1. **[4 points]** Consider the regular expression \( R = (ab + ba)^*\emptyset(aa) \). Convert \( R \) into an equivalent \( \epsilon \)-NFA using the construction provided in the textbook, and report all the intermediate steps. **Important:** do not simplify the regular expression \( R \) before applying the construction.

2. **[9 points]** Let \( \Sigma = \{a, b, c\} \). For \( w \in \Sigma^* \) and \( X \in \Sigma \), we write \( \#_X(w) \) to denote the number of occurrences of \( X \) in \( w \). Consider the following languages

\[
L_1 = \{ w \mid w \in \Sigma^*, \#_a(w) = \#_b(w) = \#_c(w) \};
\]

\[
L_2 = \{ w \mid w \in \Sigma^*, \#_a(w) = \#_c(w) \}.
\]

(a) Prove that \( L_1 \) is outside of CFL.

(b) Prove that \( L_2 \) is in CFL.

(c) Prove that \( L_2 \) is not in REG.

3. **[5 points]** Consider the CFG \( G \) implicitly defined by the following productions:

\[
S \to AAB \mid ABB \mid BBB \\
A \to aAB \mid bBB \\
B \to b \mid \epsilon
\]

Perform on \( G \) the transformations indicated below, that have been specified in the textbook, in the given order. Report the CFGs obtained at each of the intermediate steps.

(a) Eliminate the \( \epsilon \)-productions

(b) Eliminate the unary productions

(c) Eliminate the useless symbols

(d) Produce a CFG in Chomsky normal form equivalent to \( G \).

(please turn to the next page)
4. [5 points] Assess whether the following statements are true or false, providing motivations for all of your answers.

(a) Let $L_1$ be a language in $\text{REG}$ with $L_1$ non-finite, and let $L_2$ be a language in $\text{CFL} \setminus \text{REG}$. The language $L_1 \cap L_2$ may be in $\text{CFL} \setminus \text{REG}$.

(b) Let $L_1$ be a language in $\text{REG}$ with $L_1$ non-finite, and let $L_2$ be a language in $\text{CFL} \setminus \text{REG}$. The language $L_1 \cap L_2$ may be in $\text{REG}$.

(c) Let $L_1, L_2$ be languages in $\text{CFL}$. The language $L_1 \cap L_2$ belongs to $\mathcal{P}$, the class of languages that can be recognized in polynomial time by a TM.

(d) Let $R$ be the string reversal operator, which we extend to languages. Let $L$ be a language in $\text{REC}$. Then $L^R$ belongs to $\text{REC}$.

5. [4 points] Define the diagonalization language $L_d$. Show that $L_d$ is not an $\text{RE}$ language, using the proof reported in the textbook.

6. [6 points] Consider the following property of the RE languages defined over the alphabet $\Sigma = \{0, 1\}$:

$$\mathcal{P} = \{L \mid L \in \text{RE}, \text{every string in } L \text{ has even length}\}$$

and define $L_\mathcal{P} = \{\text{enc}(M) \mid L(M) \in \mathcal{P}\}$.

(a) Use Rice’s theorem to show that $L_\mathcal{P}$ is not in $\text{REC}$.

(b) Prove that $L_\mathcal{P}$ is not in $\text{RE}$. 