Derivate delle principali funzioni elementari

Nel seguito $\alpha \in \mathbb{R}$ è un numero reale, $q \in \mathbb{Q}$ è un numero razionale.

$$(e^{\alpha x})' = \alpha e^{\alpha x} \qquad (\log(\alpha x))' = \frac{1}{x} \qquad (\sinh(\alpha x))' = \alpha \cosh(\alpha x) \qquad (\cosh(\alpha x))' = \alpha \sinh(\alpha x)$$

$$(\sin(\alpha x))' = \alpha \cos(\alpha x) \qquad (\cos(\alpha x))' = -\alpha \sin(\alpha x) \qquad (\tan(\alpha x))' = \alpha(1+\tan^2(\alpha x)) = \frac{\alpha}{\cos^2(\alpha x)}$$

$$(\arcsin(\alpha x))' = \frac{\alpha}{\sqrt{1-\alpha^2 x^2}} \qquad (\arccos(\alpha x))' = -\frac{\alpha}{\sqrt{1-\alpha^2 x^2}} \qquad (\arctan(\alpha x))' = \frac{\alpha}{1+\alpha^2 x^2}$$

$$(x^q)' = qx^{q-1} \text{ (per ogni } x \text{ se } q = \frac{m}{n} \text{ con } n \text{ dispari, o } m \text{ pari, solo per } x > 0 \text{ negli altri casi)}$$

$$x^\alpha = \alpha x^{\alpha-1} \text{ (solo per } x > 0).$$

Primitive delle principali funzioni elementari

Nel seguito $\alpha, \beta \in \mathbb{R}$ (sia positivi che negativi), $c \in \mathbb{R}$, $a > 0, b > 0$ (a, b numeri reali positivi).

$$\int e^{\alpha x} dx = \frac{1}{\alpha} e^{\alpha x} + c \qquad \int \sinh(\alpha x) dx = \frac{1}{\alpha} \cosh(\alpha x) + c \qquad \int \cosh(\alpha x) dx = \frac{1}{\alpha} \sinh(\alpha x) + c$$

$$\int \sin(\alpha x) dx = -\frac{1}{\alpha} \cos(\alpha x) + c \qquad \int \cos(\alpha x) dx = \frac{1}{\alpha} \sin(\alpha x) + c$$

$$\int \frac{1}{\cos^2(\alpha x)} dx = \frac{1}{\alpha} \tan(\alpha x) + c \qquad \int \frac{1}{\sin^2(\alpha x)} dx = -\frac{1}{\alpha} \frac{1}{\tan(\alpha x)} + c$$

$$\int x^k dx = \frac{1}{k+1} x^{k+1} \text{ (per } k \neq -1) \qquad \int x^{-1} dx = \int \frac{1}{x} dx = \log|x| + c.$$

$$\int (\alpha x + \beta)^k dx = \frac{1}{\alpha(k+1)} (\alpha x + \beta)^{k+1} + c \text{ (per } k \neq -1) \qquad \int \frac{1}{\alpha x + \beta} dx = \frac{1}{\alpha} \log|\alpha x + \beta| + c$$

$$\int \frac{1}{ax^2 + b} dx = \frac{1}{\sqrt{a}\sqrt{b}} \arctan\left(\frac{\sqrt{a}}{\sqrt{b}}\, x\right) + c = \frac{1}{\sqrt{ab}} \arctan\left(\sqrt{\frac{a}{b}}\, x\right) + c \qquad \textbf{se } a, b > 0$$

$$\int \frac{1}{\sqrt{b - ax^2}} dx = \frac{1}{\sqrt{a}} \arcsin\left(\frac{\sqrt{a}}{\sqrt{b}}\, x\right) + c = \frac{1}{\sqrt{a}} \arcsin\left(\sqrt{\frac{a}{b}}\, x\right) + c \qquad \textbf{se } a, b > 0$$

$$\int \frac{x}{ax^2 + b} dx = \frac{1}{2a} \log\left|ax^2 + b\right| + c.$$

Primitive che si trovano con integrazione per parti o con sostituzioni.

$$\int \log(\alpha x) dx = x \log(\alpha x) - x + c \qquad \int \tan(\alpha x) dx = -\frac{1}{\alpha} \log|\cos(\alpha x)| + c$$

$$\int \arctan(\alpha x) dx = x \arctan(\alpha x) - \frac{1}{2\alpha} \log(1 + \alpha^2 x^2) + c$$

$$\int \arcsin(\alpha x) dx = x \arcsin(\alpha x) + \frac{1}{\alpha} \sqrt{1-\alpha^2 x^2} + c \qquad \int \arccos(\alpha x) dx = x \arccos(\alpha x) - \frac{1}{\alpha} \sqrt{1-\alpha^2 x^2} + c.$$

$$\int \sqrt{b - ax^2} dx = \frac{b}{2\sqrt{a}} \arcsin\left(\sqrt{\frac{a}{b}}\, x\right) + \frac{1}{2} x \sqrt{b - ax^2} + c$$