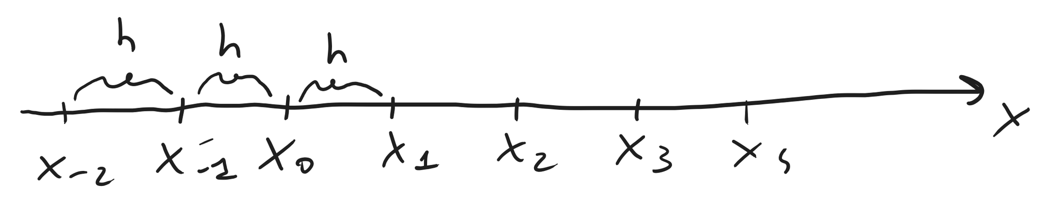
Consideriamo una funzione $f:\mathbb{R}\to\mathbb{R}$

Discretizziamo l'asse x

$$x_i = h i + x_{offset}$$



$$f_i = f(x_i)$$

$x_{i+\varepsilon}$ con $\varepsilon \in [0,1[$ $\quad x_{i+\varepsilon} = x_i + h\varepsilon$

$$f_{i+\varepsilon} = f(x_{i+\varepsilon})$$

$$f_i^n = \left.\frac{d^n f}{dx^n}\right|_{x=x_i}$$

DERIVIAMO f

$$f_i^1 = \lim_{\Delta x\to 0} \frac{f(x_i+\Delta x)-f(x_i)}{\Delta x}$$

$$f_i^1 \simeq \frac{f_{i+1}-f_i}{h} = \tilde{f}_i^1$$

determiniamo l'accuratezza di tale formula

$$f_{i+1} = f_i + f_i^1 h + O(h^2)$$

$$\tilde{f}_i^1 = \frac{\cancel{f_i} + f_i^1 h + O(h^2) - \cancel{f_i}}{h} = f_i^1 + O(h)$$

FORMULA DERIVATA DESTRA

$$f_i^1 = \frac{f_{i+1}-f_i}{h} + O(h)$$

FORMULA DERIVATA SINISTRA

$$f_i^1 = \frac{f_i - f_{i-1}}{h} + O(h)$$

FORMULA $O(h^2)$

$$\begin{cases} f_{i+1} = f_i + f_i^1 h + \frac{1}{2} f_i^2 h^2 + O(h^3) & (I) \\ f_{i-1} = f_i - f_i^1 h + \frac{1}{2} f_i^2 h^2 + O(h^3) & (II) \end{cases}$$

(I) − (II)

$$f_{i+1} - f_{i-1} = 2 f_i^1 h + O(h^3)$$

$$\boxed{f_i^1 = \frac{f_{i+1} - f_{i-1}}{2h} + O(h^2)}$$

$$f_{i+1} = f_i + f_i^1 h + \frac{1}{2} f_i^2 h^2 + \frac{1}{3!} f_i^3 h^3 + O(h^4) \qquad (I)$$

$$f_{i-1} = f_i - f_i^1 h + \frac{1}{2} f_i^2 h^2 - \frac{1}{3!} f_i^3 h^3 + O(h^4) \qquad (II)$$

(I) + (II)

$$f_{i+1} + f_{i-1} = 2 f_i + f_i^2 h^2 + O(h^4)$$

$$\boxed{f_i^2 = \frac{f_{i+1} - 2 f_i + f_{i-1}}{h^2} + O(h^2)}$$

GENERALIZZAZIONE

es.

$$\begin{cases} f_{i+1} = f_i + f_i^1 h + \frac{1}{2} f_i^2 h^2 \\ f_i = f_i \\ f_{i-1} = f_i - f_i^1 h + \frac{1}{2} f_i^2 h^2 \end{cases}$$

$$\begin{pmatrix} f_{i+1} \\ f_i \\ f_{i-1} \end{pmatrix} = \begin{pmatrix} 1 & h & h^2/2 \\ 1 & 0 & 0 \\ 1 & -h & h^2/2 \end{pmatrix} \begin{pmatrix} f_i \\ f_i^1 \\ f_i^2 \end{pmatrix}$$

$$\begin{pmatrix} f_i \\ f_i^1 \\ f_i^2 \end{pmatrix} = \begin{pmatrix} 1 & h & h^2/2 \\ 1 & 0 & 0 \\ 1 & -h & h^2/2 \end{pmatrix}^{-1} \begin{pmatrix} f_{i+1} \\ f_i \\ f_{i-1} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 1/2h & 0 & -1/2h \\ 1/h^2 & -2/h^2 & 1/h^2 \end{pmatrix} \begin{pmatrix} f_{i+1} \\ f_i \\ f_{i-1} \end{pmatrix}$$

$$\begin{pmatrix} f_i \\ f_i^1 \\ f_i^2 \end{pmatrix} = \begin{pmatrix} f_i \\ (f_{i+1} - f_{i-1})/2h \\ (f_{i+1} - 2 f_i + f_i)/h^2 \end{pmatrix}$$

f_{i+2}