The global and surface energy balance

Resources:
- Global physical climatology: chapters 2 and 4
- Global warming science: chapter 2

Energy is conserved

The earth-atmosphere energy balance is achieved as the energy received from the Sun *balances* the energy lost by the Earth back into space. In this way, the Earth maintains a stable average temperature and therefore a stable climate.

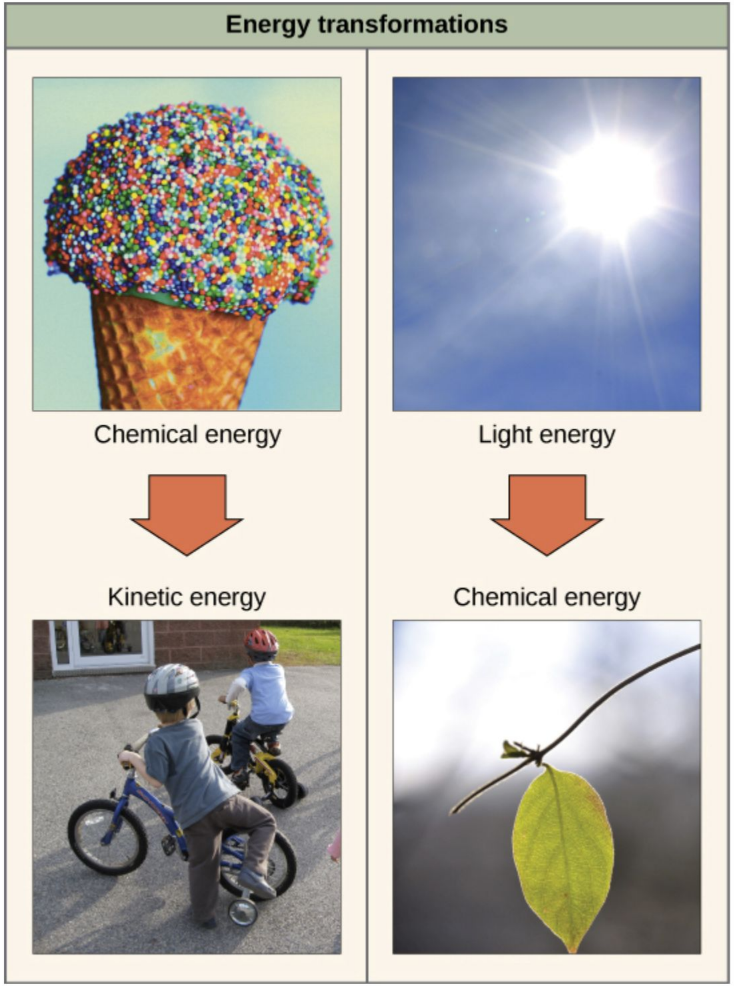
First law of thermodynamics

States that **energy is conserved.**

For a closed system: “The heat added to a system is **equal** to the change in **internal energy** minus the work exerted”.

Examples:

- *Light bulbs transforms electrical energy into light energy*
- *One pool ball hits another, transferring kinetic energy and making the second ball move*
- *You are transforming chemical energy from food into kinetic energy as you walk, breath or run*


Energy transformations
Chemical energy
Kinetic energy
Light energy
Chemical energy

Heat (dQ) can be transported to and from a system in three ways:

1. **RADIATION**: no mass is exchanged and no medium is required. Pure radiant energy moves at the speed of light.

2. **CONDUCTION**: no mass is exchanged, but a medium is required to transfer heat by collisions between atoms or molecules.

3. **CONVECTION**: mass is exchanged. A net movement of mass may occur, but more commonly parcels with different energy amounts change place, so that energy is exchanged without a net movement of mass.

Example of convection

FIGURE 2.1 **Cumulonimbus cloud over Africa photographed from the international space station expedition 16, NASA, February 5, 2008.** Note the three-dimensional structure of the clouds, the upper-level spreading anvil, its shadow, and the various shapes and sizes of the smaller clouds. Air movements associated with clouds move heat and moisture vertically by convection and also influence radiative transfer in the atmosphere. The distance across the photo is approximately 70 km and the top of the cloud is approximately 15 km high.

Emission temperature of a planet

The emission temperature of a planet is the **blackbody temperature** with which it needs to emit in order to achieve **energy balance.**

If we equate the solar energy absorbed by a planet with the energy emitted by a blackbody we can obtain the emission temperature of a planet.

The emission temperature of a planet is a **key concept** in climate.

SOLAR RADIATION ABSORBED = PLANETARY RADIATION EMITTED

Blackbody radiation

The radiation field within a closed cavity in thermodynamic equilibrium has a value that is uniquely related to the temperature of the cavity walls, regardless of the material of which the cavity is made. This is also called *blackbody radiation*.

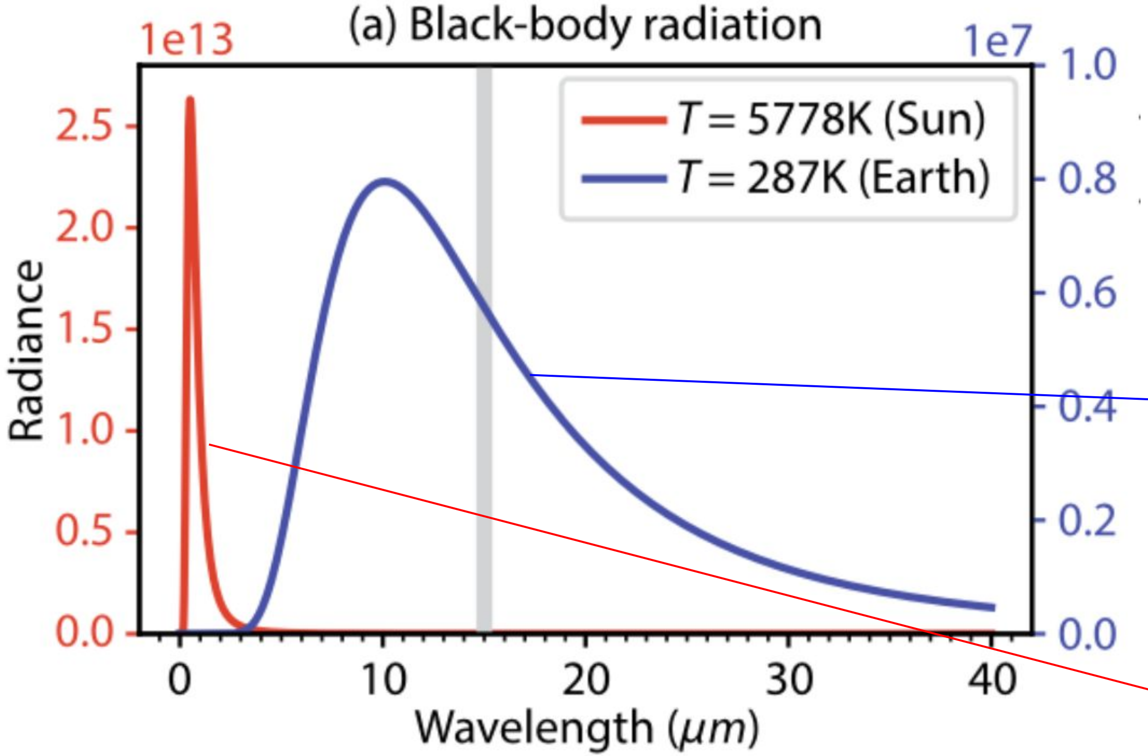
The dependence of the blackbody emission on temperature follows the **Stefan-Boltzmann law**:

$$\boxed{E_{BB} = \sigma T^4 \left[\frac{W}{m^2}\right]}$$

Stefan-Boltzmann constant:

$$\sigma = 5.67 \times 10^{-8} \left[\frac{W}{m^2 K^4}\right]$$

Temperature dependence of black-body radiation



$$E_{BB} = \sigma T^4 \left[\frac{W}{m^2}\right]$$

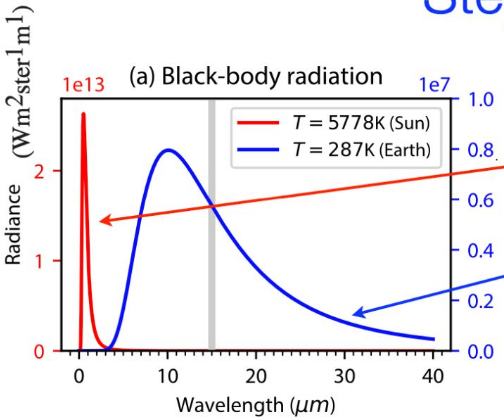
Radiation from Earth is “longwave” (LW)

Radiation from Sun is “shortwave” (SW)

Higher energy (temperature) of a blackbody → shorter wavelength → higher wavenumber (inversely proportional to the wavelength).

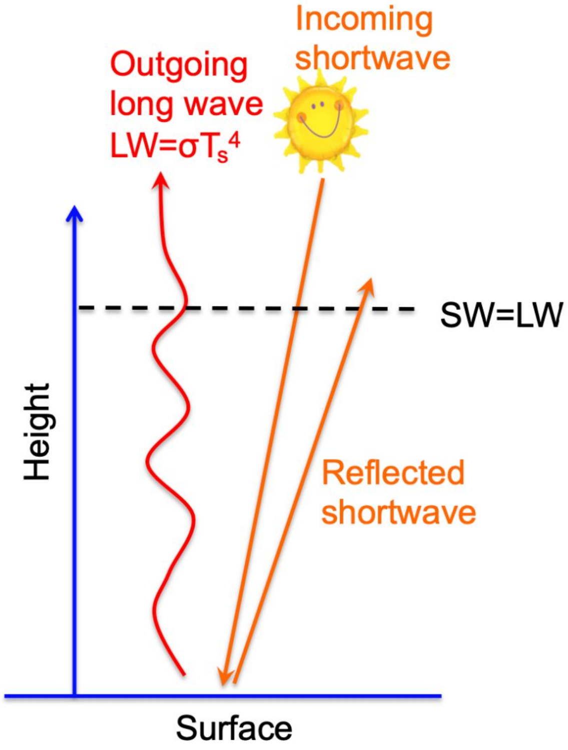
Energy balance of the Earth

Step 1: no atmo



Shortwave radiation: arriving from the sun

Longwave radiation: heat released by Ear



- Energy conservation: incoming **SW** radiation to Earth = outgoing **LW** radiation to space
- Incoming **SW** $= \dfrac{S_o}{4}(1-\alpha)$
- α = albedo = proportion SW reflected
- Outgoing **LW** $= \sigma T^4$
- Set incoming = outgoing ➡ $T = \left(\dfrac{(S_0/4)(1-\alpha)}{\sigma}\right)^{1/4} = 255\ \mathrm{K} = -18\ ^\circ\mathrm{C} \equiv T_0.$

We need to have the influence of the atmosphere on radiation!

Emissivity

We define emissivity as the ratio of the actual emission of a body (or volume of gas) to the blackbody emission at the same temperature.

$$E_R = \varepsilon E_{BB} \quad \longrightarrow \quad \varepsilon = \frac{E_R}{E_{BB}} = \frac{E_R}{\sigma T^4}$$

The Greenhouse Effect

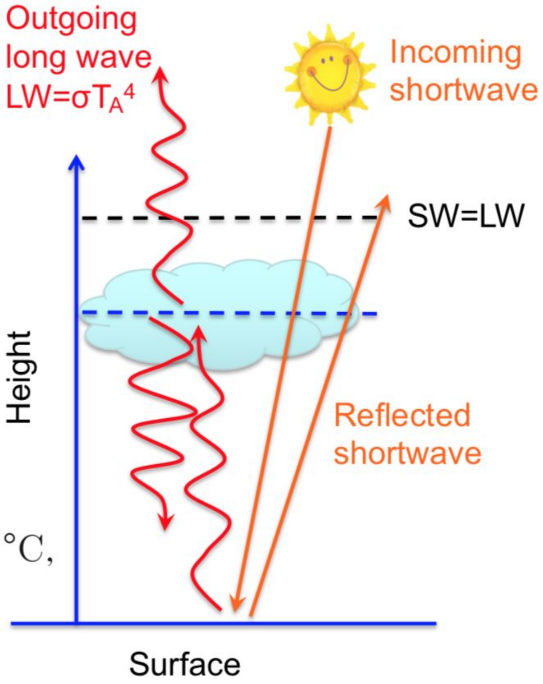
Step 2: add a 1-layer atmosphere

- Add an atmospheric layer which absorbs and re-emits LW radiation, transparent to SW
- Two unknowns: surface temperature T and (mid) atmospheric temperature T_a. Two equations (energy balance at the surface and at mid-atmosphere):

$$\frac{S_0}{4}(1-\alpha)+\epsilon\sigma T_a^4=\sigma T^4$$

$$\epsilon\sigma T^4=2\epsilon\sigma T_a^4.$$

$$T=\left(\frac{(S_0/4)(1-\alpha)}{\sigma(1-\epsilon/2)}\right)^{1/4}=T_0(1-\epsilon/2)^{-1/4}=284\,\mathrm{K}=13\,^{\circ}\mathrm{C},$$

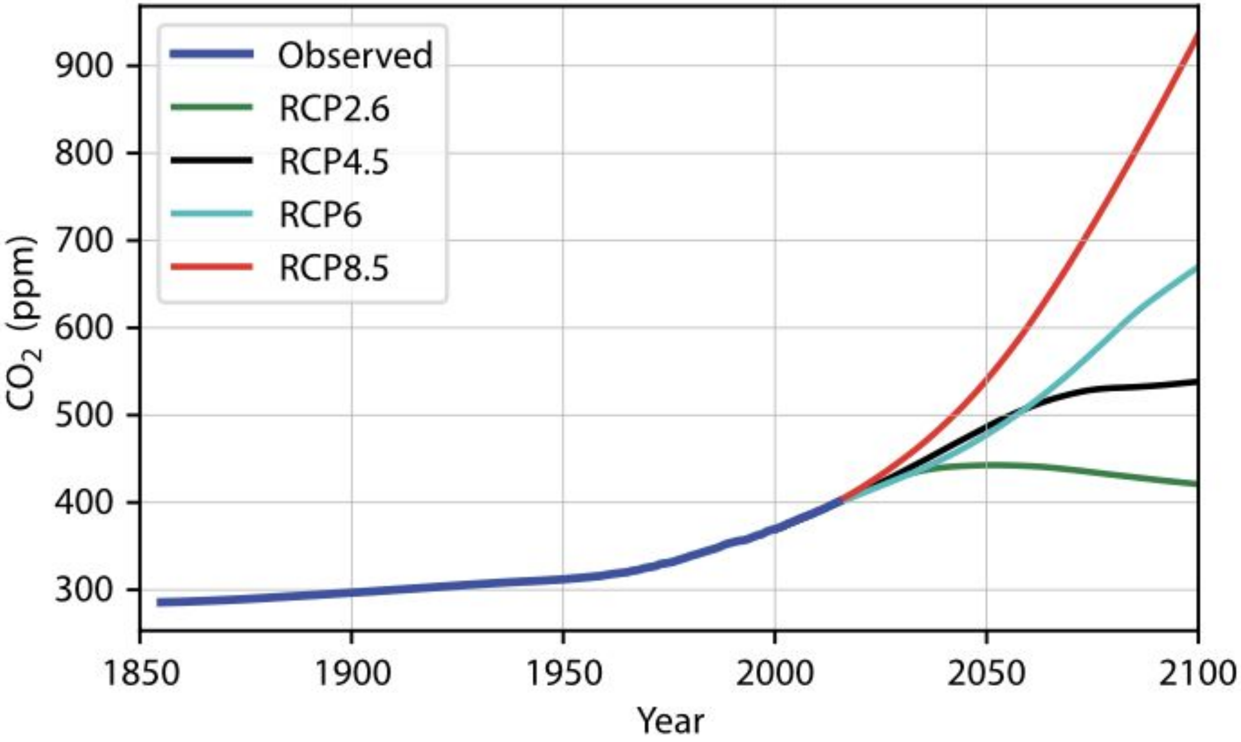


- Result: surface temperature increases due to the “greenhouse effect”

Google Colab

Notebook: “greenhouse2.ipynb”

Climate change: increasing CO2 concentration



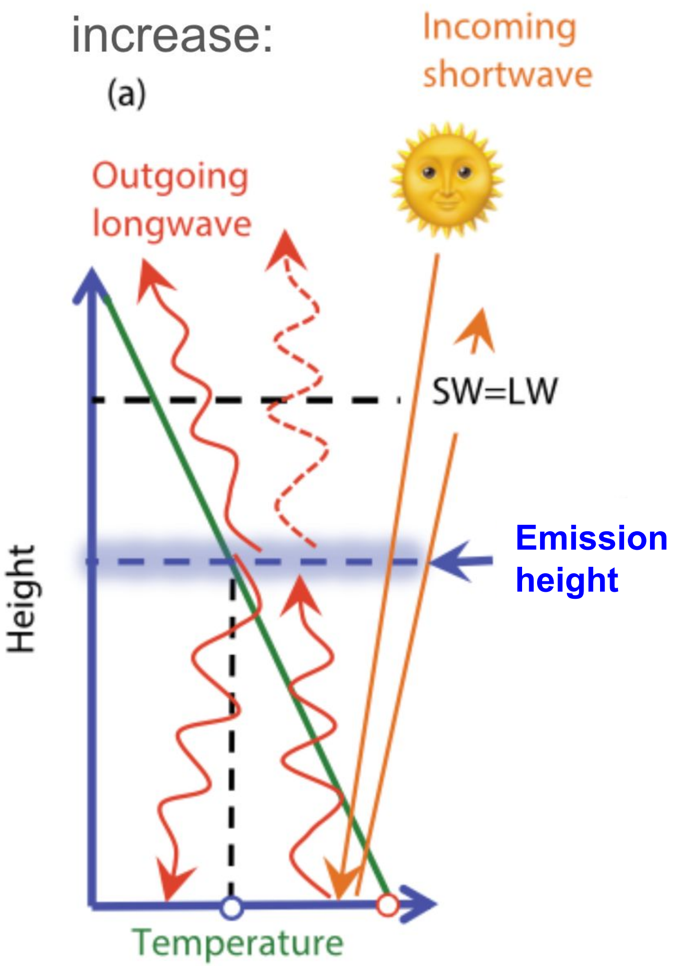
Emission height and lapse rate

The absorption by the atmosphere of LW radiation emitted by the surface depends on the wavelength of radiation, which in turn depends on the chemical composition of the atmosphere.

The atmosphere **absorbs nearly all photons of LW radiation emitted from the surface in the wavelength range corresponding to CO2 absorption**.

Why does increasing concentration of CO2 lead to further warming?
It has to do with the lapse rate, i.e. that temperature decreases with height in the troposphere.

Before CO2 increase:

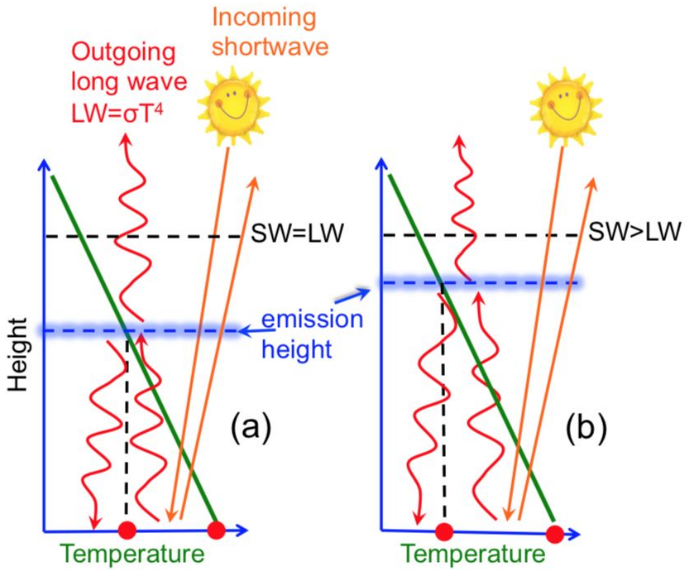


In the mean climate, the atmosphere absorbs and re-emits LW radiation: at some level the overlying air becomes thin enough that most of LW radiation emitted from that level is able to make it to the outward space without being reabsorbed again. This is called the **emission height**.

The atmosphere emits LW radiation from the emission height to space at a temperature for which the Outgoing LW Radiation (OLR) is equal to the incoming SW radiation.

The Anthropogenic Greenhouse Effect

Step 3: add a continuous atmospheric temperature profile



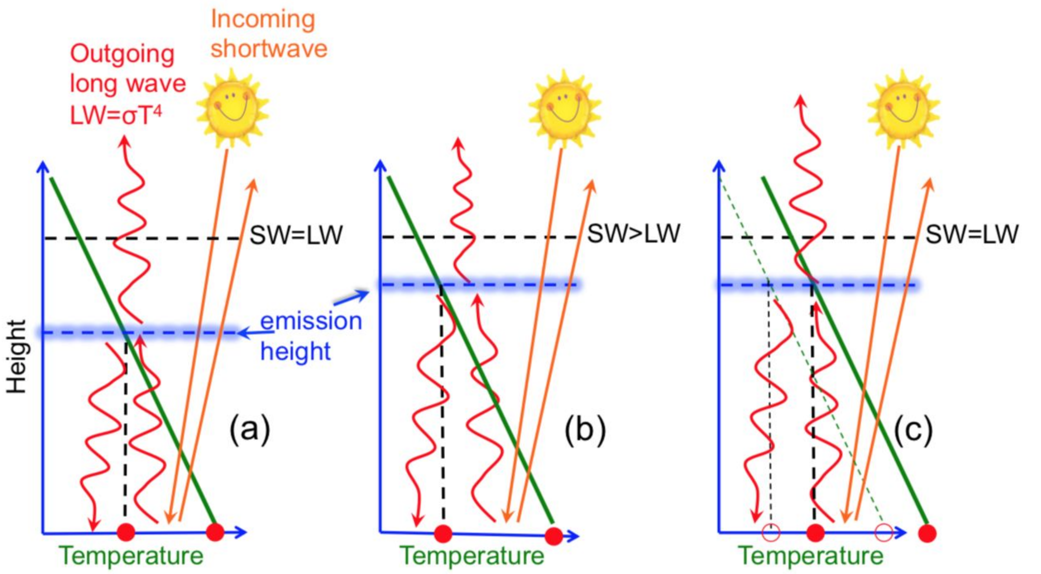
Since OLR is lower, the incoming SW radiation is > OLR causing an energy imbalance that leads to warming in the troposphere.

In addition: Higher CO2 concentration means more LW emissivity which increases surface warming too.

- Level of last absorption: where most of the radiation emitted upward escapes to space, without getting absorbed again
- Increasing greenhouse gas ➡ Rising level of last absorption ➡ Earth radiates from a colder temperature ➡ Energy balance is broken: LW < SW ➡ The temperature must adjust

The Anthropogenic Greenhouse Effect

Step 3: add a continuous atmospheric temperature profile



It's not that more photons are trapped by the added CO2, but before photons were able to escape to outer space form a lower height.

The Earth in panel c) emits at the same temperature as in the unperturbed climate in panel a), but from a higher emission height. SW=LW so the atmosphere does not warm anymore.

Google Colab

Notebook: “greenhouse3.ipynb”

Global radiative flux energy balance

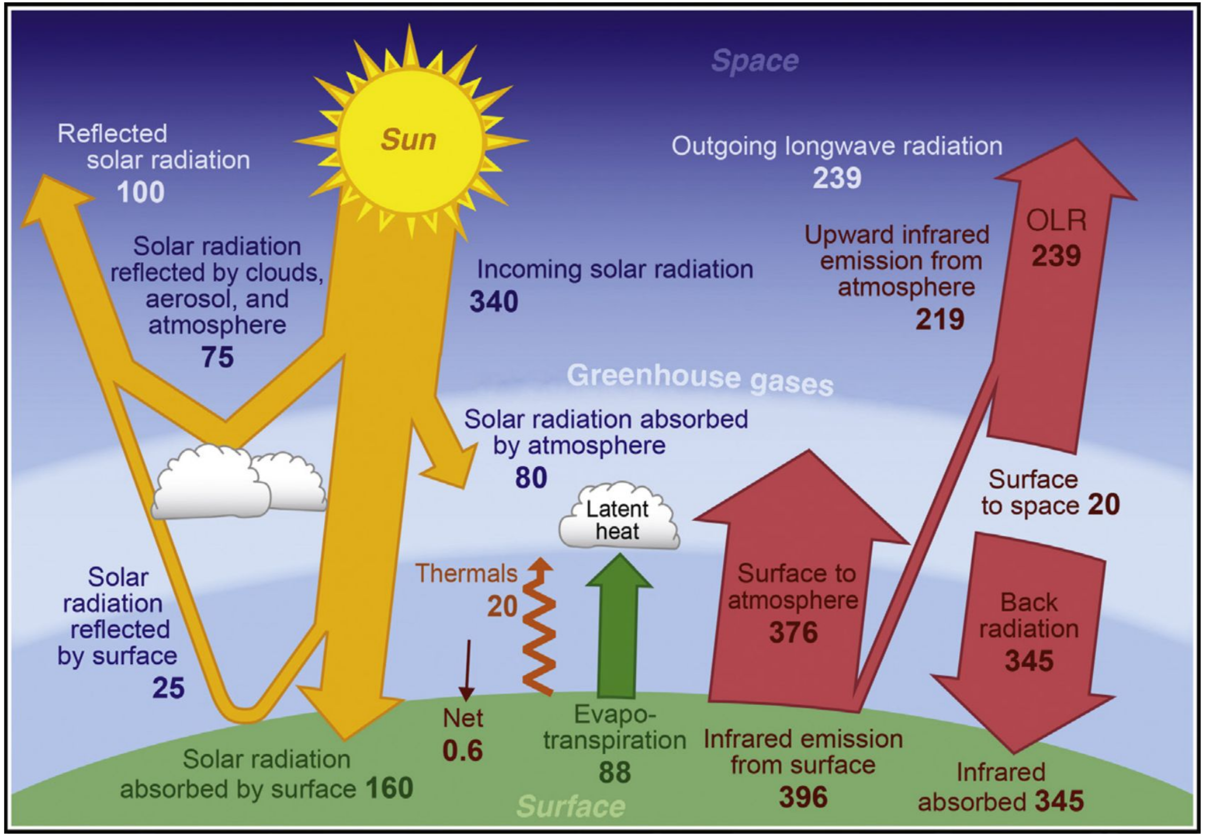


FIGURE 2.4 Global and annual average radiative and nonradiative energy-flow diagram for Earth and its atmosphere. Units are Wm^{-2}.

Global radiative flux energy balance

The vertical flux of energy in the atmosphere depends on the radiative and non-radiative fluxes between the surface, the atmosphere and space.

The strength of the greenhouse effect depends on how easy it is for the solar radiation to penetrate through the atmosphere, and on how difficult it is for the terrestrial radiation to be transmitted back to space.

- >90% of terrestrial emission to space comes from the atmosphere, only 20 W/m2 from the surface through the atmospheric IR window

- 240 W/m2 solar radiation absorbed in the climate system: ⅓ in the atmosphere, ⅔ at the surface

TOP OF ATMOSPHERE (TOA)

$$\text{Incoming solar} - \text{reflected solar} - \text{emitted terrestrial radiation} = \text{Storage}$$

$$340\ \mathrm{Wm}^{-2} - 100\ \mathrm{Wm}^{-2} - 239\ \mathrm{Wm}^{-2} = 0.6\ \mathrm{Wm}^{-2}$$

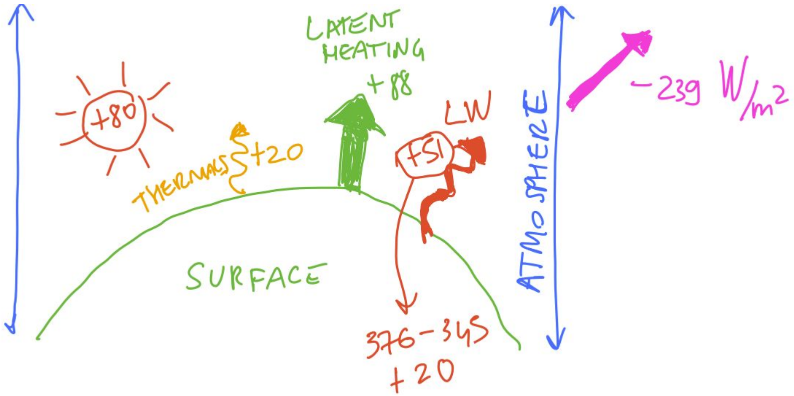
Stored in the ocean

ATMOSPHERE

Atmosphere:

$$\text{Absorbed solar} + \text{thermals} + \text{latent heating} + \text{surface longwave} - \text{outgoing longwave} = 0$$

$$80\ \mathrm{Wm^{-2}} + 20\ \mathrm{Wm^{-2}} + 88\ \mathrm{Wm^{-2}} + 51\ \mathrm{Wm^{-2}} - 239\ \mathrm{Wm^{-2}} = 0$$



ATMOSPHERE

$$\begin{array}{c}\text{Absorbed}\\\text{solar}\end{array} + \begin{array}{c}\text{surface}\\\text{longwave}\end{array} - \begin{array}{c}\text{outgoing}\\\text{longwave}\end{array} = \begin{array}{c}\text{Net}\\\text{radiative}\\\text{cooling}\end{array} = \text{Thermal} + \boxed{\begin{array}{c}\text{latent}\\\text{heating}\end{array}}$$

$$80\ \mathrm{Wm}^{-2} + 51\ \mathrm{Wm}^{-2} - 239\ \mathrm{Wm}^{-2} = 108\ \mathrm{Wm}^{-2} = 20\ \mathrm{Wm}^{-2} + 88\ \mathrm{Wm}^{-2}$$

~80% of radiative cooling is balanced by latent heating (i.e., precipitation).

Latent heating: “heating of the atmosphere by the condensation of water vapor”

SURFACE

$$\begin{array}{ccccccccc} \text{Absorbed solar} & - & \text{thermals} & - & \text{latent heating} & - & \text{surface longwave} & = & \text{Storage} \\ 160\ \mathrm{Wm}^{-2} & - & 20\ \mathrm{Wm}^{-2} & - & 88\ \mathrm{Wm}^{-2} & - & 51\ \mathrm{Wm}^{-2} & = & 0.6\ \mathrm{Wm}^{-2} \end{array}$$

Distribution of insolation

Insolation is the amount of downward solar radiation energy incident on a plane surface. Seasonal and latitudinal variations in temperature are driven primarily by variations of insolation and average solar zenith angle.

Insolation linearly depends on the cosine of latitude.

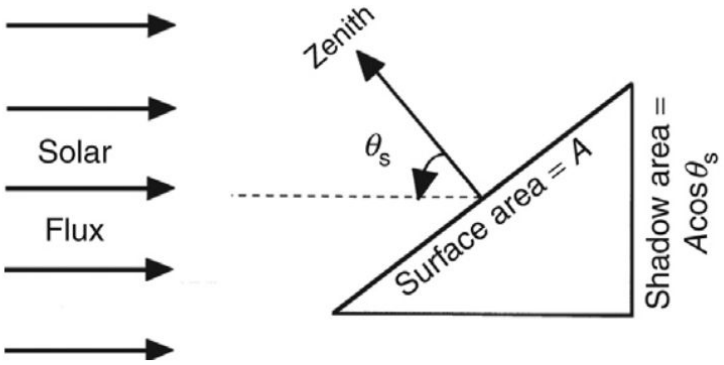
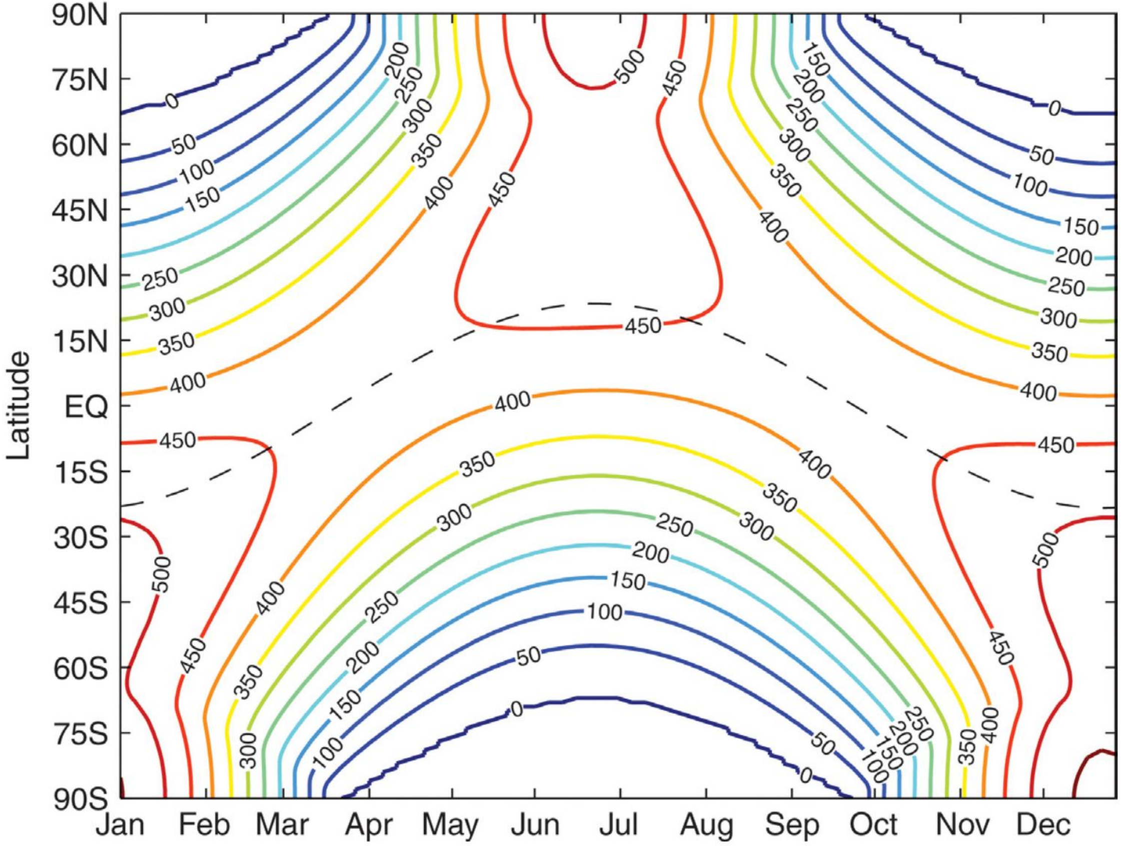


FIGURE 2.6 **Contour graph of the daily average insolation at the top of the atmosphere as a function of season and latitude.** The contour interval is 50 Wm^{-2}. The heavy dashed line indicates the latitude of the subsolar point at noon.



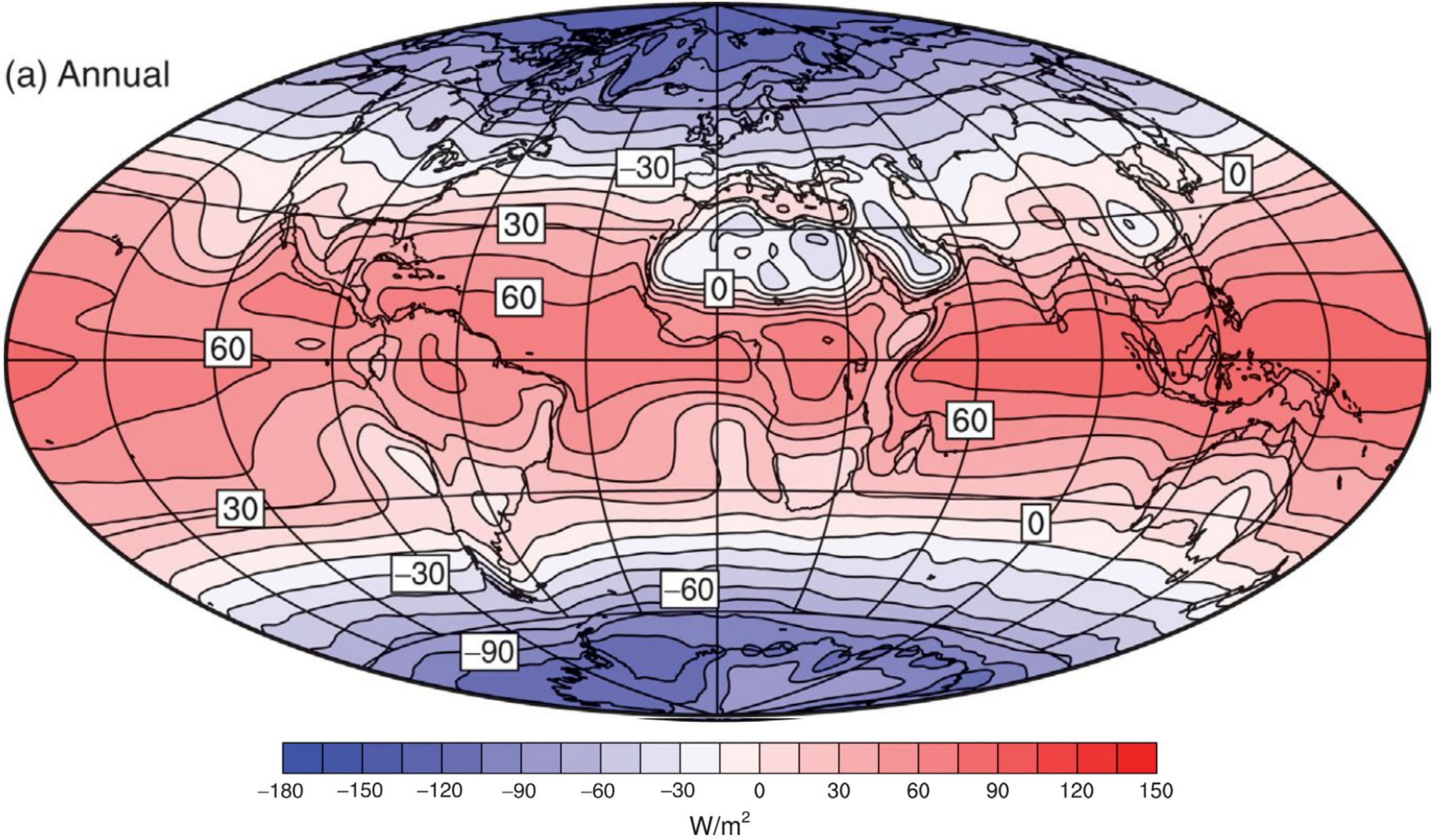


FIGURE 2.11 **Global maps of net incoming radiation.** *Data from CERES 2000–2013.*

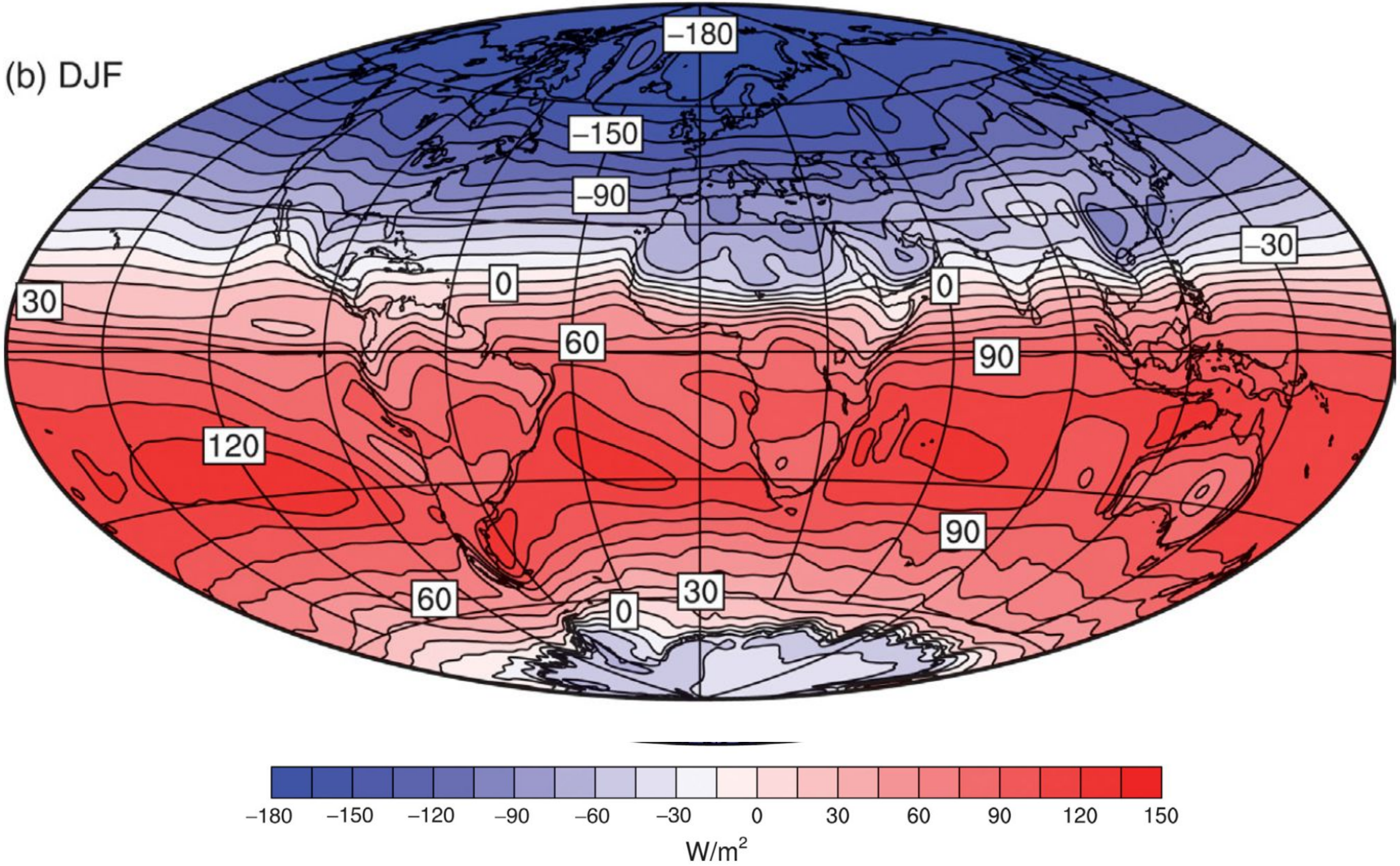


FIGURE 2.11 **Global maps of net incoming radiation.** *Data from CERES 2000–2013.*

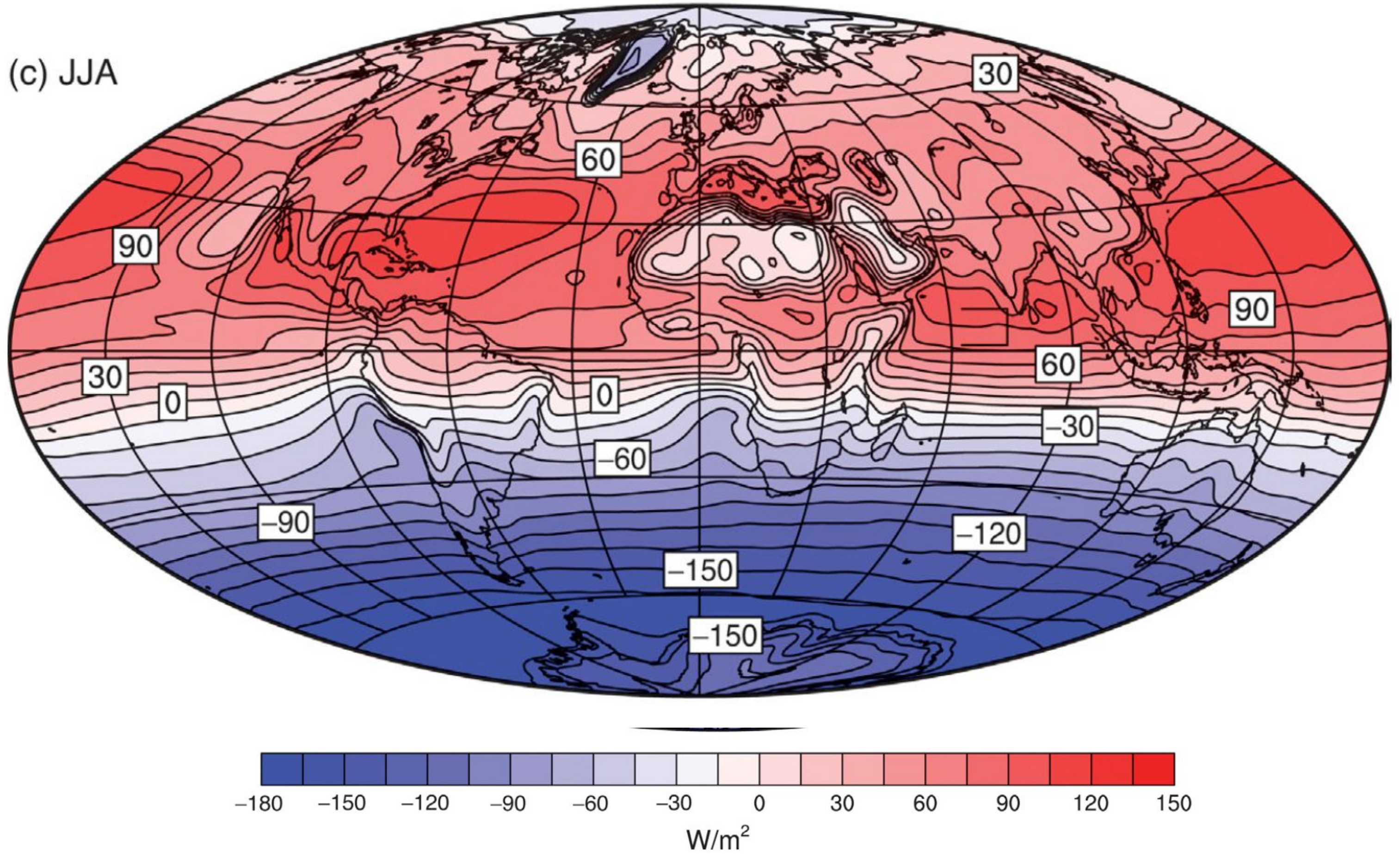


FIGURE 2.11 **Global maps of net incoming radiation.** *Data from CERES 2000–2013.*

TOA Energy Balance

- Net Radiation = Absorbed Solar – OLR

$$R_{TOA} = Q_{abs} - OLR$$

$$Q_{abs} = S_{TOA}(1-\alpha)$$

- Albedo alpha is important

Albedo

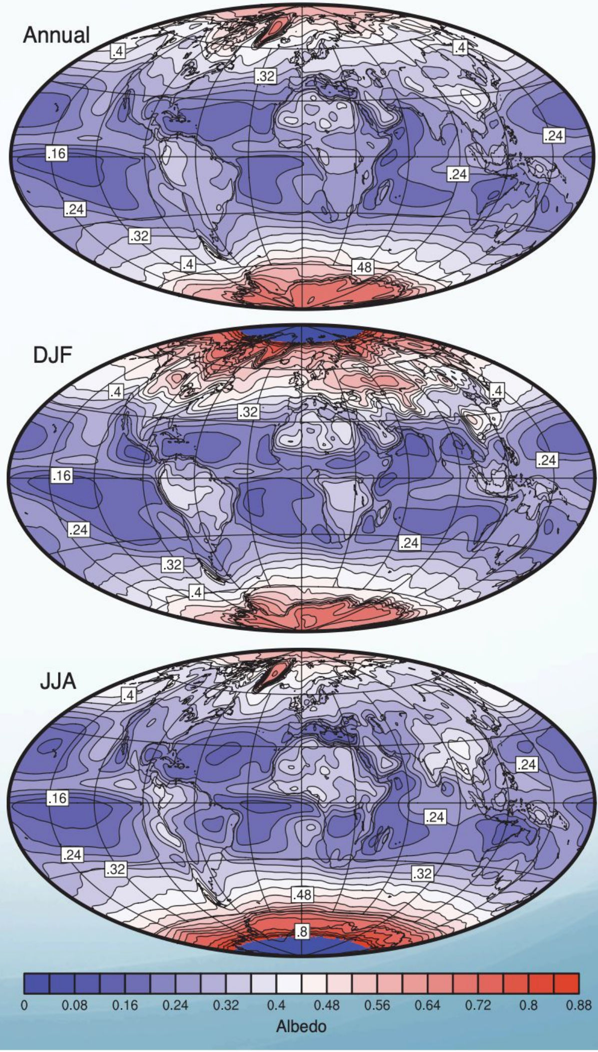
Albedo measured by satellite
at top of atmosphere.

Includes effects of surface,
clear atmosphere, and clouds

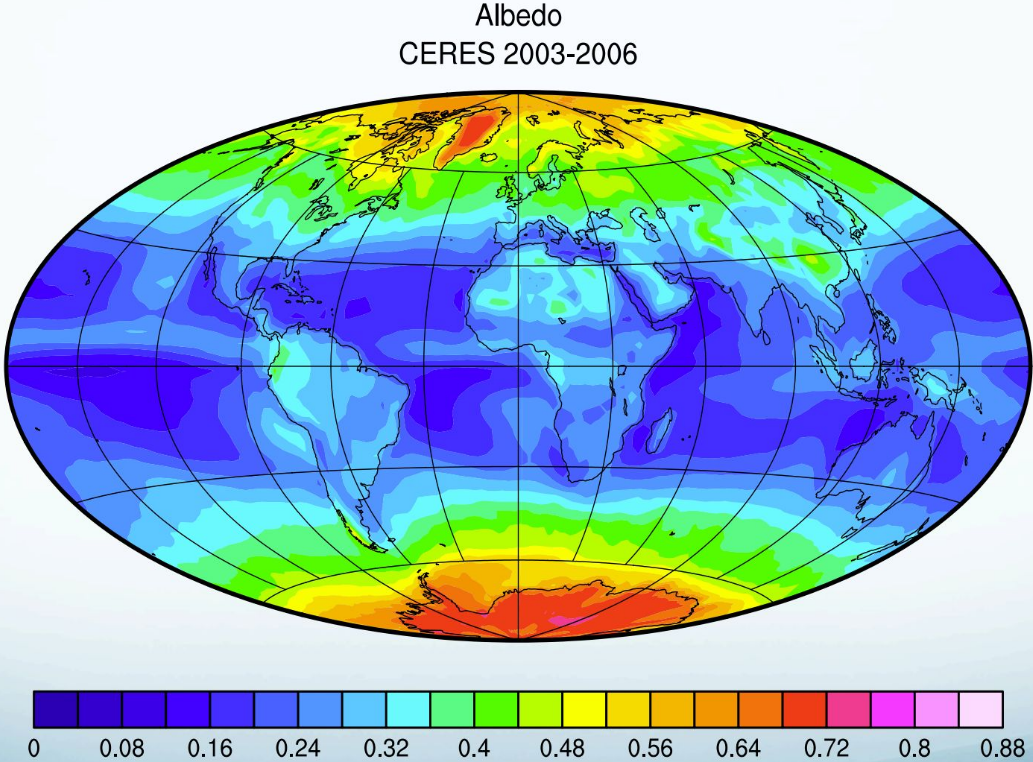
Higher over land than ocean,
because of surface albedo.

Higher in high latitudes because
of zenith angle, clouds
and surface ice.

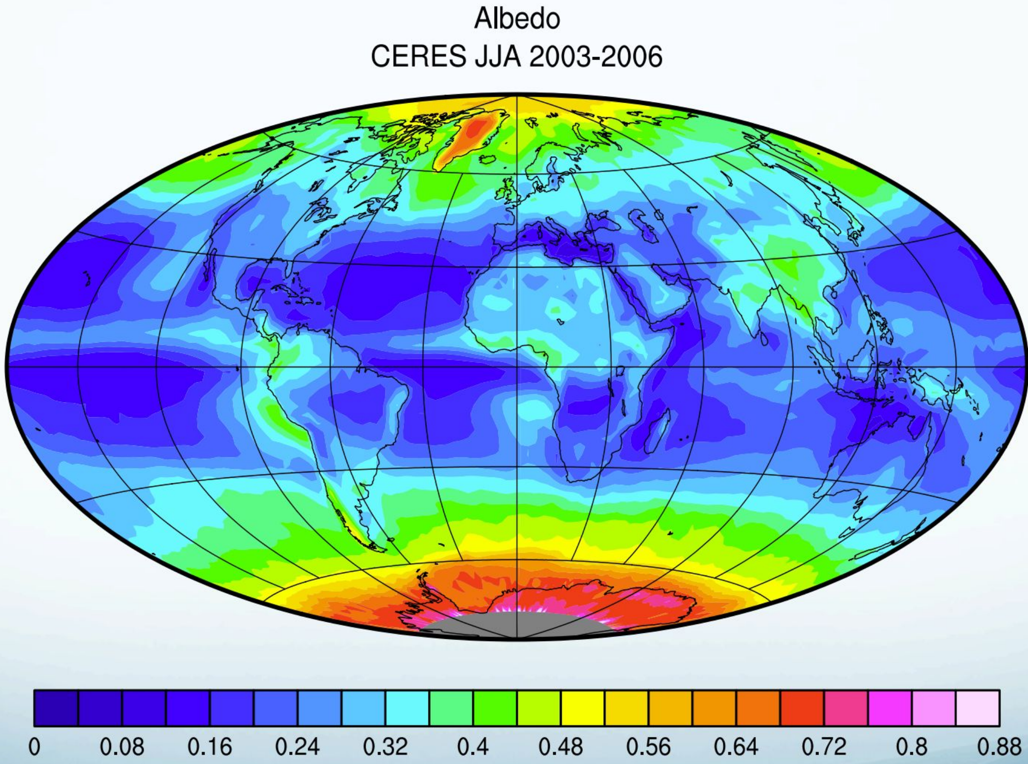
CERES data, 2000-2013.



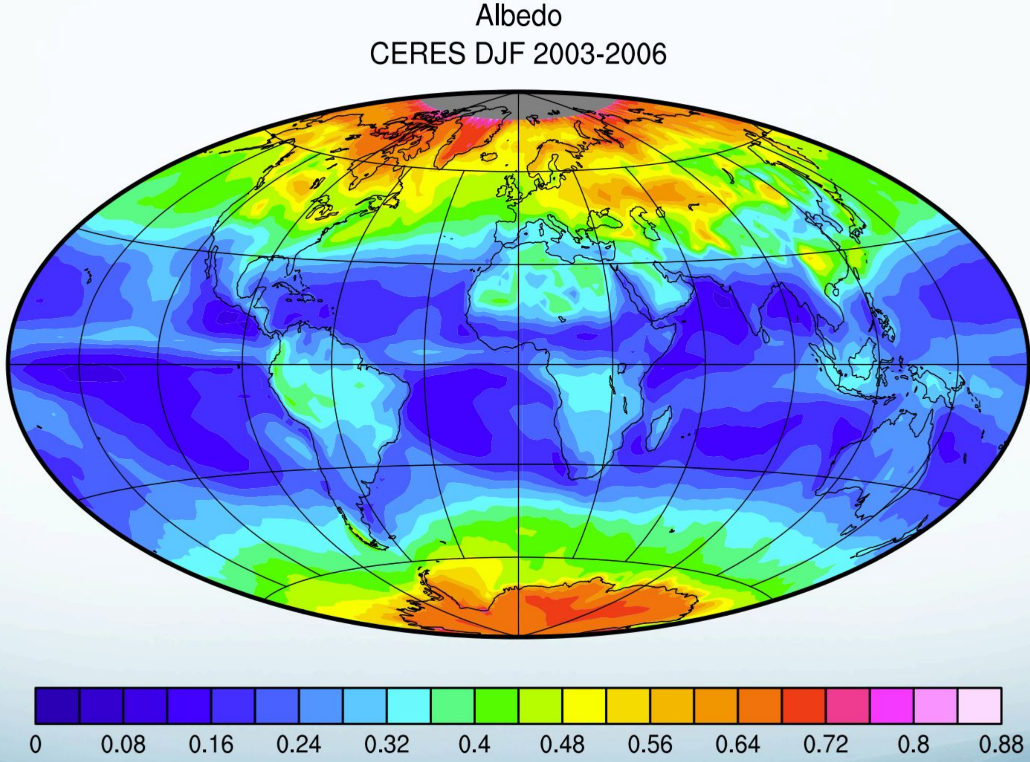
TOA Albedo – annual mean



TOA Albedo – JJA



TOA Albedo – DJF



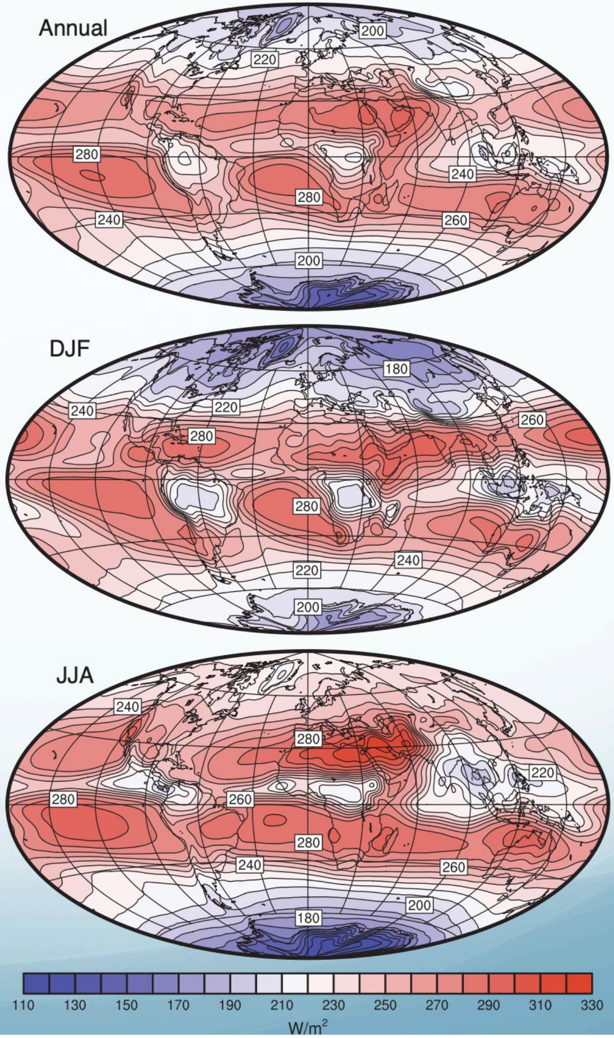
Outgoing Longwave Radiation (OLR)

Greatest in tropical latitudes that have not much cloud, since warm temperatures emit to space there.

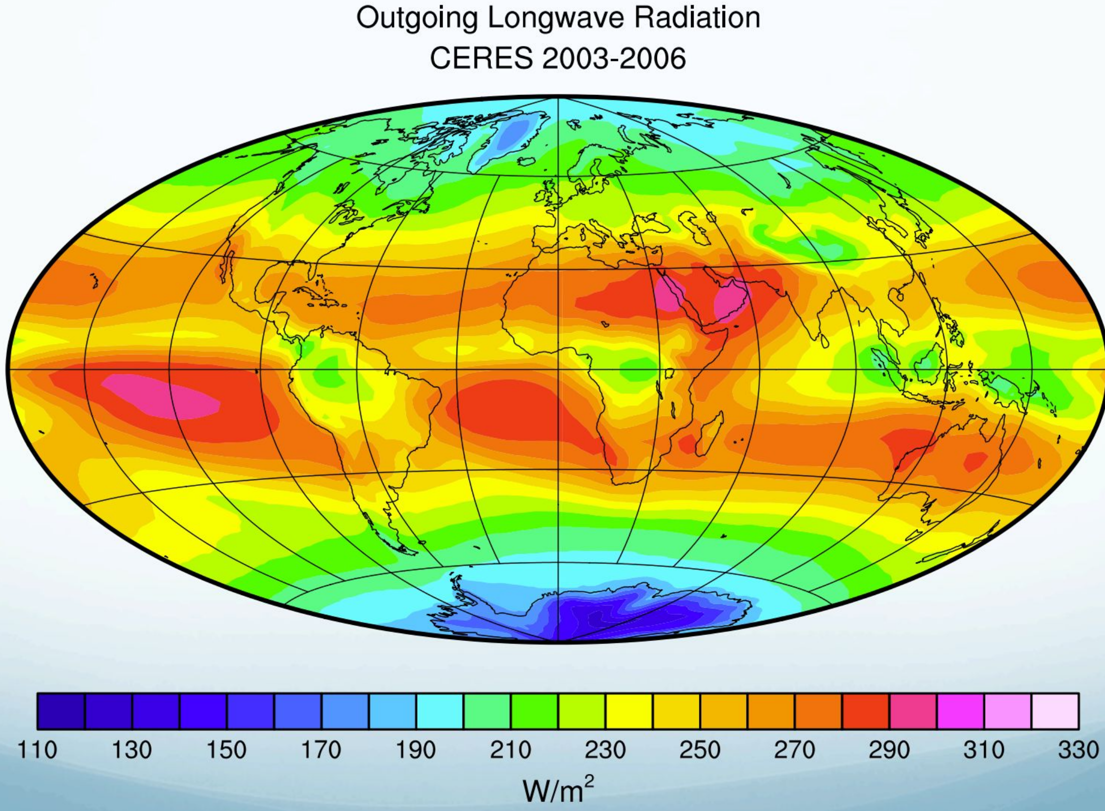
Lower over tropical cloudy regions since clouds are opaque to thermal infrared and their tops are cold.

Lower in high latitudes because it is colder there.

Super high over tropical deserts in summer because the surface is hot and there are few clouds or water vapor to trap IR.



OLR = Outgoing Longwave Radiation

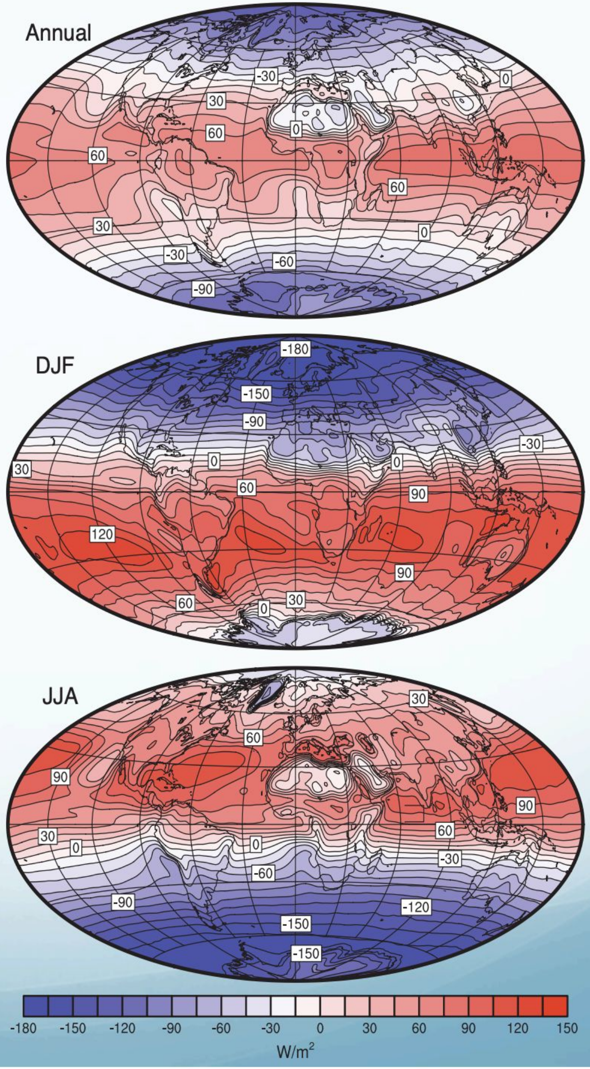


Net Incoming Radiation Absorbed Solar minus OLR

More near equator than in high latitudes

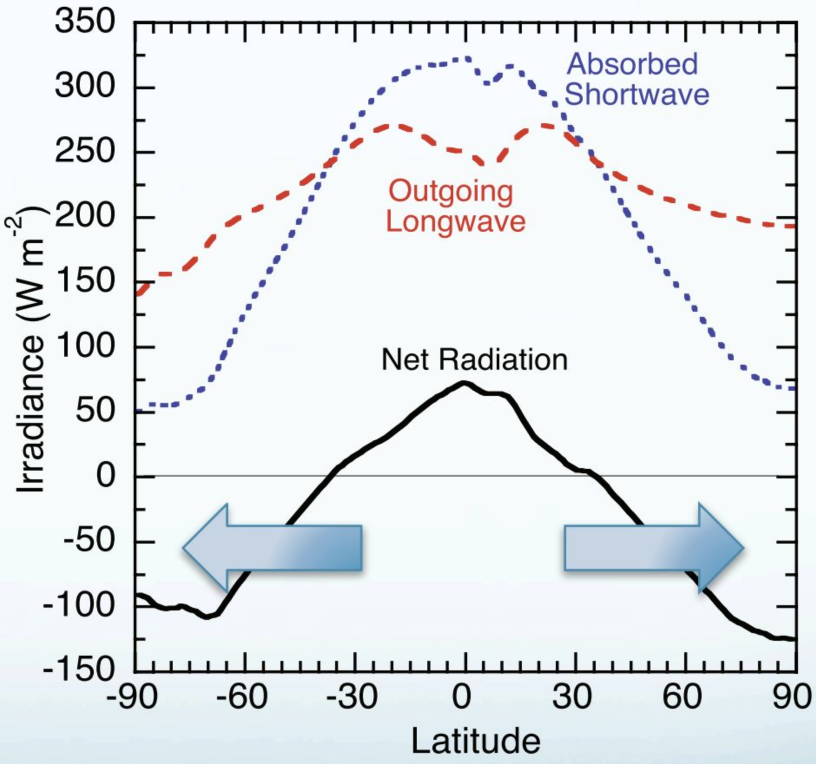
Much more in summer hemisphere than winter hemisphere

Note annual mean is negative over the Sahara/Arabian Desert. How does this support dryness there?



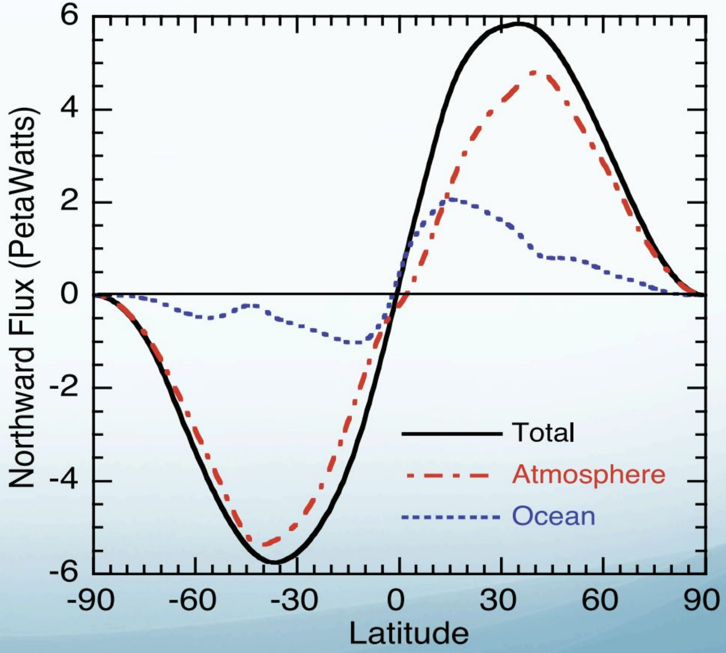
Zonal Average Top-of-Atmosphere (TOA) – Annual mean

Need to move Energy Poleward – in both hemispheres



Poleward Heat Flux

- Contributions of atmosphere and ocean are both important, but are different functions of latitude.
- Total from TOA net radiation satellite data.
- Atmosphere from atmosphere measurements
- Ocean flux is residual in energy budget



Greenhouse gases

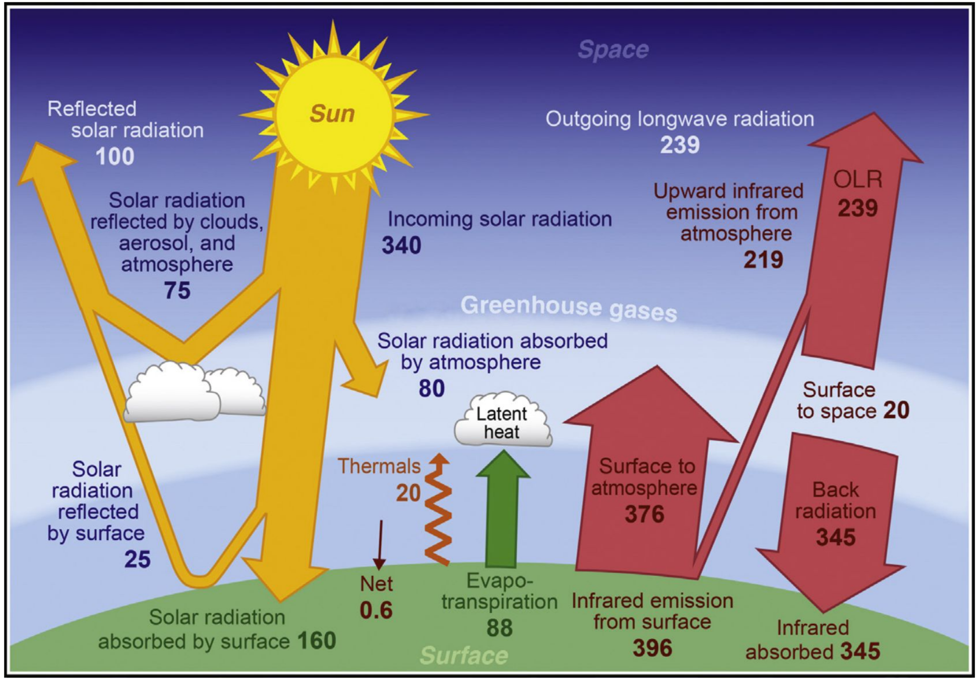
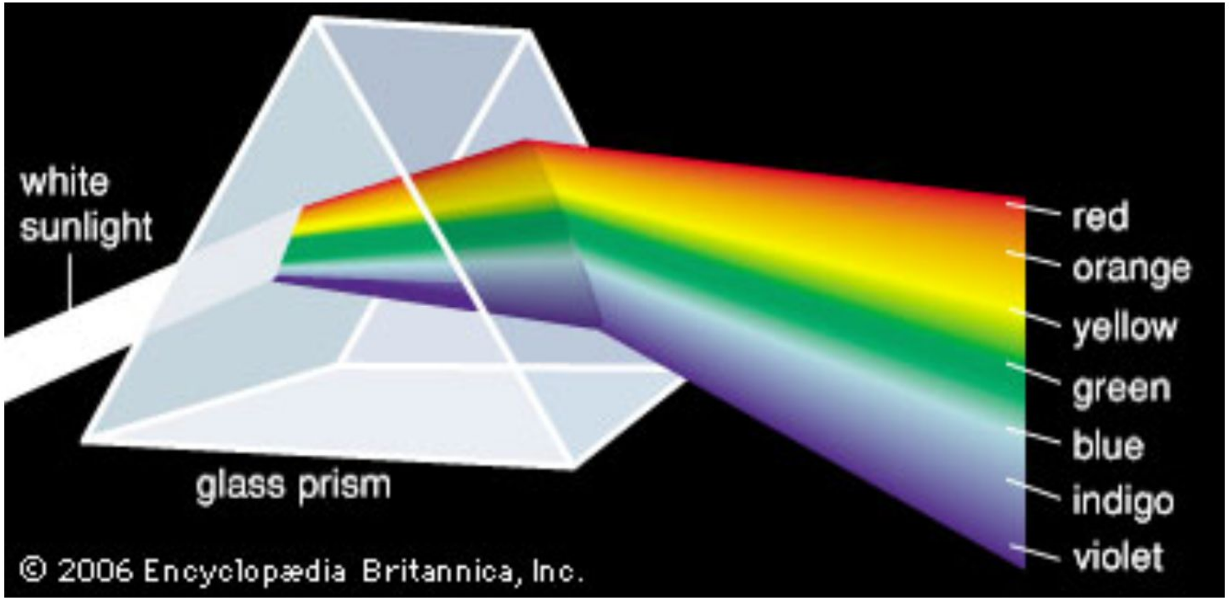


FIGURE 2.4 **Global and annual average radiative and nonradiative energy-flow diagram for Earth and its atmosphere.** Units are Wm^{-2}.

OLR is not a single entity: it is made of different wavelengths of electromagnetic radiation

How greenhouse gasses work: wavelength-dependent radiation & molecular energy levels



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White sunlight is made of all colors: different molecules absorb at different wavelengths, why?

Black body radiation

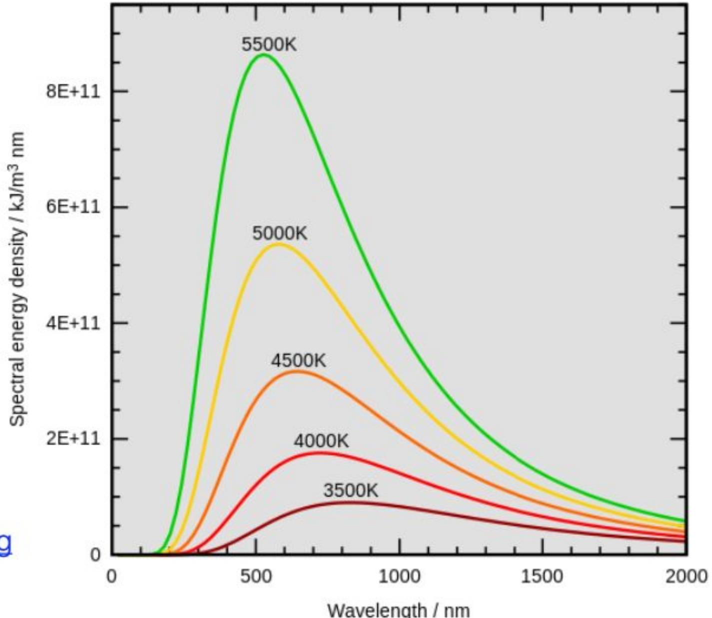
Planck's law of black-body radiation: $$B(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

$B(\lambda,T)d\lambda$ is the energy per area/ time/ angle emitted between wavelengths λ & $\lambda+d\lambda$; T=temperature; h=Planck's const; c=speed of light; k=Boltzmann's const.

Total emitted radiation per area/ time: σT^4 Stefan–Boltzmann constant: $\sigma = 5.670367\times10^{-8}\ \mathrm{W\cdot m^{-2}\cdot K^{-4}}$

Blackbody is a body that absorbs all of the incoming radiation and emits according to the Planck's law.

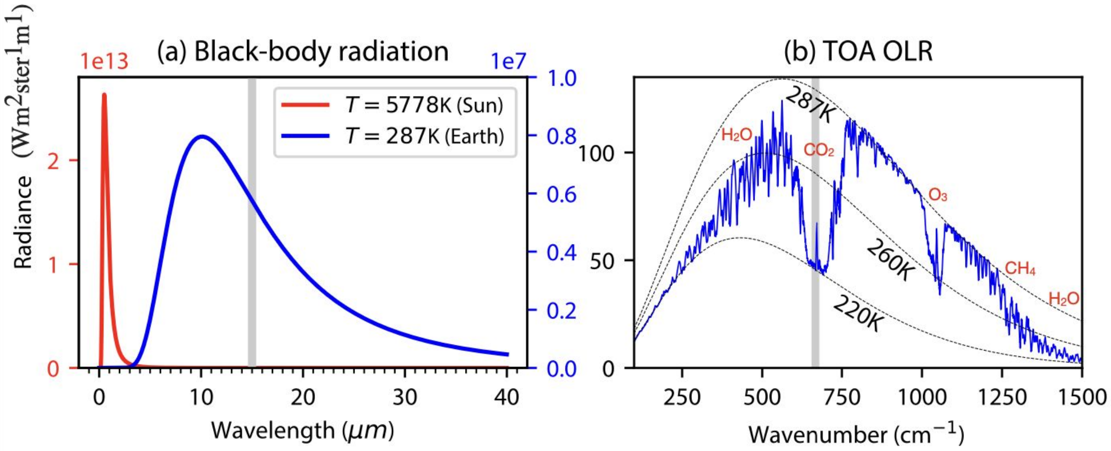
Radiation is a function of wavelength and temperature.

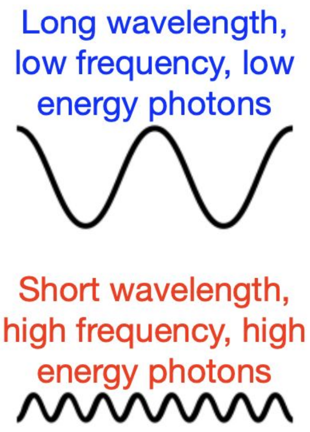


https://en.wikipedia.org/wiki/File:Wiens_law.svg

Shortwave vs longwave radiation

Earth's surface and the sun both emit blackbody radiation according to Planck's function — they radiate over the full spectrum





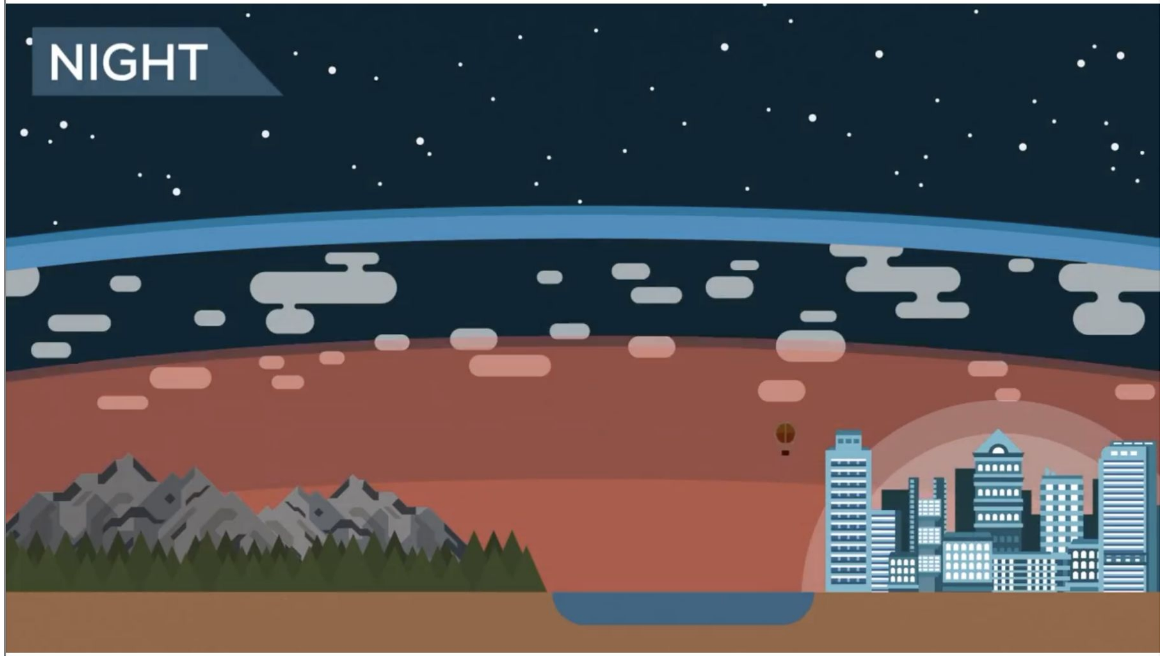
Long wavelength, low frequency, low energy photons

Short wavelength, high frequency, high energy photons

Figure 2.5: Blackbody radiation.
(a) Planck's black-body spectral radiance for the emission temperatures of the Earth (blue) and Sun (red), as a function of wavelength. (b) blue curve shows estimated outgoing longwave radiation at the top of the atmosphere as a function of wavenumber, with black-body radiation curves at different temperatures shown by dashed lines. The deviations from the 287 K black body radiation curve indicate absorption bands due to CO_2, CH_4, H_2O, and O_3; the central CO_2 absorption line is shown on both panels as a vertical gray bar.

Global Warming Science 101, Greenhouse, Camille Hankel & Eli Tziperman

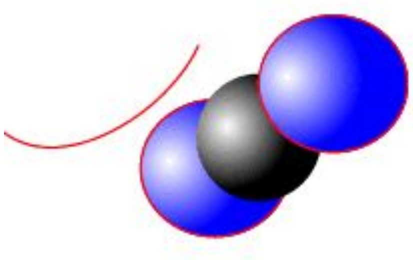
How do greenhouse gases work?



https://www.youtube.com/watch?v=sTvqIijqvTg

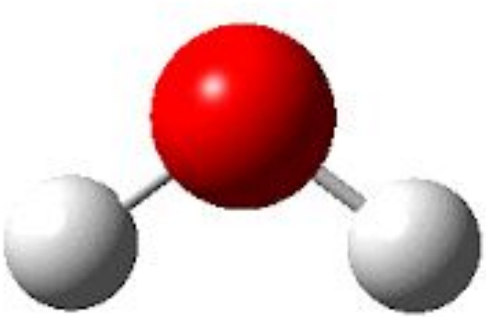
Vibrations due to absorption of photon

Carbon dioxide: CO_2

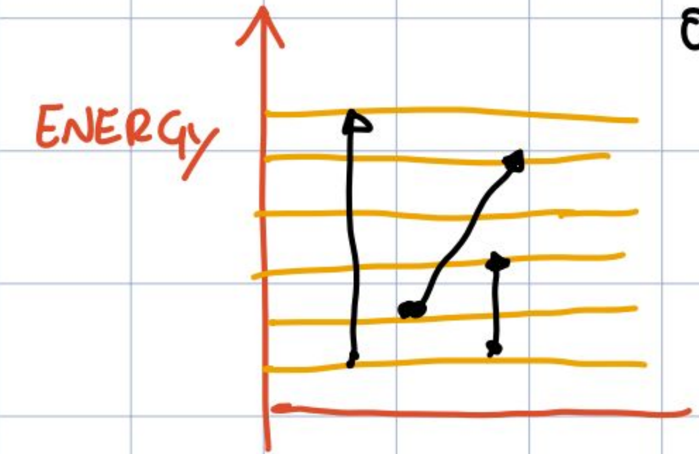


http://www.dynamicscience.com.au/tester/solutions1/chemistry/greenhouse/co2andtheghe.htm

Water: H_2O



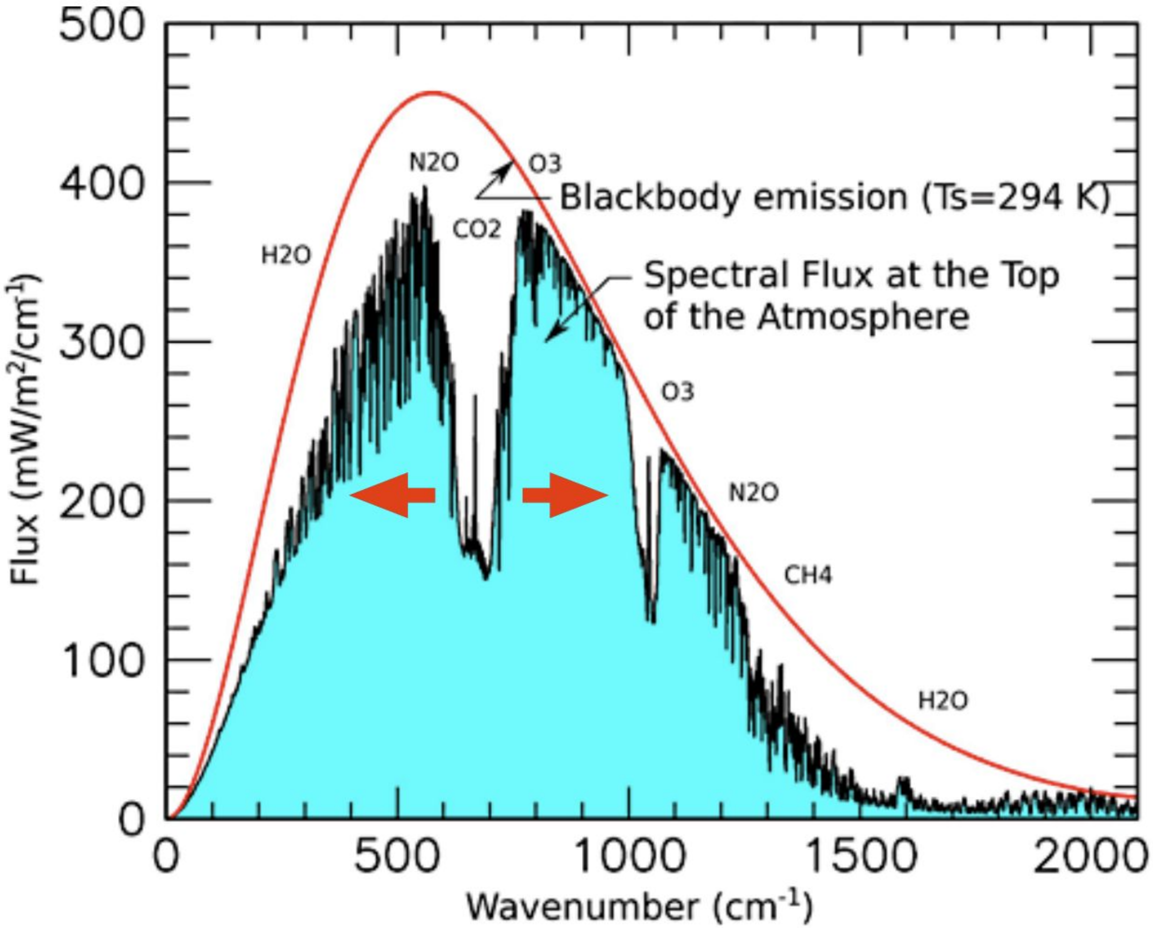
THE MOLECULE IS A QUANTUM SYSTEM



JUMPS OF ENERGY WHEN PHOTON IS ABSORBED OR RELEASED

$E = h\nu$ ↓ FREQUENCY

$\lambda = \frac{c}{\nu}$ $c \rightarrow$ speed of light



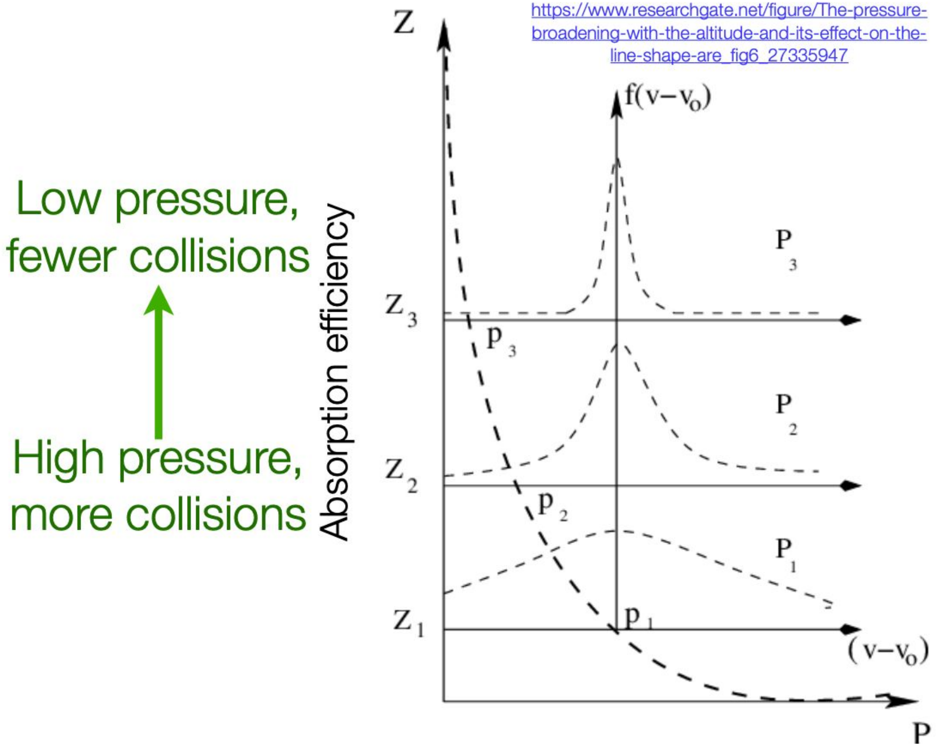
https://www.giss.nasa.gov/research/briefs/schmidt_05/

Broadening

Molecules can absorb photons with energy in a broader range around the precise energies corresponding to transitions between pairs of molecular energy levels due to two main effects.

1) Pressure broadening

When two molecules collide while one of them is absorbing a photon. As one molecule absorbs a photon, some excess energy can be transferred to/from the colliding molecule. This broadening is enhanced at higher pressures, as the frequency of molecular collisions increases.



Low pressure, fewer collisions

High pressure, more collisions

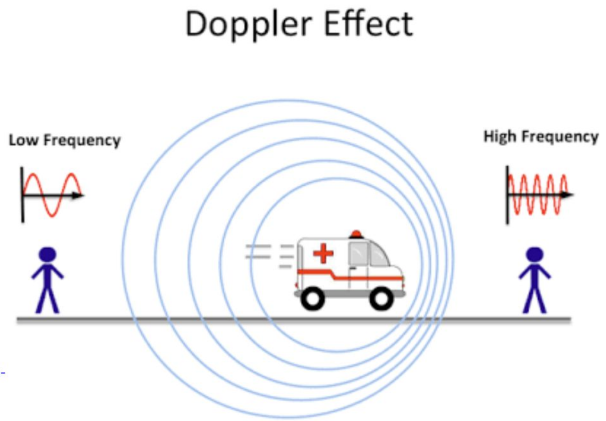
https://www.researchgate.net/figure/The-pressure-broadening-with-the-altitude-and-its-effect-on-the-line-shape-are_fig6_27335947

Broadening

Molecules can absorb photons with energy in a broader range around the precise energies corresponding to transitions between pairs of molecular energy levels due to two main effects.

2) Doppler broadening (much weaker)

The thermal motion of molecules leads to a shift in the frequency of emitted/absorbed photons as seen by the moving molecule due to the Doppler effect. A photon with a slightly different frequency than that needed to induce a transition can be absorbed by the moving molecule.



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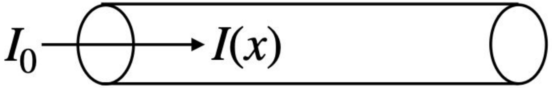
5

Given these two effects, absorption is maximal at the absorption lines, at the frequencies/wavelengths corresponding to molecular energy transitions, and it gradually decays away from these lines. Even when the atmosphere is saturated with respect to absorption at the CO_2 absorption lines, increasing CO_2 concentration leads to stronger absorption at wavelengths near the absorption lines that are not saturated. The increased LW absorption near these lines again explains the greenhouse effect due to increasing CO_2

Logarithmic dependence of RF on CO_2

The absorption is typically proportional to the radiation intensity at any given location, and therefore the equation for the transmitted radiation, I(x), may be written as $dI/dx = -\mu I$, where μ depends on the absorbing medium. The solution for the radiation intensity along the pipe is, therefore,

$$I(x) = I_0 e^{-\mu x}$$



For CO_2, the RF varies logarithmically with concentration. We showed in section 3.2 that the warming and radiative forcing are approximately linearly related. This implies that the temperature response also depends logarithmically on the CO_2 concentration. We, therefore, write, schematically,

$$T = T_0 + A\log_2(\mathrm{CO}_2/280)$$

which implies:

$$T_{\times 2} - T_{\times 1} = A\log_2(2\mathrm{CO}_2/280) - A\log_2(\mathrm{CO}_2/280) = A\log_2 2 = A,$$

$$T_{\times 4} - T_{\times 2} = A\log_2(4\mathrm{CO}_2/280) - A\log_2(2\mathrm{CO}_2/280) = A\log_2 2 = A.$$

That is, each doubling of CO_2 leads to the same increase in temperature!

The **radiative forcing** due to a given atmospheric CO_2 increase is the radiation [W/m^2] that is absorbed by the added gas and is prevented to be transmitted to outer space, over all wavelengths, assuming the tropospheric temperature, moisture and clouds are kept at their values before the CO_2 increase.

Radiative forcing depends logarithmically on CO_2 rather than exponentially. Why?

Because the atmosphere is SATURATED with respect to CO_2 absorption.

$\Updownarrow$

LW photons emitted from the surface are already absorbed by the atmosphere at preindustrial CO_2 concentration.

What about other greenhouse gases?

How do we quantify their effect relative to that of CO_2?

Global warming potential

CH_4 is a more powerful greenhouse gas than CO_2 but has a shorter lifetime in the atmosphere (“time horizon”)

How important is one more kg of CH_4 in the atmosphere compared to one more kg of CO_2 in terms of radiative forcing?

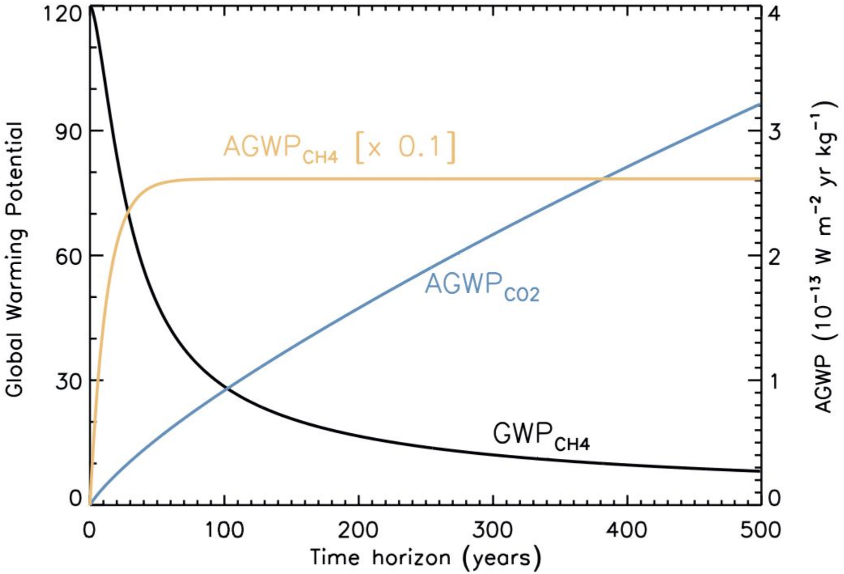
Other GHG: Global Warming Potential (GWP)

GWP: the time-integrated RF due to a pulse emission of a GHG, relative to a pulse emission of an equal mass of CO_2

$$GWP(x) = \frac{\int_0^{TH} a_x \cdot [x(t)]\, dt}{\int_0^{TH} a_r \cdot [r(t)]\, dt}.$$

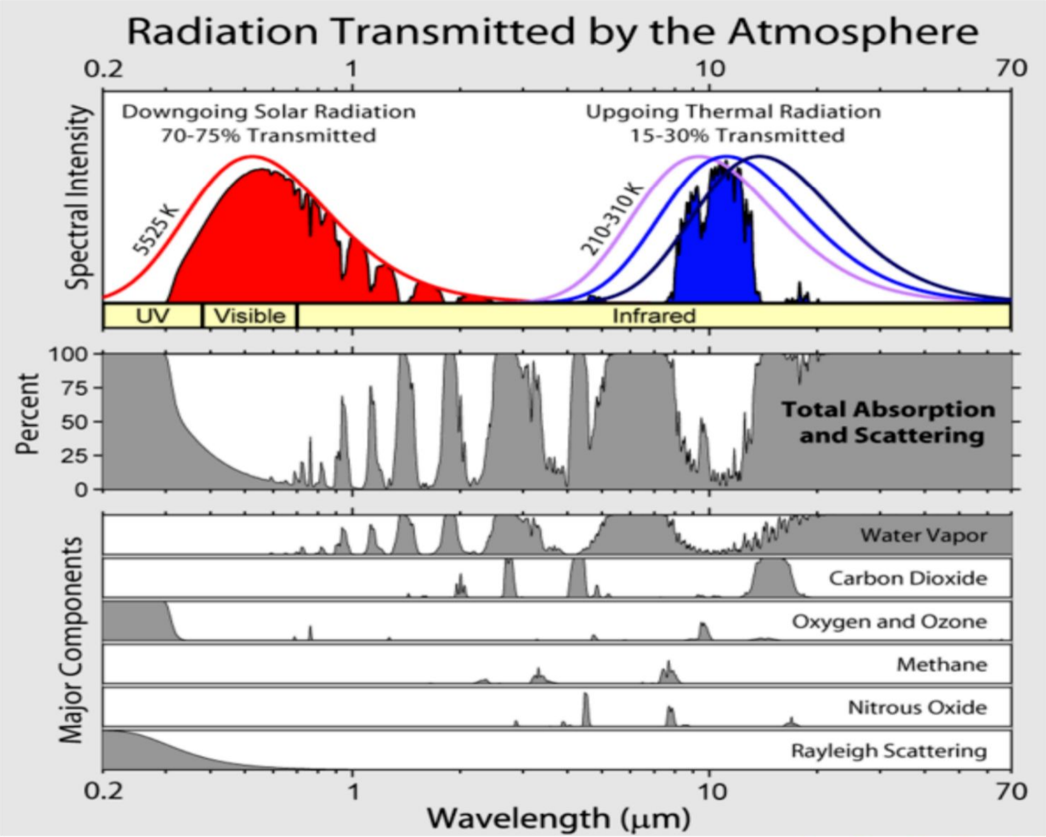
The GWP of a greenhouse gas is its radiative forcing effect relative to CO_2, given its lifetime and strength as an absorber over a specified time horizon (TH).

$x(t)$: time-dependent decay of a GHG; $r(t)$: that of CO_2; $a_{x,r}$: RF per 1 kg ($W/m^2/kg$).



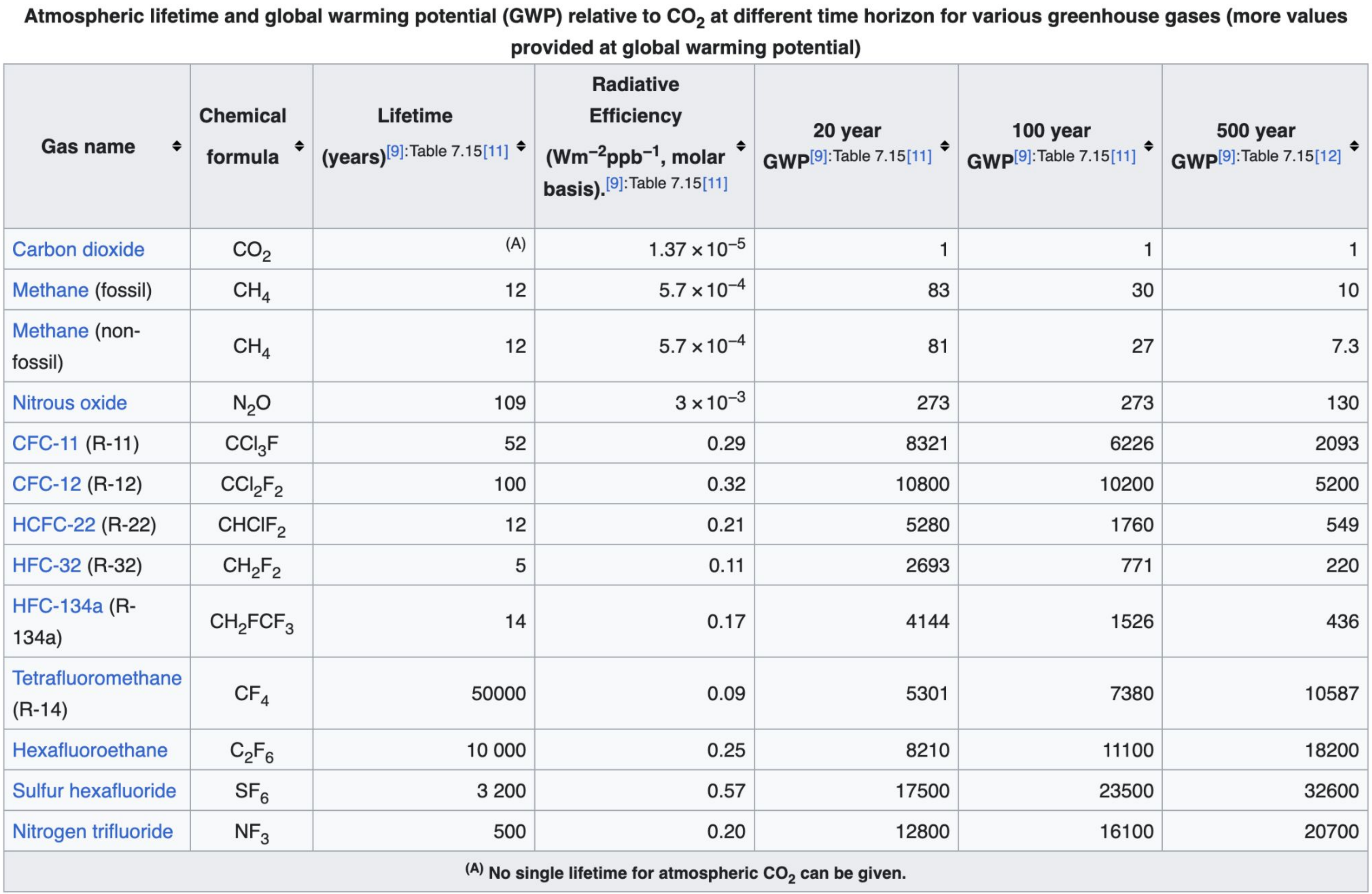
IPCC, Climate Change 2013, Chapter 8

IR absorption of the major greenhouse gases



https://en.wikipedia.org/wiki/Absorption_band

CO_2 & water vapor absorbs the most IR, at different wavelengths



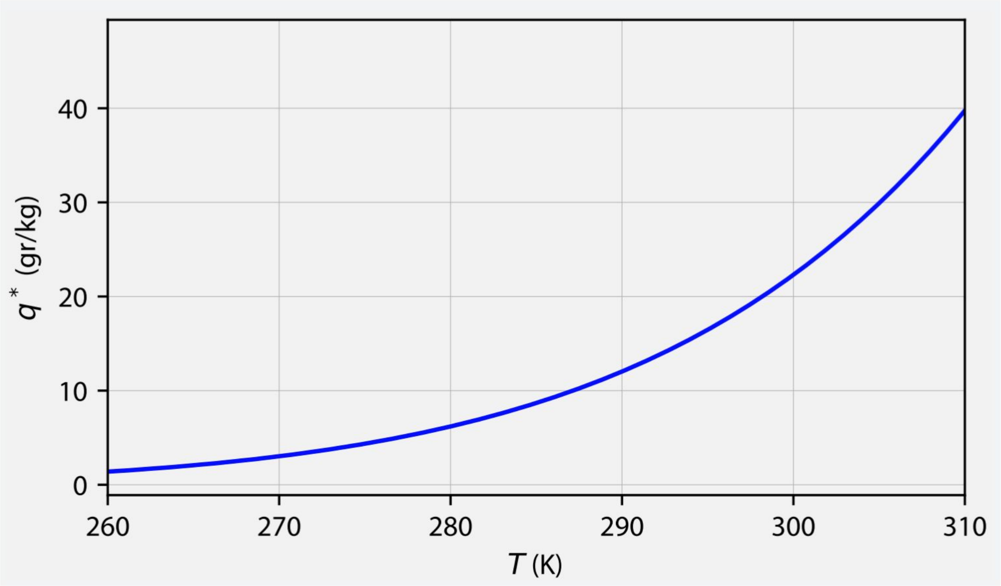
Atmospheric lifetime and global warming potential (GWP) relative to CO_2 at different time horizon for various greenhouse gases (more values provided at global warming potential)

Gas name	Chemical formula	Lifetime (years)[9]:Table 7.15[11]	Radiative Efficiency ($Wm^{-2}ppb^{-1}$, molar basis).[9]:Table 7.15[11]	20 year GWP[9]:Table 7.15[11]	100 year GWP[9]:Table 7.15[11]	500 year GWP[9]:Table 7.15[12]
Carbon dioxide	CO_2	(A)	1.37×10^{-5}	1	1	1
Methane (fossil)	CH_4	12	5.7×10^{-4}	83	30	10
Methane (non-fossil)	CH_4	12	5.7×10^{-4}	81	27	7.3
Nitrous oxide	N_2O	109	3×10^{-3}	273	273	130
CFC-11 (R-11)	CCl_3F	52	0.29	8321	6226	2093
CFC-12 (R-12)	CCl_2F_2	100	0.32	10800	10200	5200
HCFC-22 (R-22)	$CHClF_2$	12	0.21	5280	1760	549
HFC-32 (R-32)	CH_2F_2	5	0.11	2693	771	220
HFC-134a (R-134a)	CH_2FCF_3	14	0.17	4144	1526	436
Tetrafluoromethane (R-14)	CF_4	50000	0.09	5301	7380	10587
Hexafluoroethane	C_2F_6	10 000	0.25	8210	11100	18200
Sulfur hexafluoride	SF_6	3 200	0.57	17500	23500	32600
Nitrogen trifluoride	NF_3	500	0.20	12800	16100	20700

(A) No single lifetime for atmospheric CO_2 can be given.

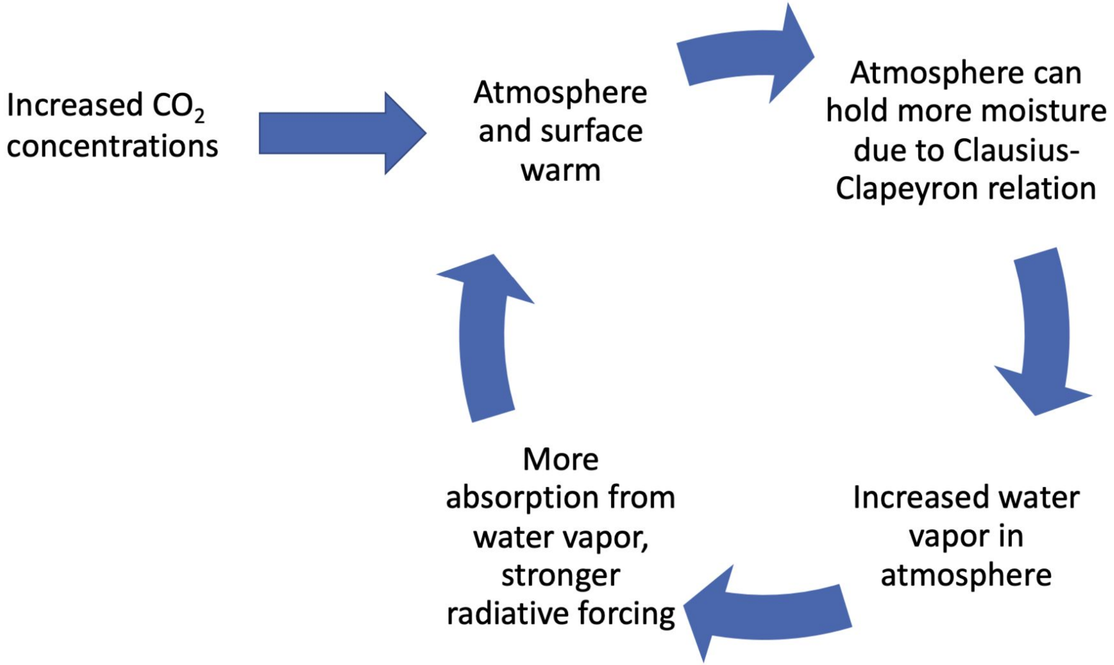
The water vapor feedback

First, a reminder: the Clausius-Clapeyron relation



The saturation water vapor increases exponentially with temperature, at about 6%/ °C

The water vapor feedback



Direct radiative forcing of absorption by water vapor molecules reinforces that by CO_2 via the water vapor feedback

- Climate cools/warms to reach radiative equilibrium: incoming SW = outgoing LW

- Adding greenhouse gases to the atmosphere traps LW radiation, so that incoming SW > outgoing LW, the Earth must warm to get back to equilibrium

- Atmosphere is mostly saturated wrt absorption by CO_2. Anthropogenic CO_2 causes warming because temperature decreases with height (lapse rate), raising the level of last absorption; & bec of increased absorption in margins of absorption bands

- CO_2 is a particularly effective greenhouse gas because it absorbs wavelengths of radiation at which the Earth emits most strongly and where H_2O is less effective

- Surface temperature response scales (only/weakly) logarithmically with CO_2 concentration: warming due to each CO_2 doubling is approximately the same

- Greenhouse effect of water vapor leads to a positive feedback on CO_2 increase

- The Global warming potential of other GHGs depends on both their efficiency and life time, CO_2 has an especially long life time in the atmosphere

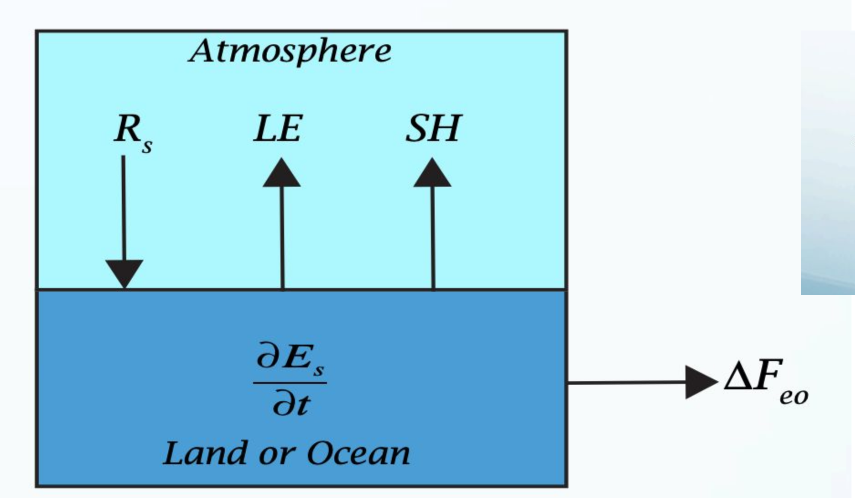
Workshop

Surface energy balance

Surface energy balance

The energy budget can be written in terms of energy flux per unit area [W/m²]

The processes that determine the energy transfer between the surface and the atmosphere are solar and infrared radiation, plus fluxes of energy associated with fluid motions of the atmosphere and oceans. The storage of heat below the surface is also important.



$$\frac{\partial E_s}{\partial t} = G = R_s - \mathrm{LE} - \mathrm{SH} - \Delta F_{\mathrm{eo}}$$

- **In equilibrium**, the storage is zero and the radiative heating of the surface is balanced by three terms.
- Storage = radiation – latent – sensible – horizontal

$$\frac{\partial E_s}{\partial t} = G = R_s - \mathrm{LE} - \mathrm{SH} - \Delta F_{\mathrm{eo}}$$

- becomes

$$R_s = LE + SH + \Delta F_{eo}$$

Heat Capacity

- The amount of energy it takes to heat up the surface by 1°K
- Energy in surface is heat capacity $\bar{C}_{\mathrm{eo}}$ ($\mathrm{J\ m^{-2}\ K^{-1}}$) times bulk surface temperature

$$E_s = \bar{C}_{\mathrm{eo}} T_{\mathrm{eo}}$$

Heat Capacity: Atmosphere

- The amount of energy it takes to heat up the surface by 1°K
- Heat Capacity of Atmosphere = total mass times specific heat at constant pressure

$$\overline{C}_a = c_p \frac{p_s}{g} = \frac{1004\,\mathrm{J\,K^{-1}\,kg^{-1}} \cdot 10^5\,\mathrm{Pa}}{9.81\,\mathrm{m\,s^{-2}}} = 1.02 \times 10^7\,\mathrm{J\,K^{-1}\,m^{-2}}$$

- So, if you apply 100 Wm^{-2} it heats up at rate of nearly one degree per day.

$$\boxed{\frac{dT_a}{dt} = \frac{F}{C_a} = \frac{100\,Wm^{-2}}{1.02 \times 10^7\,J\,K^{-1} m^{-2}} = 10^{-5}\,K\,s^{-1} \times 86400\,s\,day^{-1} = 0.86\,^\circ K\,day^{-1}}$$

Heat Capacity: Ocean

- Heat Capacity of Ocean = total mass times specific heat for some depth of ocean d_w

$$\bar{C}_o = \rho_w c_w d_w = 10^3 \mathrm{kg\,m^{-3}} \cdot 4218 \mathrm{J\,K^{-1}\,kg^{-1}} \cdot d_w$$
$$= d_w \cdot 4.2 \times 10^6 \mathrm{J\,K^{-1}\,m^{-2}\,m^{-1}}$$

- So, to have the same heat capacity as the atmosphere, you a little over two meters of ocean.

$$\bar{C}_a = 1.02 \times 10^7 \mathrm{J\,K^{-1}\,m^{-2}} = d_w \cdot 4.2 \times 10^6 \mathrm{J\,K^{-1}\,m^{-2}\,m^{-1}}$$

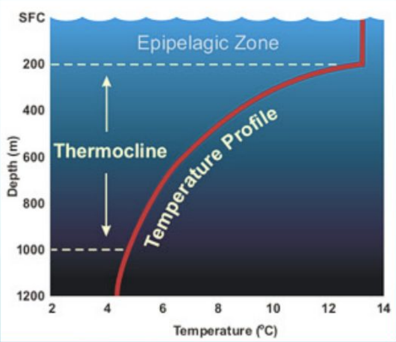
- when $d_w = 2.43 m$

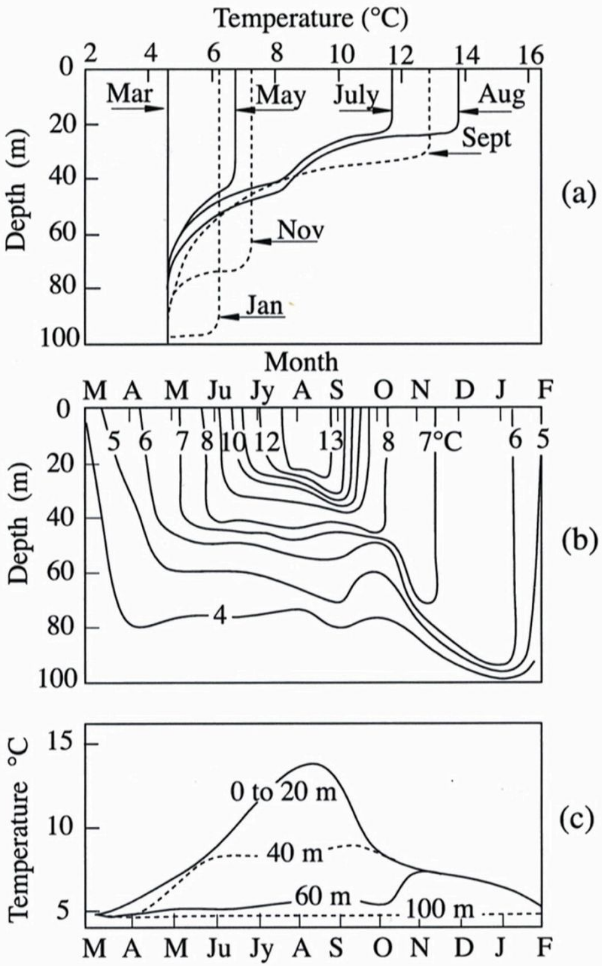
The Ocean Heat Capacity

- Covers the Earth to an average depth of 2650 meters, so about 1000 times the heat capacity of the atmosphere,
- But only about the first 70 meters of the ocean is well-mixed on time scales of seasons,
- So on time scales of seasons, the ocean mixed-layer has about 30 times the heat capacity of the atmosphere
- But the ocean has another longer time scale that it takes to heat the entire 2650 meters equivalent of global ocean heat capacity.

Ocean Mixed Layer

- From Chapter 7
- There’s always a layer on top that has almost the same temperature – well-mixed layer
- This is shallower in summer and deeper in winter





Land Heat Capacity

- Although the land is the most massive part of the climate system, because it is a solid, it can only transfer heat through conduction.
- As a result of it's low thermal transmissivity, the effective heat capacity of the land is generally less than that of the atmosphere.

Land Heat Capacity

- Diffusive heat flux goes down the gradient of temperature.

$$F_s = -K_T \frac{\partial T}{\partial z}$$

- The convergence of this heat flux drives a temperature tendency.

$$C_s \frac{\partial T}{\partial t} = -\frac{\partial}{\partial z}\left(F_s\right) = \frac{\partial}{\partial z}\left(K_T \frac{\partial T}{\partial z}\right)$$

- Define the thermal diffusivity of the surface material, and assume it is a constant

$$D_T = K_T / C_s$$

Land Heat Capacity

- Assuming that the thermal diffusivity is a constant

$$D_T = K_T / C_s$$

- we get,

$$\frac{\partial T}{\partial t} = D_T \frac{\partial^2 T}{\partial z^2}$$

- Do a little scale analysis

$$\frac{\Delta T}{\tau} = D_T \frac{\Delta T}{{h_T}^2}$$

- to get a characteristic depth

$$h_T = \sqrt{D_T \tau}$$

- Which depends on the square root of diffusivity and characteristic time scale

Characteristic Depth

- The characteristic depth of penetration is

$$h_T = \sqrt{D_T \tau}$$

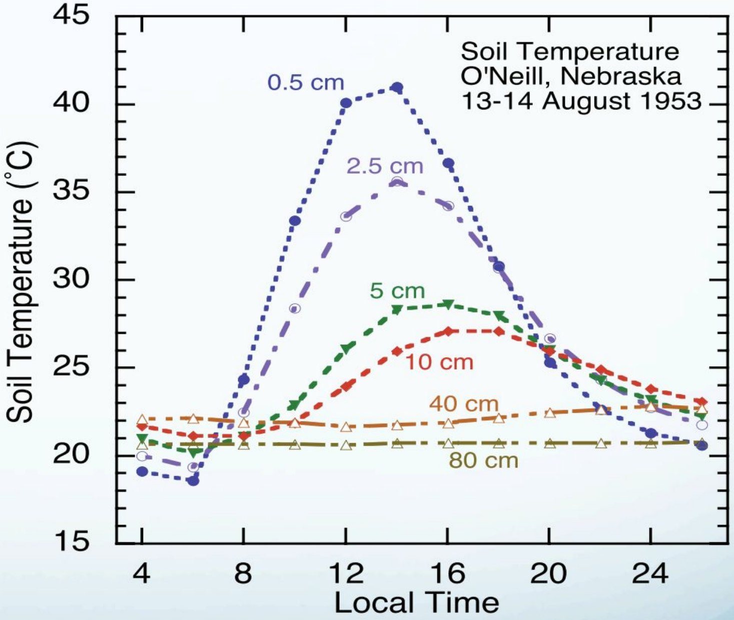
- For a diffusivity of $D_T = 5 \times 10^{-7}$ m^2 s^{-1}
- And a time period of one year
- We get a characteristic penetration depth of

$$h_T = \sqrt{D_T \tau} = \sqrt{5 \times 10^{-7}\, m^2 s^{-1} \times 86400\, s\, day^{-1} \times 365 days / 2\pi} = 1.6m$$

- for a diurnal variation of temperature this would be about 8 cm or 3 inches.

Heat Diffusion into soil

- For a grass field in Nebraska
- Signal amplitude is about e^{-1} by 10cm depth.
- Signal is delayed with depth



Surface Radiation

- The primary variable here is the surface albedo, which varies a lot.
- Surface emissivity is mostly high for natural surfaces.
- S = solar, and F = longwave

$$R_s = S^{\downarrow}(0) - S^{\uparrow}(0) + F^{\downarrow}(0) - F^{\uparrow}(0)$$

$$S^{\downarrow}(0) - S^{\uparrow}(0) = S^{\downarrow}(0)(1 - \alpha_s)$$

Surface Albedos

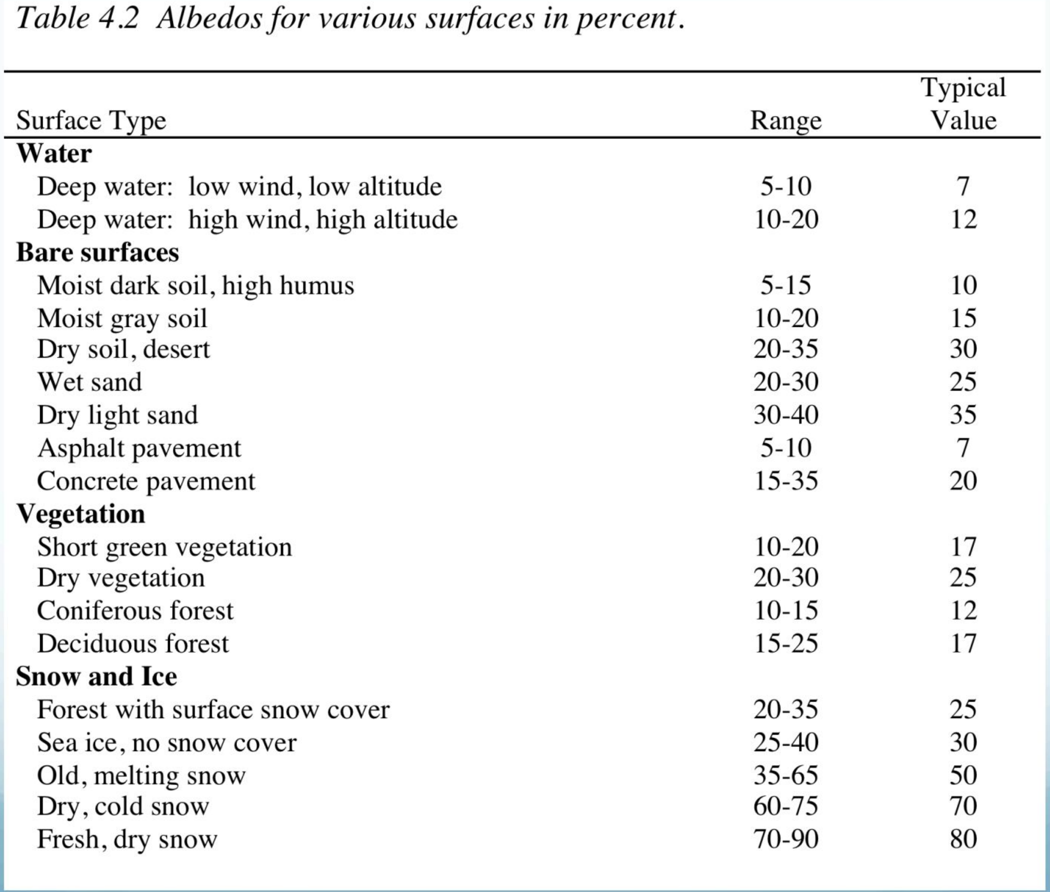
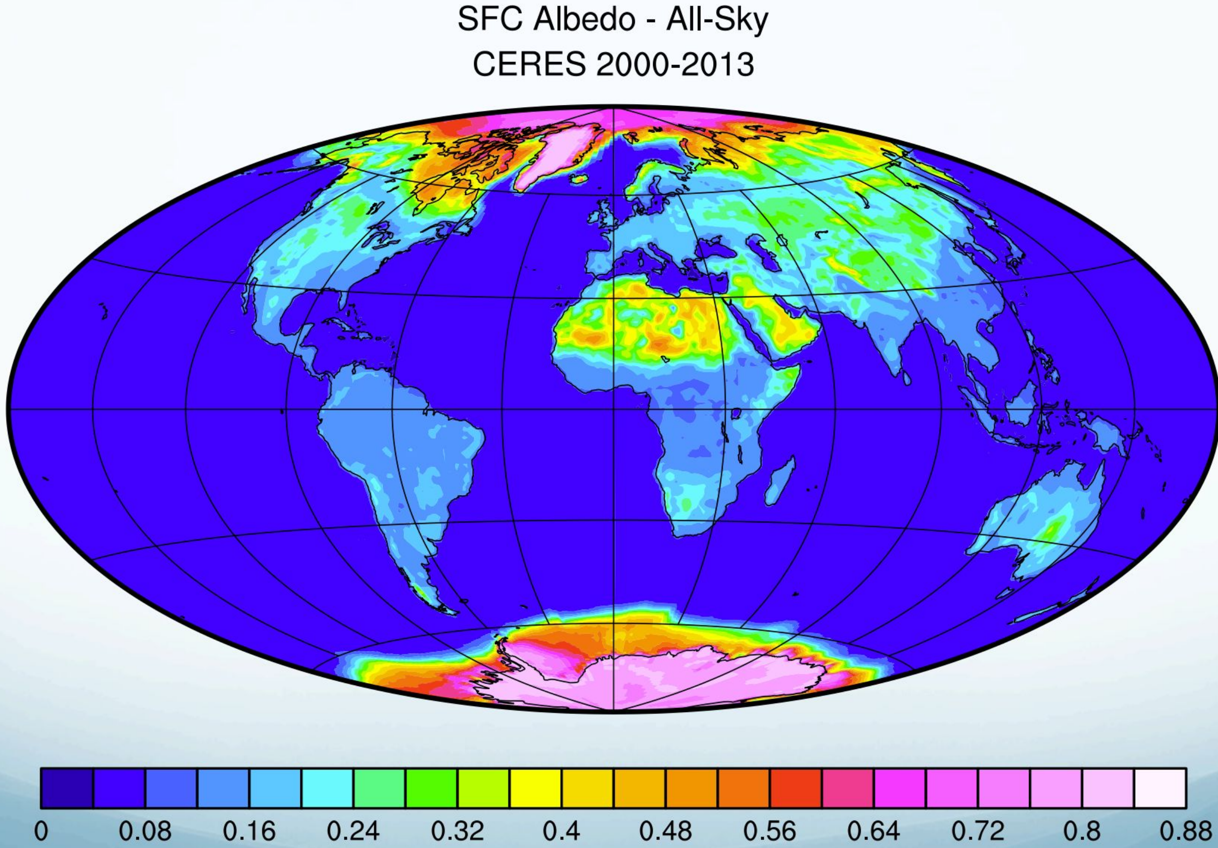


Table 4.2 Albedos for various surfaces in percent.

Surface Type	Range	Typical Value
Water		
Deep water: low wind, low altitude	5-10	7
Deep water: high wind, high altitude	10-20	12
Bare surfaces		
Moist dark soil, high humus	5-15	10
Moist gray soil	10-20	15
Dry soil, desert	20-35	30
Wet sand	20-30	25
Dry light sand	30-40	35
Asphalt pavement	5-10	7
Concrete pavement	15-35	20
Vegetation		
Short green vegetation	10-20	17
Dry vegetation	20-30	25
Coniferous forest	10-15	12
Deciduous forest	15-25	17
Snow and Ice		
Forest with surface snow cover	20-35	25
Sea ice, no snow cover	25-40	30
Old, melting snow	35-65	50
Dry, cold snow	60-75	70
Fresh, dry snow	70-90	80

Surface Albedo – “Observed”



Net Longwave at the ground

- The upward longwave flux at the surface is

$$F^{\uparrow}(0) = (1-\varepsilon)F^{\downarrow}(0) + \varepsilon\sigma T_s^4$$

- where ε is the emissivity of the surface
- The net downward longwave a the surface is

$$F^{\downarrow}(0) - F^{\uparrow}(0) = \varepsilon(F^{\downarrow}(0) - \sigma T_s^4)$$

Non-Radiative Fluxes

- Turbulence transports sensible and latent heat vertically in the boundary layer and above.
- The upward sensible heat flux is

$$\text{Upward sensible heat flux} = c_p \rho \overline{wT}$$

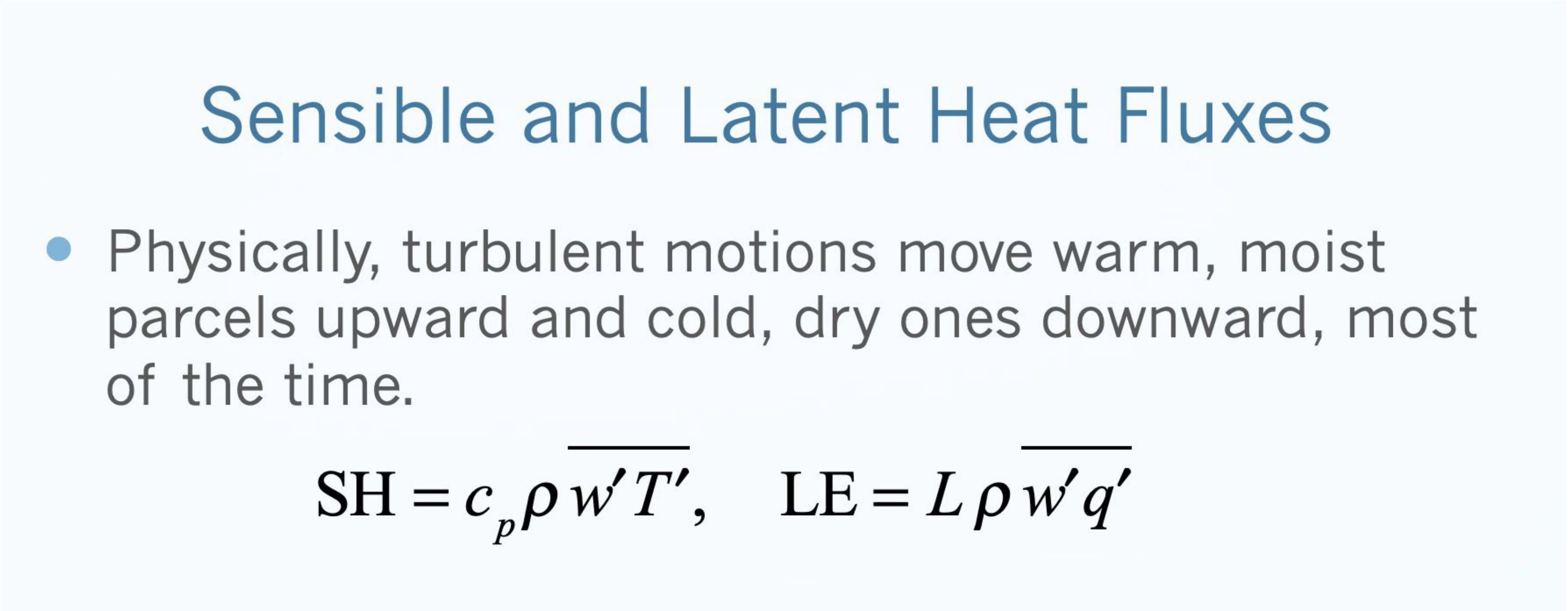
- Decompose variables into time mean and deviations.

$$w = \bar{w} + w', \quad T = \bar{T} + T'$$

- then,

$$\overline{wT} = \bar{w}\bar{T} + \overrightarrow{w'T'}$$

$$\text{Total} = \text{mean} + \text{eddy}$$



Sensible and Latent Heat Fluxes

- Physically, turbulent motions move warm, moist parcels upward and cold, dry ones downward, most of the time.

$$\mathrm{SH} = c_p \rho \overline{w'T'}, \quad \mathrm{LE} = L \rho \overline{w'q'}$$

https://www.youtube.com/watch?v=STkDRoaTfPs&t=1s

Sensible heat is heat exchanged by a body or thermodynamic system in which the exchange of heat changes the temperature of the body or system, and some macroscopic variables of the body or system, but leaves unchanged certain other macroscopic variables of the body or system, such as volume or pressure.

Latent heat (also known as **latent energy** or **heat of transformation**) is energy released or absorbed, by a body or a thermodynamic system, during a constant-temperature process—usually a first-order phase transition, like melting or condensation.

Latent heat can be understood as hidden energy which is supplied or extracted to change the state of a substance without changing its temperature or pressure. This includes the latent heat of fusion (solid to liquid), the latent heat of vaporization (liquid to gas) and the latent heat of sublimation (solid to gas).

The terms *sensible heat* and *latent heat* refer to energy transferred between a body and its surroundings, defined by the occurrence or non-occurrence of temperature change; they depend on the properties of the body. Sensible heat is sensed or felt in a process as a change in the body's temperature. Latent heat is energy transferred in a process without change of the body's temperature, for example, in a phase change (solid/liquid/gas).

Sensible heat flux

Sensible heat flux represents the heat exchanged between the surface and the atmosphere due to temperature differences.

$$Q_s = \rho\, C_p \,\frac{(T_s - T_a)}{r_s}$$

Where:

- Q_s = sensible heat flux (W/m²)
- ρ = air density (kg/m³)
- C_p = specific heat capacity of air at constant pressure (J/(kg·K))
- T_s = surface temperature (K)
- T_a = air temperature at a reference height (K)
- r_s = resistance to heat transfer (s/m)

$$Q_s = C_H\, \rho\, u\, (T_s - T_a)$$

Where:

- C_H = bulk transfer coefficient for heat (m/s)
- u = wind speed at the surface (m/s)

Latent heat flux

Latent heat flux refers to the heat exchanged due to the phase change of water (e.g., evaporation or condensation) at the surface.

$$Q_l = \rho \, L_v \, \frac{(q_s - q_a)}{r_l}$$

Where:

- Q_l = latent heat flux (W/m²)
- ρ = air density (kg/m³)
- L_v = latent heat of vaporization (J/kg)
- q_s = specific humidity at the surface (kg/kg)
- q_a = specific humidity at a reference height (kg/kg)
- r_l = resistance to water vapor transfer (s/m)

$$Q_l = C_E \, \rho \, u \, (q_s - q_a)$$

Where:

- C_E = bulk transfer coefficient for water vapor (m/s)
- q_s = specific humidity at the surface (kg/kg)
- q_a = specific humidity at a reference height (kg/kg)
- u = wind speed at the surface (m/s)

Summary

- **Sensible Heat Flux (Q_s)** is mainly influenced by the temperature difference between the surface and the air, along with the wind speed and air density.
- **Latent Heat Flux (Q_l)** depends on the difference in specific humidity between the surface and the air, and is also influenced by wind speed and air density.

Both fluxes play critical roles in the energy balance at the Earth's surface and contribute to atmospheric processes like convection, cloud formation, and weather patterns.