1/(1.5T_s) = f_s/1.5$ [rad/s]

In termini di frequenza, il polo è alla frequenza $f_c = \nu_c/(2\pi) = f_s/(1.5 \cdot 2\pi) \cong f_s/10$ [Hz]