Le27

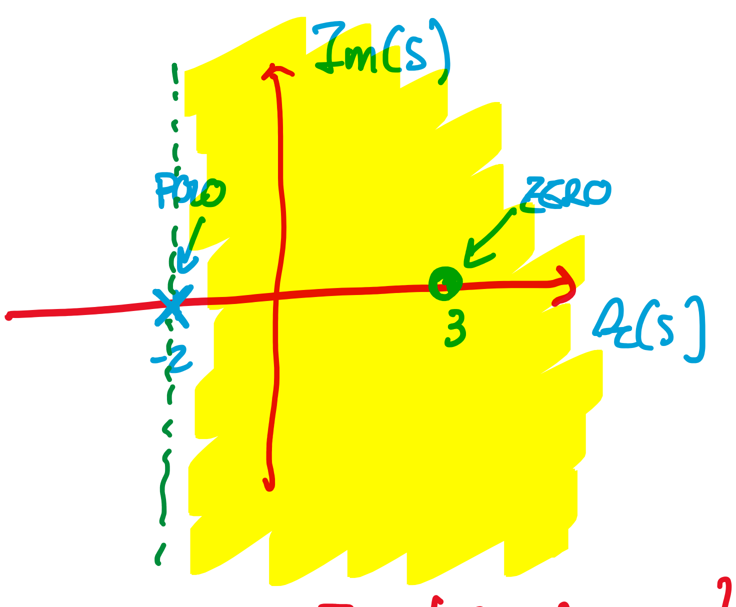
Friday, 24 May 2024 10:39

Es 1a

TROVARE L'ANTITRASFORMATA CAUSALE DI

$$H(s) = \frac{s-3}{s+2}$$

IMPROPRIA



$$\Gamma_h = \{Re(s) > -2\}$$

$$\begin{array}{l} s-3 \; : \; s+2 = 1 \\ -s-2 \\ \hline -\;-5 \end{array}$$

QUOZIENTE

RESTO

$$H(s) = 1 - \frac{5}{s+2} = \frac{s+\cancel{2}\cancel{-5}\,-3}{s+2} \checkmark$$

$$\mathcal{L}^{-1} \qquad \mathcal{L}^{-1}$$

POLO

$$h(t) = \delta(t) - 5e^{-2t}1(t)$$

$$\frac{1}{s+2} = \frac{1}{s-\underbrace{(-2)}_{s_0}}$$

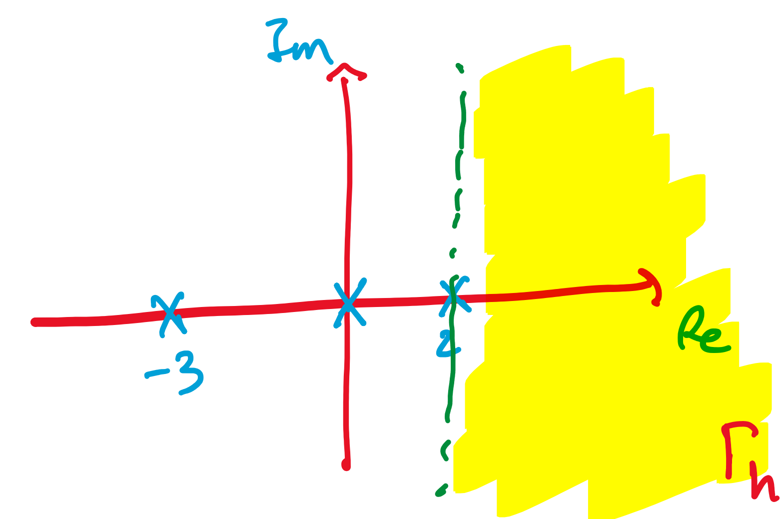
$$e^{s_0 t}1(t) \xrightarrow{\mathcal{L}} \frac{1}{s-s_0}$$

Es 1b

TROVARE ANTITRASF. CAUSALE DI

$$H(s) = \frac{1}{s^3+s^2-6s} = \frac{1}{s(s^2+s-6)} = \frac{1}{s(s-2)(s+3)}$$

$$p_{1,2} = \frac{-1 \pm \sqrt{\cancel{1+24}}^{\,5}}{2} = \begin{cases} 2 \\ -3 \end{cases}$$



$$\Gamma_h = \{Re(s) > 2\}$$

$$H(s) = \frac{1}{s(s-2)(s+3)} = \frac{R_0}{s} + \frac{R_1}{s-2} + \underbrace{\frac{R_2}{s+3}}_{\text{FRATTO SEMPLICE}}$$

POLI DIVERSI

CERCHIAMO R_1

$$H(s)(s-2)\Big|_{s=2} = \cancel{R_0 \frac{(s-2)}{s}} + R_1 + \cancel{R_2 \frac{(s-2)}{s+3}}$$

CASO GENERALE

polo p_k $\rightarrow$ $R_k = H(s)(s-p_k)\Big|_{s=p_k}$

QUESTO PRODOTTO ELIMINA LA PRESENZA DEL POLO p_k IN $H(s)$

$$H(s) = \frac{1}{s(s-2)(s+3)} = \frac{R_0}{s} + \frac{R_1}{s-2} + \frac{R_2}{s+3}$$

$$R_0 = H(s)\,s\Big|_{s=0} = \frac{1}{(s-2)(s+3)}\Big|_{s=0} = -\frac{1}{6}$$

$$R_1 = H(s)(s-2)\Big|_{s=2} = \frac{1}{s(s+3)}\Big|_{s=2} = \frac{1}{10}$$

$$R_2 = H(s)(s+3)\Big|_{s=-3} = \frac{1}{s(s-2)}\Big|_{s=-3} = \frac{1}{15}$$

$$h(t) = -\frac{1}{6}1(t) + \frac{1}{10}e^{2t}1(t) + \frac{1}{15}e^{-3t}1(t)$$

POLO POLO

NOTA

POLO p_0 ABBIA MOLT. 3

$$H(s) = \frac{R_0}{(s-p_0)} + \frac{R_1}{(s-p_0)^2} + \frac{R_2}{(s-p_0)^3} + \dots \frac{K}{(s-p_1)^n} + \dots$$

$$R_2 = H(s)(s-p_0)^3\Big|_{s=p_0} = \cancel{R_0(s-p_0)^2} + \cancel{R_1(s-p_0)} + R_2 + \dots + \cancel{K\frac{(s-p_0)^3}{(s-p_1)^n}}$$

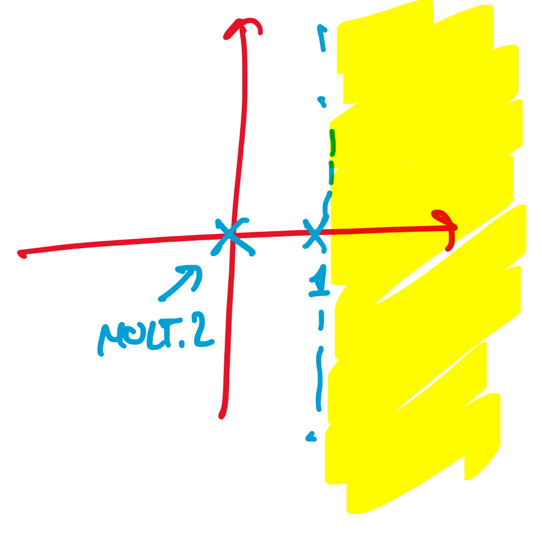
$$R_1 = \frac{\partial}{\partial s}H(s)(s-p_0)^3\Big|_{s=p_0} = \cancel{2R_0(s-p_0)} + R_1 + \cancel{0} + \dots \cancel{\frac{3K(s-p_0)^2}{(s-p_1)^n}} \cancel{-nK\frac{(s-p_0)^3}{(s-p_1)^{n+1}}}$$

$$2R_0 = \frac{\partial^2}{\partial s^2}H(s)(s-p_0)^3\Big|_{s=p_0} = 2R_0 + \cancel{0} + \cancel{0} + \dots + \cancel{\frac{(s-p_0)^e}{(s-p_1)^q}}$$

$$\boxed{\frac{1}{k!}\frac{\partial^k}{\partial s^k}H(s)(s-p_0)^{\mu_0}\Big|_{s=p_0} = \cancel{k!}\,R}$$

Es 1c

$$H(s) = \frac{4s-1}{2s^2(s-1)}$$



$$= \frac{R_0}{s} + \frac{R_1}{s^2} + \frac{R_2}{s-1}$$

$$R_2 = H(s)(s-1)\Big|_{s=1} = \frac{4s-1}{2s^2}\Big|_{s=1} = \frac{3}{2}$$

$$R_1 = H(s)\,s^2\Big|_{s=0} = \frac{4s-1}{2(s-1)}\Big|_{s=0} = \frac{1}{2}$$

$$R_0 = \frac{\partial}{\partial s}H(s)s^2\Big|_{s=0} = \frac{\partial}{\partial s}\left(\frac{4s-1}{2(s-1)}\right)\Big|_{s=0}$$

$$= \frac{4}{2(s-1)} - \frac{(4s-1)}{2(s-1)^2}\Big|_{s=0} = -2 + \frac{1}{2} = -\frac{3}{2}$$

$$h(t) = -\frac{3}{2}1(t) + \frac{1}{2}t\,1(t) + \frac{3}{2}e^{t}1(t)$$