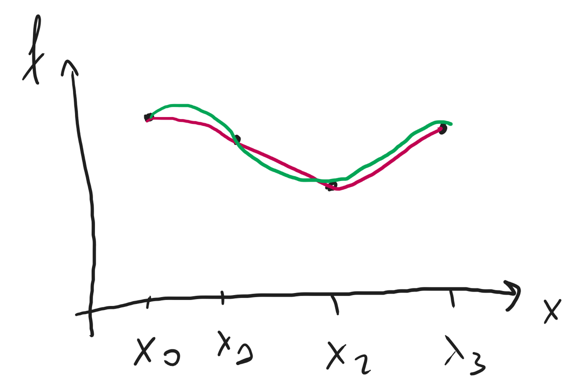
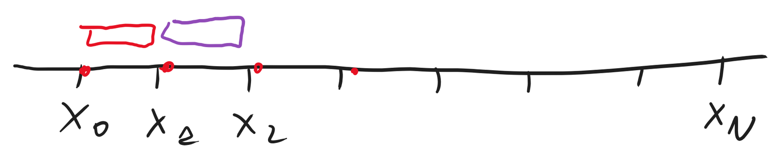
INTERPOLAZIONE



SPLINES

$\forall\ [x_i, x_{i+1}]$ $\quad f(x) = \sum_{k=0,m} c_{i,k} x^k \quad$ per $x \in [x_i, x_{i+1}]$

imponiamo che $f(x)$ sia continua con derivate continue fino all'ordine $m-1$, $f(x)$ passa per i punti dati



Condizioni sulla $f(x)$

se $x \in [x_i, x_{i+1}]$ $\quad f(x) = P_i(x) = \sum_{k=0,m} c_{i,k} x^k$

$$P_i(x_i) = f_i \qquad i = 0, N-1$$

$$P_{N-1}(x_N) = f_N$$

$$P_i^{(\ell)}(x_{i+1}) = P_{i+1}^{(\ell)}(x_{i+1}) \qquad P_i^{(\ell)}(x) = \frac{d^\ell}{dx^\ell} P_i(x)$$

$\ell = 0, m-1$

$i = 0, N-2$

Numero parametri liberi $(m+1)N$

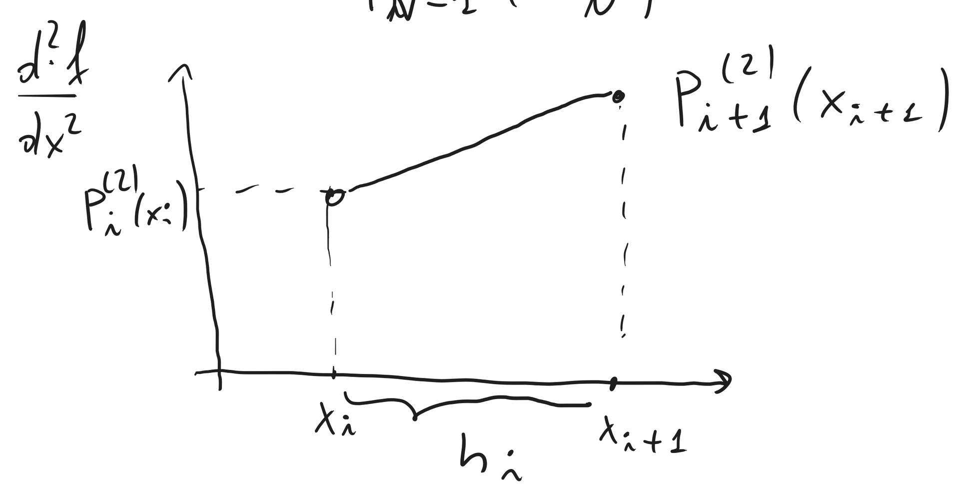
Numero dei vincoli: $N+1 + m(N-1) = (m+1)N - (m-1)$

Abbiamo $m-1$ parametri liberi

SPPLINES cubiche con scelta NATURALE delle condizioni aggiuntive

$$P_0^{(2)}(x_0) = 0$$

$$\text{e}\ \ P_{N-1}^{(2)}(x_N) = 0$$



$$h_i = x_{i+1} - x_i$$

$$P_i^{(2)}(x) = \frac{P_{i+1}^{(2)}}{h_i}(x - x_i) - \frac{(x - x_{i+1})}{h_i} P_i^{(2)}$$

$$P_i(x) = \alpha_i (x - x_i)^3 + \beta_i (x - x_{i+1})^3 + \gamma_i (x - x_i) + \eta_i (x - x_{i+1})$$

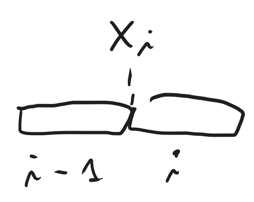
$$\frac{P_{i+1}^{(2)}}{h_i} = 6\alpha_i \qquad 6\beta_i = -\frac{P_i^{(2)}}{h_i}$$

$$P_i(x_{i+1}) = P_{i+1}(x_{i+1}) = f_{i+1}$$

$$h_i \gamma_i + \alpha_i h_i^3 = f_{i+1} \implies \gamma_i = \frac{f_{i+1}}{h_i} - \alpha_i h_i^2$$

$$P_i(x_i) = f_i$$

$$f_i = -\beta_i h_i^3 - \eta_i h_i$$



$$\eta_i = -\beta_i h_i^2 - \frac{f_i}{h_i}$$

$$3\alpha_{i-1} h_{i-1}^2 + \gamma_{i-1} + \eta_{i-1} = 3\beta_i h_i^2 + \gamma_i + \eta_i$$

$$h_{i-1} \underbrace{P_{i-1}^{(2)}} + 2(h_{i-1} + h_i) \underbrace{P_i^{(2)}} + h_i \underbrace{P_{i+1}^{(2)}} = 6\left(\frac{y_i}{h_i} - \frac{y_{i-1}}{h_{i-1}}\right)$$

con $y_i = f_{i+1} - f_i$

$P_0^{(2)} = 0$

$$\begin{pmatrix} d_1 & h_1 & & \\ h_1 & d_2 & h_2 & \\ & & \ddots & \\ & & & \end{pmatrix} \begin{pmatrix} P_1^{(2)} \\ P_2^{(2)} \\ \vdots \\ P_{N-1}^{(2)} \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ \vdots \\ b_N \end{pmatrix}$$

con $d_i = 2(h_{i-1} + h_i$

con $b_i = 6\left(\frac{y_i}{h_i} - \frac{y_{i-1}}{h_{i-1}}\right)$

$$\vec{A} \cdot \vec{P} = \vec{B} \qquad \vec{P} = A^{-1} \vec{B}$$