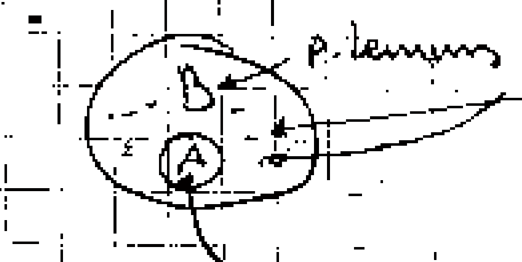
TH. 4.1: Let L be a regular language. Then there exist a constant n (which depends on L) such that for every string w in L such that $|w| \geq n$, we can break w into three strings $w = xyz$:

= FACTORIZE

- $y \neq \epsilon$
- $|xy| \leq n$
- For all $k \geq 0$, the string xy^kz is also in L $(xy^kz \in L, \forall k \geq 0)$

NOTE: if L is REG (A) then $(\Rightarrow)$ p. Lemma property (B) $\{A \Rightarrow B\}$

Never: p. lemma (B) $\Rightarrow$ Reg (A) (NOT TRUE) $\{$ ~~B ⇒ A~~ not true $\}$



Some languages that has pumping lemma are not REG

If the language it's regular so it must satisfy the pumping lemma.

+ PROOF PUMPING LEMMA

- Since works for any string when apply choose every string
- show that exist at least one

I: We assume the language has the Reg. ℓ, but we use counter-proof to show that hypothesis it's wrong

- We have to proove $\forall k \geq 0$ s.t. $w_k \in L$

+ COUNTER PROOF^I $(\neg B \Rightarrow \neg A)$

- we have $\forall w$, so we need to find just one that doesn't work
- we have $\exists w$, so we need to show that for every doesn't work. (to do it make sure to choose a w in good way, it means you have to manage 4/3 case maximum)
- we have to proove $\exists k \geq 0$ s.t. $w_k \notin L$

PROOF

Suppose L is a regular language. Then L is recognized by some DFA A, say, n states.

Now consider any string of length n or more, say $w = a_1 a_2 \dots a_m \in L$ with $m \geq n$

Let $p_i = \hat{\delta}(q_0, a_1 a_2 \dots a_i)$ for each $i = 0, 1 \dots n$

There exists $i \leq j \leq n$ such that $p_i = p_j$, because for the pigeonhole principle it's not possible for the $n+1$ different p_i's for $i = 0, 1 \dots n$ to be distinct since there are only n different states.

Let us write $w = xyz$, where:
1) $x = a_1 a_2 \dots a_i$
2) $y = a_{i+1} a_{i+2} \dots a_j$
3) $z = a_j a_{j+1} a_{j+2} \dots a_m$

NOTE:
- x maybe empty if $i = 0$
- y can not empty since $i < j$
- z maybe empty if $j = n = m$