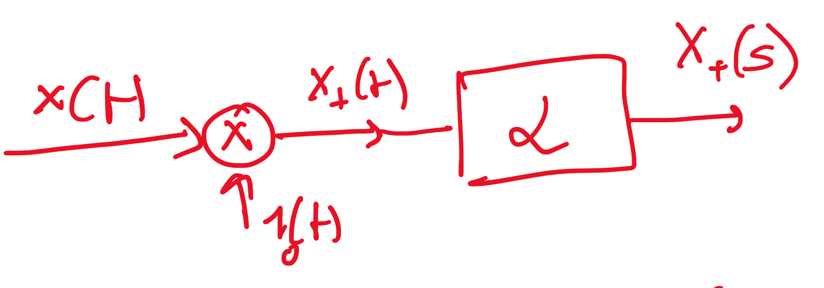
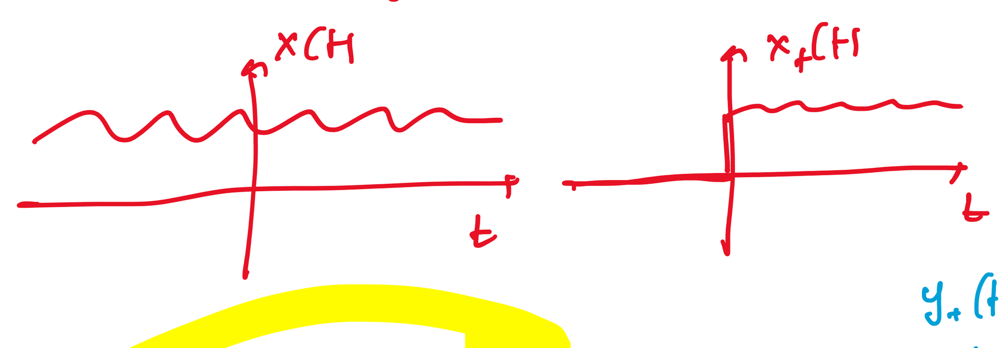
Le28

Monday, 27 May 2024 00:01

TRASF. UNILATERA

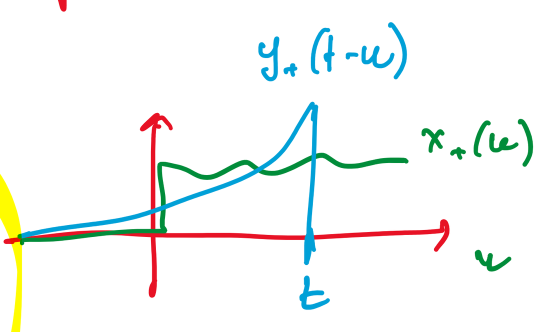




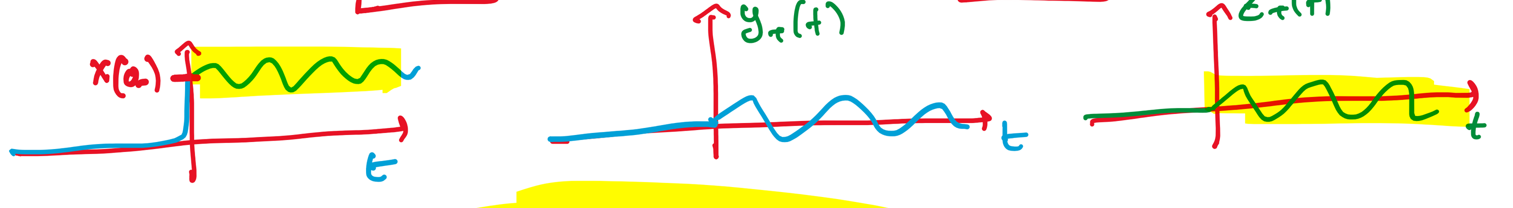
$$z_+(t) = x_+ * y_+(t)$$

CAUSALI

$$Z_+(s) = X_+(s)\,Y_+(s)$$



$$x_+(t) \longrightarrow \boxed{d/dt} \xrightarrow{y_+(t) = \frac{\partial x_+(t)}{\partial t}} \boxed{\int_0^t} \longrightarrow z_+(t) = x_+(t) - x(0_-)1(t)$$



$$Y_+(s) = s X_+(s) - x(0_-)$$

$$Z_+(s) = \frac{1}{s} Y_+(s) = X_+(s) - \frac{1}{s} x(0_-)$$

$$x_+'(t) \longrightarrow s X_+(s) - x(0_-) = X_+'(s)$$

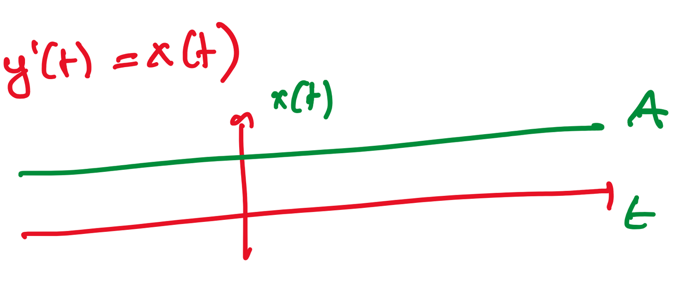
$$x_+''(t) \longrightarrow s X_+'(s) - x'(0_-)$$
$$= s^2 X_+(s) - s\,x(0_-) - x'(0_-) = X_+''(s)$$

$$x_+'''(t) \longrightarrow s X_+''(s) - x''(0_-)$$
$$= s^3 X_+(s) - s^2 x(0_-) - s\,x'(0_-) - x''(0_-)$$

Es1 FILTRO RC

$$y(t) + RC\,y'(t) = x(t)$$

$x(t) = A$



$y(0^-) = V_0$

$y(t) = ?$ $t > 0$

$$y_+(t) + RC\,y_+'(t) = x_+(t) = A\,1(t)$$

$$Y_+(s) + RC\big(sY_+(s) - \underbrace{y(0_-)}_{V_0}\big) = X_+(s) = \frac{A}{s}$$

$$Y_+(s)\,\cancel{(1+RCs)} - \cancel{RCV_0} = \frac{X_+(s) + RCV_0}{1+RC\cdot s}$$

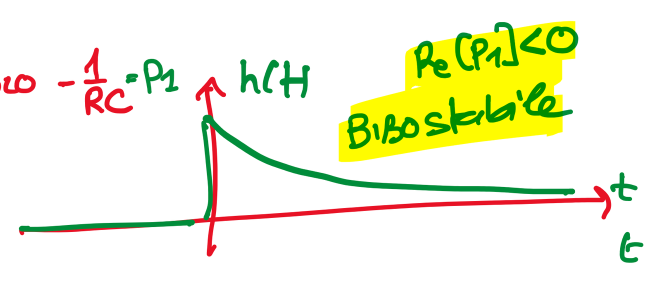
$$Y_+(s) = \underbrace{\overbrace{\frac{1}{1+RC\cdot s}}^{H(s)} X_+(s)}_{y_f(t)\ \text{FORZATA}} + \underbrace{\frac{RCV_0}{1+RCs}}_{y_\ell(t)\ \text{LIBERA}}$$

$$H(s) = \frac{1}{1+RCs} = \frac{1}{RC}\cdot\frac{1}{s+\frac{1}{RC}} \xrightarrow{\mathcal{L}^{-1}} h(t) = \frac{1}{RC}e^{-\frac{1}{RC}t}1(t)$$

polo $-\frac{1}{RC} = P_1$

$Re[P_1] < 0$

Bibostabile



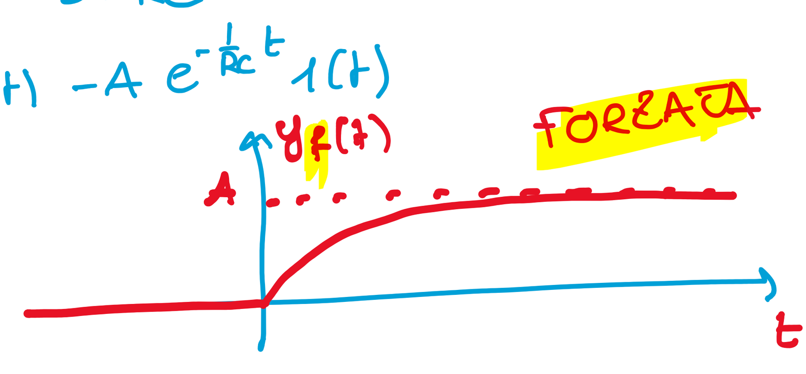
$$Y_f(s) = H(s)\,X_+(s) = \frac{A}{RC}\cdot\frac{1}{s(s+\frac{1}{RC})} = \frac{R_0}{s} + \frac{R_1}{s+1/RC}$$

$$R_0 = Y_f(s)\,s\big|_{s=0} = \frac{A}{RC}\cdot\frac{1}{s+\frac{1}{RC}}\Big|_{s=0} = A$$

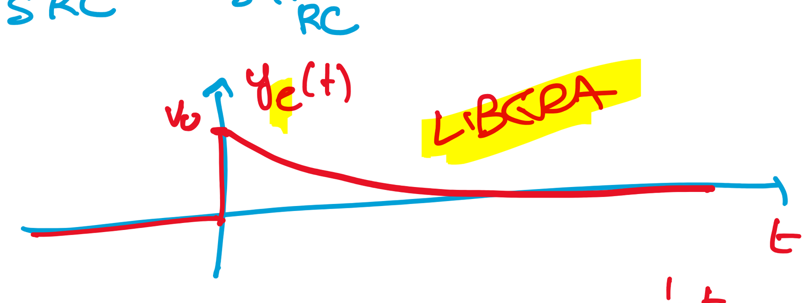
$$R_1 = Y_f(s)\left(s+\tfrac{1}{RC}\right)\Big|_{s=-\frac{1}{RC}} = \frac{A}{RC}\cdot\frac{1}{s}\Big|_{s=-\frac{1}{RC}} = -A$$

$$Y_f(s) = \frac{A}{s} - \frac{A}{s+1/RC}$$

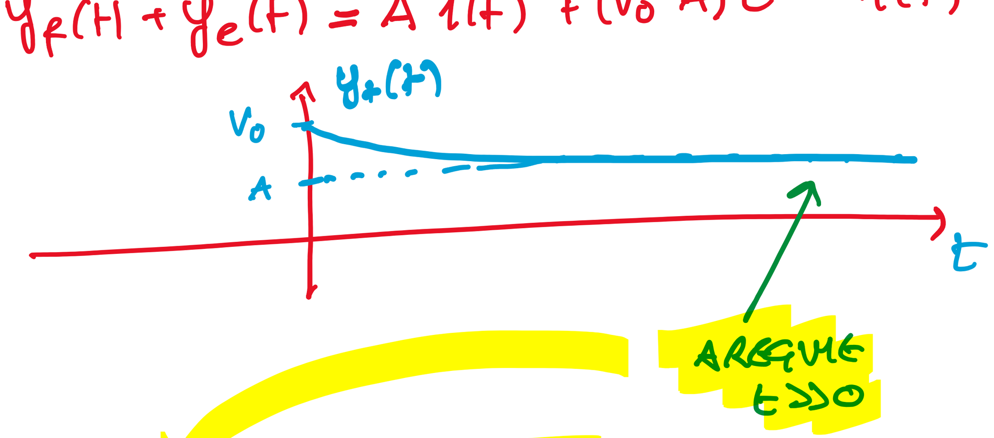
$$y_f(t) = A\,1(t) - A\,e^{-\frac{1}{RC}t}1(t)$$



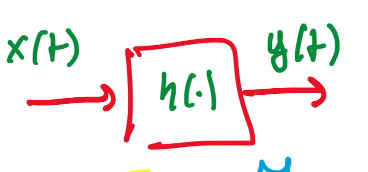
$$Y_\ell(s) = \frac{RCV_0}{1+sRC} = \frac{V_0}{s+\frac{1}{RC}} \xrightarrow{\mathcal{L}^{-1}} y_\ell(t) = V_0 e^{-\frac{1}{RC}t}1(t)$$



$$y_+(t) = y_f(t) + y_\ell(t) = A\,1(t) + (V_0 - A)\,e^{-\frac{1}{RC}t}1(t)$$



$y(t) = A$ t >> 0



$$x(t) = A \xrightarrow{\mathcal{F}} X(j\omega) = A\,\delta(\omega)\cdot 2\pi$$

$$Y(j\omega) = X(j\omega)\,H(j\omega)$$
$$= A\,\delta(\omega)\cdot 2\pi\cdot\frac{1}{1+\cancel{RCj\omega}}$$
$$= A\,\delta(\omega)\,2\pi$$

$y(t) = A$ $\xleftarrow{\mathcal{F}^{-1}}$

Es3

$$\underset{s^2}{y''(t)} - \underset{s}{y'(t)} - \underset{1}{6y(t)} = \underset{s}{x'(t)} - \underset{1}{3x(t)}$$

1) $H(s) = ?$
2) BIBO STABILE?
3) RISPOSTA FORZATA CON $x(t) = 1(t)$
4) SIA $x(t) = A\cos(\omega_0 t + \varphi_0)$ E CONDIZIONI INIZIALI NULLE TROVARE ω_0 CHE GARANTISCA UN COMPORTAMENTO A REGIME DEL TIPO $y(t) = \frac{1}{5}x(t - t_0)$ $t >> 0$

$$H(s) = \frac{b(s)}{a(s)} = \frac{s-3}{s^2-s-6}$$

$$P_{1,2} = \frac{1 \pm \sqrt{\cancel{1+24}}^{\,5}}{2} = \begin{cases} 3 \\ -2 \end{cases}$$

$$= \frac{\cancel{s-3}}{\cancel{(s-3)}(s+2)} = \frac{1}{s+2}$$

$$h(t) = e^{-2t}1(t)$$

BIBOSTABILE

$Re[-2] = -2 < 0$

eq. diff. di riferimento COMPATTA

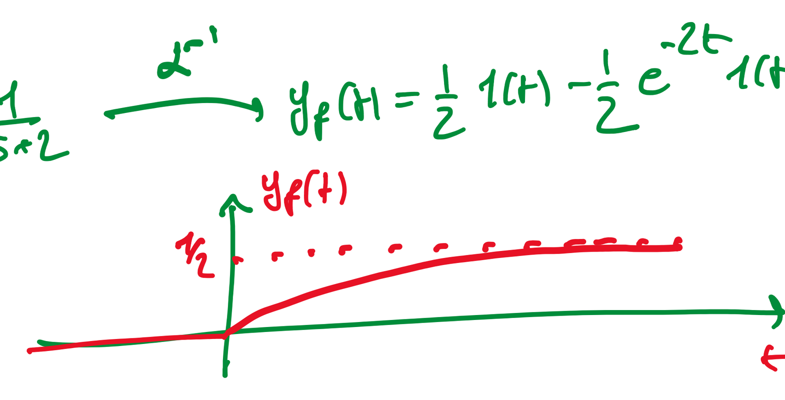
$$x(t) = y'(t) + 2y(t)$$

$$Y_f(s) = H(s)\,X_+(s)$$
$$= \frac{1}{s(s+2)} = \frac{R_0}{s} + \frac{R_1}{s+2} = \frac{R_0(s+2) + R_1 s}{s(s+2)}$$
$$= \frac{s(R_0+R_1) + 2R_0}{s(s+2)}$$

$R_0 + R_1 = 0$ $R_1 = -R_0 = -1/2$

$2R_0 = 1$ $R_0 = 1/2$

$$Y_f(s) = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{1}{s+2} \xrightarrow{\mathcal{L}^{-1}} y_f(t) = \frac{1}{2}1(t) - \frac{1}{2}e^{-2t}1(t)$$



A REGIME t >> 0

$$x(t) = A\cos(\omega_0 t + \varphi_0) \longrightarrow \boxed{h(\cdot)} \longrightarrow y(t) = A\,|H(j\omega_0)|\cos(\omega_0 t + \varphi_0 + \angle H(j\omega_0))$$

MOLT. x COSTANTE

TRASLAZIONE

$$= \frac{1}{5}x(t - t_0)$$

$$H(s) = \frac{1}{s+2}$$

$$H(j\omega) = \frac{1}{2+j\omega}$$

$h(t) = e^{-2t}1(t)$

REALE

$$|H(j\omega_0)| = \left|\frac{1}{2+j\omega_0}\right| = \frac{1}{\sqrt{4+\omega_0^2}} = \frac{1}{5}$$

$$\cancel{\sqrt{4+\omega_0^2}} = \cancel{5}\ \cancel{25}\ 21$$

$\omega_0^2 = 21$ OVVERO $\omega_0 = \pm\sqrt{21}$

$$(2+j\omega_0)(2-j\omega_0) = 2^2 - (j\omega_0)^2 = 4 + \omega_0^2$$