Le31

Thursday, 6 June 2024 08:44

APPELLO 12 LUGLIO 2021

ES 1 LAPLACE

$x(t) = \delta(t) + 2e^{-t}1(t)$

$Y_f(s) = \frac{1}{s(s+a)}$

$y_f(t) = 1 \quad t \gg 0$

1) IDENTIFICARE a
2) TROVARE $H(s)$
3) TROVARE L'EQ. DIFFERENZIALE ASSOCIATA
4) DIRE SE IL SISTEMA È BIBO STABILE
5) EVOLUZIONE LIBERA CON $y(0_-) = y'(0_-) = 0$, $y''(0_-) = 1$

COND. INIZIALI NULLE! $y_\ell(t) = 0$

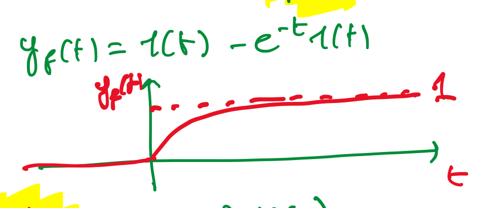
$Y_f(s) = \frac{1}{s(s+a)} = \frac{R_0}{s} + \frac{R_1}{s+a} = \frac{1/a}{s} - \frac{1/a}{s+a}$ polo $-a$

$y_f(t) = \frac{1}{a}1(t) - \frac{1}{a}e^{-at}1(t)$

$\lim_{t\to\infty} y_f(t) = 1 \rightarrow a \neq 0, \; a > 0$

$a = 1$

$y_f(t) = 1(t) - e^{-t}1(t)$



$Y_f(s) = \frac{1}{s(s+1)} = H(s)X(s)$

$x(t) = \delta(t) + 2e^{-t}1(t)$

$X(s) = 1 + 2\frac{1}{s+1} = \frac{s+3}{s+1}$

$H(s) = \frac{Y_f(s)}{X(s)} = \frac{1}{s(s+1)} \cdot \frac{1}{\frac{s+3}{s+1}} = \frac{1}{s(s+3)}$

$p_1 = 0$, $p_2 = -3$

NON BIBO STABILE

NOTA: CHE INGRESSO LIMITATO MI PUÒ DARE UN'USCITA NON LIMITATA?

$x_1(t) = e^{p_2 t}1(t) \longrightarrow X_1(s) = \frac{1}{s-p_2}$ $\quad Re[p_2] \le 0$

$X_1(s)H(s) = \frac{1}{s(s+3)(s-p_2)} = \frac{R_0}{s} + \frac{R_1}{s+3} + \frac{R_2}{s-p_2}$ NON DIVERGE

(A) $p_2 \neq 0, -3$

(B) $p_2 = 0$

$X_1(s)H(s) = \frac{1}{s^2(s+3)} = \frac{R_0}{s} + \frac{R_1}{s^2} + \frac{R_2}{s+3}$

$\to R_1 t 1(t)$ DIVERGE!

(C) $p_2 = -3$

$X_1(s)H(s) = \frac{1}{s(s+3)^2} = \frac{R_0}{s} + \frac{R_1}{s+3} + \frac{R_2}{(s+3)^2}$

$\to te^{-3t}1(t)$ NON DIVERGE

$H(s) = \frac{1}{s(s+3)} = \frac{1}{s^2+3s}$ GRADO $n=2$

$x(t) = y''(t) + 3y'(t)$

ES 2 TEOREMA DEL CAMPIONAMENTO

$x(t) = \text{sinc}\left(\frac{t+1}{3}\right)\cos(2t) - \frac{\pi}{12}\sin(4t)$

($x_1(t)$ = primo termine, $x_2(t)$ = secondo termine)

1) $X(j\omega) = ?$
2) PASSO CAMPIONAMENTO T PER POTER RICOSTRUIRE IL SEGNALE
3) T SE $x(t)$ VIENE PREFILTRATO CON $H(j\omega) = \begin{cases} 1 & 1<|\omega|<3 \\ 0 & \text{altrove} \end{cases}$

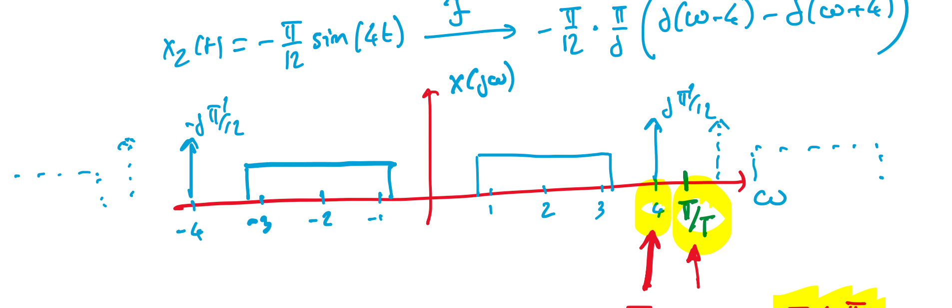
$\text{sinc}(t/3) \xrightarrow{\mathcal{F}} 3\,\text{rect}\left(\frac{3\omega}{2\pi}\right)$

$\text{sinc}\left(\frac{t+1}{3}\right) \xrightarrow{\mathcal{F}} 3\,\text{rect}\left(\frac{3\omega}{2\pi}\right)e^{+j\omega \cdot 1}$

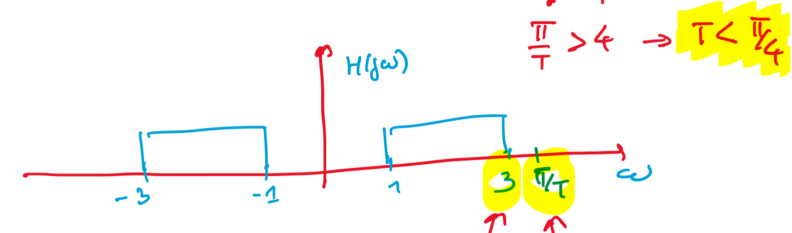
TRASL $t_0 = -1$

$x_1(t) \xrightarrow{\mathcal{F}} \frac{3}{2}\text{rect}\left(\frac{3(\omega-2)}{2\pi}\right)e^{j(\omega-2)} + \frac{3}{2}\text{rect}\left(\frac{3(\omega+2)}{2\pi}\right)e^{j(\omega+2)}$

$x_2(t) = -\frac{\pi}{12}\sin(4t) \xrightarrow{\mathcal{F}} -\frac{\pi}{12}\cdot\frac{\pi}{j}\left(\delta(\omega-4) - \delta(\omega+4)\right)$



$\frac{\pi}{T} > 4 \rightarrow T < \frac{\pi}{4}$



$\frac{\pi}{T} > 3 \rightarrow T < \frac{\pi}{3}$

ES 3 TRASF. DI FOURIER DISCRETA

$x(n) = \left(\frac{1}{4}\right)^{|n|}\cos\left(\frac{\pi}{5}n\right)$ (con $y(n) = \left(\frac{1}{4}\right)^{|n|}$)

TROVARE $X(e^{j\theta}) = ?$ È REALE? SÌ. È PARI? SÌ.

$X(e^{j\theta}) = \frac{1}{2}Y(e^{j(\theta-\pi/5)}) + \frac{1}{2}Y(e^{j(\theta+\pi/5)})$

$Y(e^{j\theta}) = \sum_{n=-\infty}^{+\infty}\left(\frac{1}{4}\right)^{|n|}e^{-jn\theta}$

$= \sum_{n=0}^{+\infty}\left(\frac{1}{4}\right)^{n}e^{-jn\theta} + \sum_{n=-\infty}^{-1}\left(\frac{1}{4}\right)^{-n}e^{-jn\theta}$

$\left(\frac{1}{4}e^{-j\theta}\right)^n \qquad \left(\frac{1}{4}e^{j\theta}\right)^{-n}$

$= \sum_{n=0}^{+\infty}\left(\frac{1}{4}e^{-j\theta}\right)^n + \sum_{m=0}^{\infty}\left(\frac{1}{4}e^{j\theta}\right)^m - 1$

$= \frac{4}{4-e^{-j\theta}} + \frac{4}{4-e^{j\theta}} - 1$

$= \frac{4(4-e^{j\theta}) + 4(4-e^{-j\theta}) - (4-e^{-j\theta})(4-e^{j\theta})}{(4-e^{-j\theta})(4-e^{j\theta})}$

$= \frac{16 - 4e^{j\theta} - 4e^{-j\theta} - 1}{16+1-4(e^{j\theta}+e^{-j\theta})}$

$= \frac{15}{17-8\cos(\theta)}$

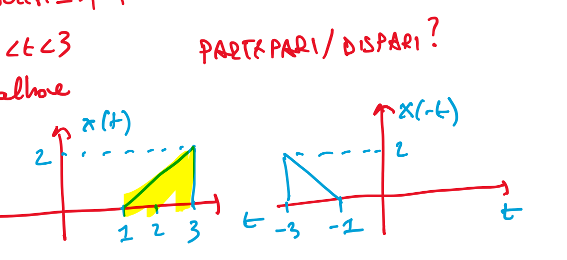
$X(e^{j\theta}) = \frac{15/2}{17-8\cos(\theta-\pi/5)} + \frac{15/2}{17-8\cos(\theta+\pi/5)}$

$X(e^{-j\theta}) = \frac{15/2}{17-8\cos(-\theta+\pi/5)} + \frac{15/2}{17-8\cos(-\theta-\pi/5)} = X(e^{j\theta})$ PARI!

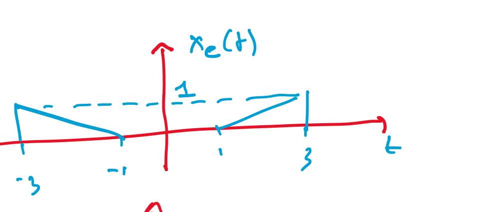
ES SIMMETRIE 301 di esercizi1.pdf

PARTE PARI / DISPARI?

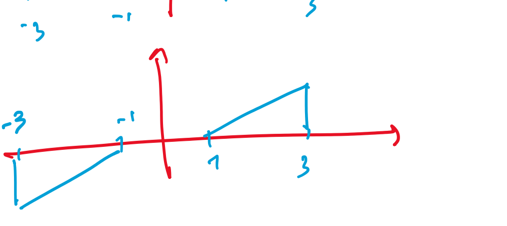
1) $x(t) = \begin{cases} t-1 & 1<t<3 \\ 0 & \text{altrove} \end{cases}$



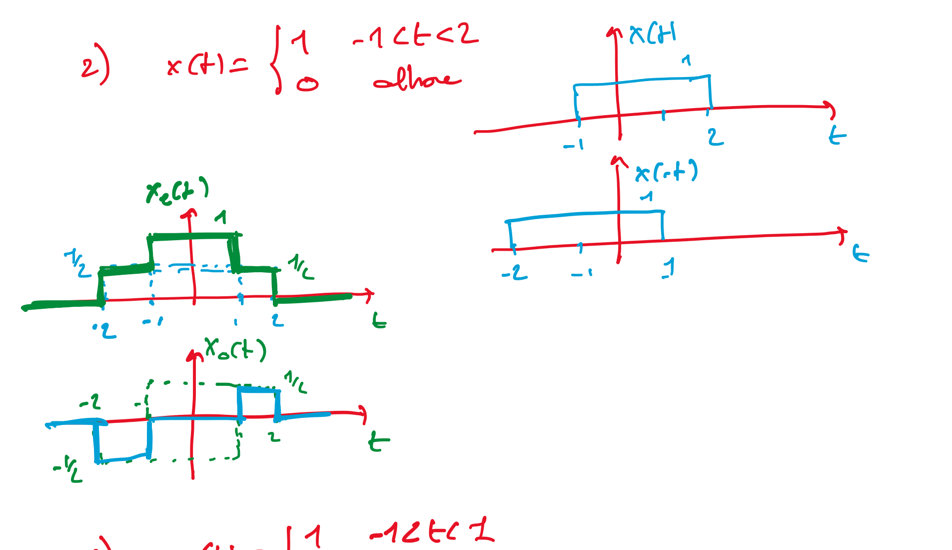
$x_e(t) = \frac{x(t)+x(-t)}{2}$



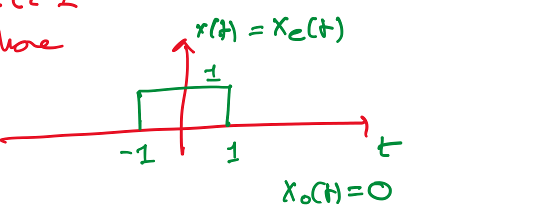
$x_o(t) = \frac{x(t)-x(-t)}{2}$



2) $x(t) = \begin{cases} 1 & -1<t<2 \\ 0 & \text{altrove} \end{cases}$



4) $x(t) = \begin{cases} 1 & -1<t<1 \\ 0 & \text{altrove} \end{cases}$



$x_o(t) = 0$

ES MATLAB

T passo camp.

x vettore campioni

```
N = length(x)
t = (0:N-1)*T
X = T*fft(x)
om = (0:N-1)*2*pi/T/N

X = fftshift(T*fft(x))
om = (-N/2 : N/2-1)*2*pi/T/N
```

assumiamo N pari