Le29

Thursday, 30 May 2024 08:46

Es2

$x(t) = K y(t) + m y''(t)$ $\quad x(t) = F_0 \cos(\omega_0 t)$

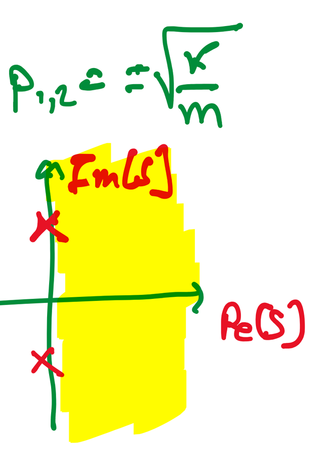
$y(0_-) = y_0$

$y'(0_-) = v_0$

$y(t) = ?\quad t>0$

$$H(s) = \frac{1}{K + ms^2} = \frac{\frac{1}{\sqrt{mK}}}{} \frac{1}{\sqrt{\frac{K}{m}}}\frac{1}{m} \cdot \frac{1\sqrt{\frac{K}{m}}}{s^2 + \frac{K}{m}}$$

$p_{1,2} = \pm\sqrt{\frac{K}{m}}$

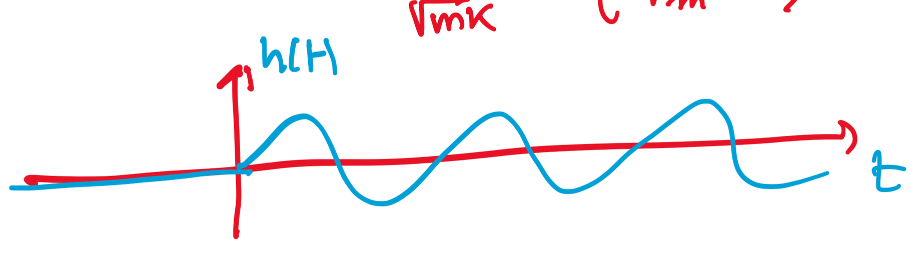


NON BIBO STABILE

$\cos(\omega_0 t)1(t) \rightarrow \frac{s}{s^2+\omega_0^2}$

$\sin(\omega_0 t)1(t) \rightarrow \frac{\omega_0}{s^2+\omega_0^2}$

$$h(t) = \frac{1}{\sqrt{mK}} \sin\left(\sqrt{\frac{K}{m}}\, t\right) 1(t)$$



$x(t) = K y(t) + m y''(t)$

$\downarrow \mathcal{L}^+$

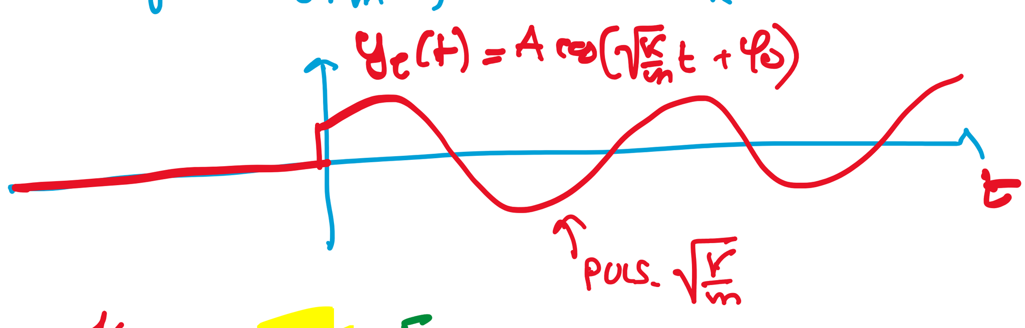
$$\frac{1}{m} X(s) = \frac{K}{m} Y(s) + \cancel{m}\left(s^2 Y(s) - s\, \cancel{y(0_-)}^{\,y_0} - \cancel{y'(0_-)}^{\,v_0}\right)$$

$$Y(s) = \underbrace{\frac{1/m}{s^2+K/m} X(s)}_{\text{RISP. FORZATA}} + \underbrace{\frac{s y_0 + v_0}{s^2+K/m}}_{\text{EV. LIBERA}}$$

$$Y_\ell(s) \text{ (LIBERA)} = y_0 \cdot \underbrace{\frac{s}{s^2+K/m}}_{\cos} + \frac{v_0}{\sqrt{K/m}} \underbrace{\frac{\sqrt{K/m}}{s^2+K/m}}_{\sin}$$

$$y_\ell(t) = y_0 \cos\left(\sqrt{\frac{K}{m}}\,t\right)1(t) + v_0\sqrt{\frac{m}{K}} \sin\left(\sqrt{\frac{K}{m}}\,t\right)1(t)$$

$y_\ell(t) = A\cos\left(\sqrt{\frac{K}{m}}\,t + \varphi_0\right)$



$$Y_f(s) = \frac{1/m}{s^2+K/m} \cdot \cancel{X(s)}\, \frac{F_0 s}{s^2+\omega_0^2}$$

$x(t) = F_0\cos(\omega_0 t)$

$x^+(t) = F_0\cos(\omega_0 t)1(t)$

$$= \frac{F_0}{m} \cdot \frac{s}{\left(s^2+\frac{K}{m}\right)\left(s^2+\omega_0^2\right)}$$

$$Z(s^2) = \frac{1}{\left(s^2+\frac{K}{m}\right)\left(s^2+\omega_0^2\right)} = \frac{R_0}{s^2+\frac{K}{m}} + \frac{R_1}{s^2+\omega_0^2}$$

$$R_0 = Z(s^2)\left(s^2+\frac{K}{m}\right)\Big|_{s^2=-\frac{K}{m}} = \frac{1}{s^2+\omega_0^2}\Big|_{s^2=-\frac{K}{m}} = \frac{1}{\omega_0^2-\frac{K}{m}}$$

$$R_1 = Z(s^2)\left(s^2+\omega_0^2\right)\Big|_{s^2=-\omega_0^2} = \frac{1}{s^2+\frac{K}{m}}\Big|_{s^2=-\omega_0^2} = -\frac{1}{\omega_0^2-\frac{K}{m}}$$

$$Y_f(s) = \frac{F_0/m}{\omega_0^2-K/m}\left(\frac{s}{s^2+K/m} - \frac{s}{s^2+\omega_0^2}\right)$$

$$y_f(t) = \frac{F_0/m}{\omega_0^2-K/m}\left(\cos\left(\sqrt{K/m}\,t\right) - \cos(\omega_0 t)\right)1(t)$$

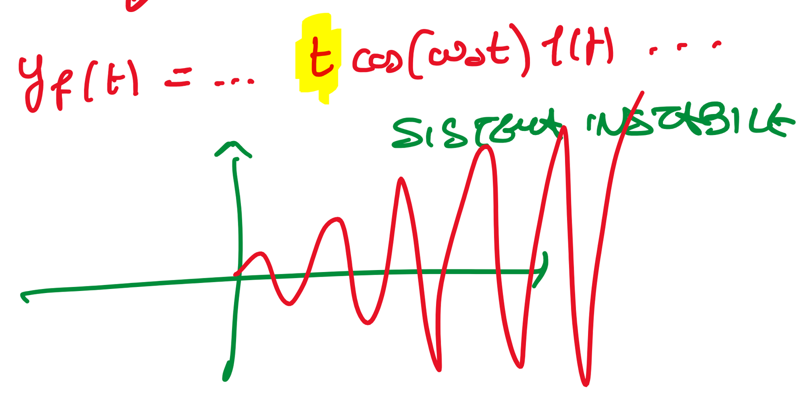
cosa succede se $\sqrt{K/m} = \omega_0$?

$$Y_f(s) = F_0/m \frac{s}{(s^2+\omega_0^2)^2}$$

$\downarrow \mathcal{L}_+^{-1}$

$y_f(t) = \ldots\ t\cos(\omega_0 t)1(t) \ldots$

SISTEMA INSTABILE



Es1

$x(n) = p_0^n 1_0(n) \quad p_0 \in \mathbb{C}$

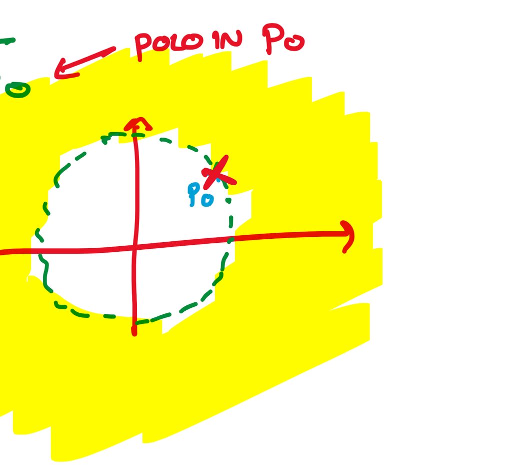
$$X(z) = \sum_{n=\cancel{-\infty}\ 0}^{+\infty} p_0^n \cancel{1_0(n)} \cdot z^{-n} = \sum_{n=0}^{+\infty} (p_0 z^{-1})^n$$

$$= \frac{1}{1-p_0 z^{-1}} \qquad |p_0 z^{-1}| < 1$$

$\frac{|p_0|}{|z|} < 1$

$|z| > |p_0|$ Γ_x

$$X(z) = \frac{z}{z-p_0} \leftarrow \text{POLO IN } p_0$$



Es1b

$x(n) = p_0^n 1_0(-n)$

$$X(z) = \sum_{n=-\infty}^{\cancel{+\infty}\ 0} p_0^n \cancel{1_0(-n)}\, z^{-n} = \sum_{n=-\infty}^{0} (z p_0^{-1})^{-n}$$

$$= \sum_{m=0}^{\infty} (z p_0^{-1})^m = \frac{1}{1-z p_0^{-1}} \qquad |z p_0^{-1}| < 1$$

$\frac{|z|}{|p_0|} < 1$

$|z| < |p_0|$ Γ_x

$$X(z) = \frac{-p_0}{z-p_0} \leftarrow \text{POLO } p_0$$

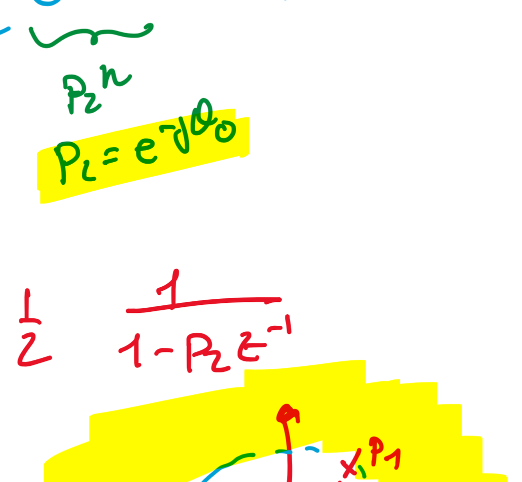
Es1c

$x(n) = \cos(\theta_0 n) 1_0(n)$

$$= \frac{1}{2}\underbrace{e^{j\theta_0 n}}_{p_1^n} 1_0(n) + \frac{1}{2}\underbrace{e^{-j\theta_0 n}}_{p_2^n} 1_0(n)$$

$p_1 = e^{j\theta_0}$ $\qquad p_2 = e^{-j\theta_0}$

$$X(z) = \frac{1}{2}\frac{1}{1-p_1 z^{-1}} + \frac{1}{2}\frac{1}{1-p_2 z^{-1}}$$



$$X(z) = \frac{1}{2}\frac{1}{1-z^{-1}e^{j\theta_0}} + \frac{1}{2}\frac{1}{1-z^{-1}e^{-j\theta_0}}$$

$$= \frac{1}{2}\frac{1-z^{-1}e^{-j\theta_0}+1-z^{-1}e^{j\theta_0}}{(1-z^{-1}e^{j\theta_0})(1-z^{-1}e^{-j\theta_0})} = \frac{1-z^{-1}\cos(\theta_0)}{1-z^{-1}2\cos(\theta_0)+z^{-2}}$$

X CASA $\quad x(n) = \sin(\theta_0 n) 1_0(n)$

$X(z) = ?$

Es 2a

$x_0(n) = -p_0^{n+1} 1_0(n) = -p_0 \cdot p_0^n 1_0(n)$

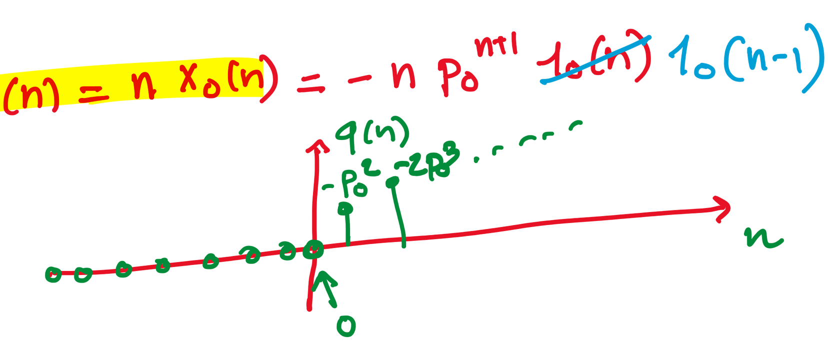
$$X(z) = -p_0 \frac{1}{1-z^{-1}p_0} = \frac{1}{z^{-1}-\frac{1}{p_0}}$$

$$= \frac{1}{z^{-1}-p_0^{-1}} = \frac{p_0 z}{p_0 - z}$$

Es 2b

$x_1(n) = (n+1)\, p_0^{n+2} 1_0(n)$

$q(n) = n\, x_0(n) = -n\, p_0^{n+1} \cancel{1_0(n)}\ 1_0(n-1)$



$x_1(n) = -q(n+1)$

$Q(z) = -z\, X_0'(z)$

$X_1(z) = -z\, Q(z) = z^2 X_0'(z)$

$$X_0(z) \quad \frac{1}{z^{-1}-p_0^{-1}}$$

$$X_1(z) = \cancel{z^2} \cdot \frac{\cancel{1}}{(z^{-1}-p_0^{-1})^2} \cdot \cancel{-1} \cdot \cancel{z^{-2}} = \frac{1}{(z^{-1}-p_0^{-1})^2}$$

Es 2c

$x_2(n) = -\frac{1}{2}(n+1)(n+2)p_0^{n+3} 1_0(n)$

$\downarrow$

$$X_2(z) = \frac{1}{(z^{-1}-p_0^{-1})^3}$$

X CASA