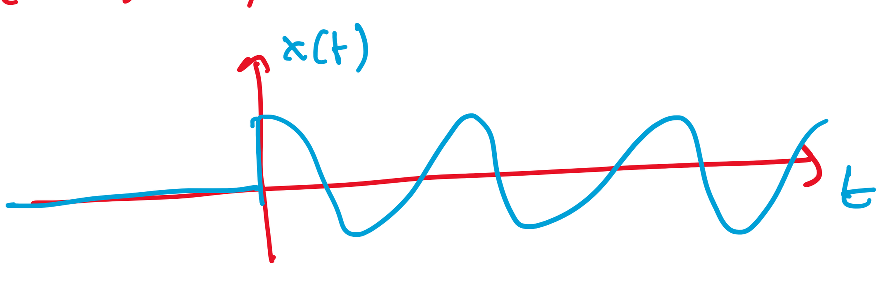
Le26

Thursday, 23 May 2024 08:44

Es 1a $x(t) = \cos(\omega_0 t)\,1(t)$

$X(s) = ?$

$\Gamma_x = ?$



$$X(s) = \int_{-\infty}^{+\infty} x(t)\, e^{-st}\, dt$$

$$x(t) = \frac{1}{2} e^{\overset{s_1 = j\omega_0}{j\omega_0} t}\, 1(t) + \frac{1}{2} e^{\overset{s_2 = -j\omega_0}{-j\omega_0} t}\, 1(t)$$

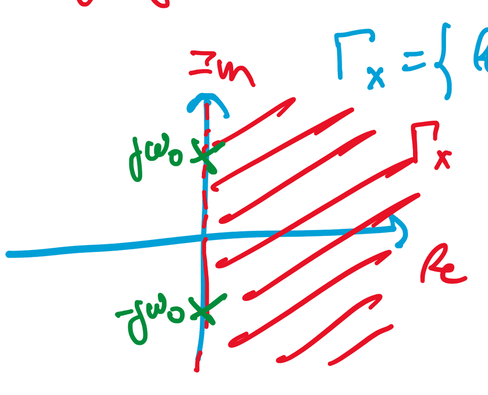
$$\boxed{e^{s_0 t}\, 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s - s_0} \qquad \{Re[s] > Re[s_0]\}}$$

$$X(s) = \frac{1}{2}\frac{1}{s - j\omega_0} + \frac{1}{2}\frac{1}{s + j\omega_0}$$

$$\{Re[s] > \underbrace{\cancel{Re[j\omega_0]}}_{0}\} \cap \{Re[s] > \underbrace{\cancel{Re[-j\omega_0]}}_{0}\}$$

$$X(s) = \frac{1}{2}\frac{s \cancel{+ j\omega_0} + s \cancel{- j\omega_0}}{(s - j\omega_0)(s + j\omega_0)} = \frac{s}{s^2 + \omega_0^2}$$

$\Gamma_x = \{Re[s] > 0\}$



$s^2 = -\omega_0^2$

$s = \pm j\omega_0$

~~XCASA~~ **Es 1b**

$$\sin(\omega_0 t)\, 1(t) \xrightarrow{\mathcal{L}} \frac{\omega_0}{s^2 + \omega_0^2} \qquad \Gamma = \{Re[s] > 0\}$$

Es 1c $x(t) = 1(t)$

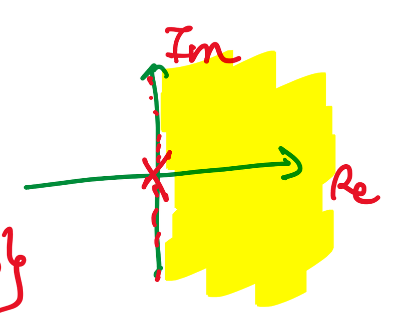
$X(s) = ?$

$\cos(\omega_0 t)\, 1(t) \longrightarrow \frac{s}{s^2 + \omega_0^2}$

$e^{s_0 t}\, 1(t) \longrightarrow \frac{1}{s - s_0}$

$X(s) = \frac{1}{s}$

$\Gamma_x = \{Re[s] > 0\}$



Es 1d $x(t) = rect(t)$

$$X(s) = \int_{\cancel{-\infty}\; -1/2}^{\cancel{+\infty}\; 1/2} \cancel{rect(t)}\, e^{-st}\, dt = \left.\frac{e^{-st}}{-s}\right]_{-1/2}^{1/2}$$

$$= \frac{e^{-s/2} - e^{s/2}}{-s} = \frac{e^{s/2} - e^{-s/2}}{s}$$

$$X(0) = \int_{-\infty}^{+\infty} rect(t)\, \cancel{e^{-0\cdot t}}\, dt = 1$$

$$X(s) = \begin{cases} 1 & s = 0 \\ \dfrac{e^{s/2} - e^{-s/2}}{s} & s \neq 0 \end{cases} \qquad \Gamma_x = \mathbb{C}$$

Es 1e $x(t) = \delta(t)$

$$X(s) = \int_{-\infty}^{+\infty} \delta(t)\, e^{-st}\, dt = 1 \qquad \Gamma_x = \mathbb{C}$$

Es 1f $x(t) = \delta(t - t_0)$

REGOLA DI TRASL.

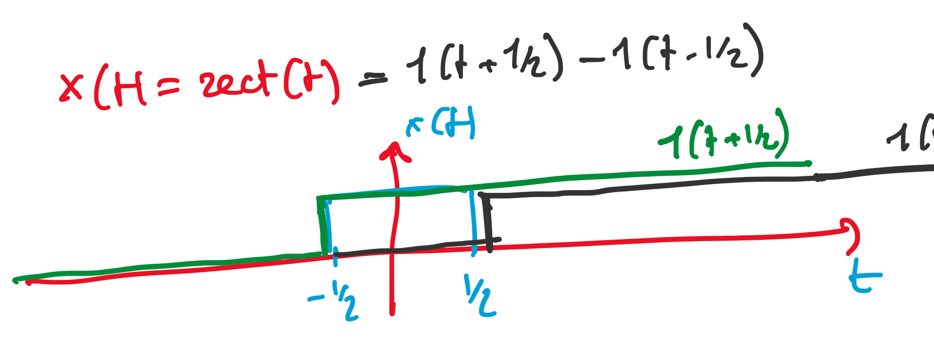
$$X(s) = 1 \cdot e^{-st_0} = e^{-st_0}$$

INTEGRALE

$$X(s) = \int_{-\infty}^{+\infty} \delta(t - t_0)\, e^{-st}\, dt = e^{-st_0}$$

Es 1d CON TRASL.

$$x(t) = rect(t) = 1(t + 1/2) - 1(t - 1/2)$$



$$1(t) \xrightarrow{\mathcal{L}} \frac{1}{s}$$

$$x(t - t_0) \xrightarrow{\mathcal{L}} X(s)\, e^{-st_0}$$

TRASL -1/2 TRASL =1/2

$$rect(t) \xrightarrow{\mathcal{L}} \frac{1}{s} \cdot e^{s/2} - \frac{1}{s} e^{-s/2}$$

$$= \frac{e^{s/2} - e^{-s/2}}{s}$$

REG. DI CONV.

$\{Re[s] > 0\}$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \dots$$

$$e^{s/2} = 1 + \frac{s}{2} + \frac{(s/2)^2}{2} + \frac{(s/2)^3}{3!} + \dots$$

$$e^{-s/2} = 1 - \frac{s}{2} + \frac{(s/2)^2}{2} - \frac{(s/2)^3}{3!} \dots$$

$$e^{s/2} - e^{-s/2} = s + 2\frac{(s/2)^3}{3!} + 2\frac{(s/2)^5}{5!} + \dots$$

$$\frac{e^{s/2} - s^{-s/2}}{s} = 1 + \frac{(s/2)^2}{3!} + \frac{(s/2)^4}{5!} + \frac{(s/2)^6}{7!} + \dots$$

Es DI APPLICAZ. REGOLA DI MODULAZIONE

$$1(t) \xrightarrow{\mathcal{L}} \frac{1}{s} \qquad \Gamma = \{Re[s] > 0\}$$

$$x(t)\, e^{s_0 t} \xrightarrow{\mathcal{L}} X(s - s_0)$$

$$e^{s_0 t}\, 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s - s_0}$$

Es 1g $x(t) = t^k e^{s_0 t}\, 1(t)$ $\quad k \geq 0$

$X(s) = ?$

$k=0 \quad x_0(t) = e^{s_0 t}\, 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s - s_0} \qquad Re[s] > Re[s_0]$

$k=1 \quad x_1(t) = t\, x_0(t) \xrightarrow{\mathcal{L}} -X_0'(s) = \cancel{-1} \cdot \cancel{-1}\, \frac{1}{(s - s_0)^2}$

$k=2 \quad x_2(t) = t\, x_1(t) \xrightarrow{\mathcal{L}} -X_1'(s) = \cancel{-1} \cdot \frac{+2}{(s - s_0)^3}$

$k=3 \quad x_3(t) = t\, x_2(t) \xrightarrow{\mathcal{L}} -X_2'(s) = \cancel{-1} \cdot \frac{+3 \cdot 2}{(s - s_0)^4} = \frac{3!}{(s - s_0)^4}$

$k=4 \quad x_4(t) = t\, x_3(t) \xrightarrow{\mathcal{L}} -X_3'(s) = \cancel{-1} \cdot \frac{+4 \cdot 3!}{(s - s_0)^5} = \frac{4!}{(s - s_0)^5}$

$k \quad x_k(t) \xrightarrow{\mathcal{L}} \frac{k!}{(s - s_0)^{k+1}}$

$$\frac{t^k e^{s_0 t}\, 1(t)}{k!} \xrightarrow{\mathcal{L}} \frac{\cancel{k!}\,1}{(s - s_0)^{k+1}} \qquad \{Re[s] > Re[s_0]\}$$

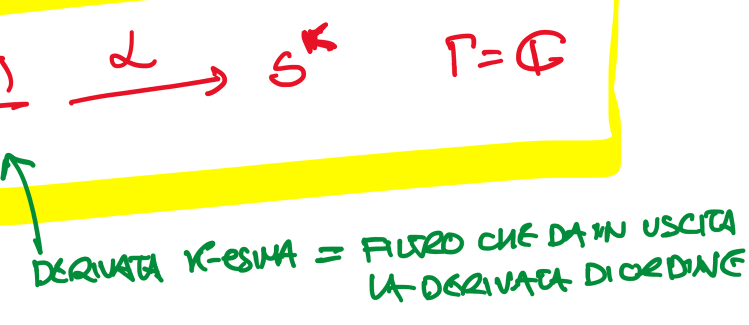
ESEMPIO DERIVAZIONE NEL TEMPO

$$x(t) = 1(t) \xrightarrow{\mathcal{L}} X(s) = \frac{1}{s} \qquad \Gamma_x = \{Re(s) > 0\}$$

$$y(t) = x'(t) = \delta(t) \xrightarrow{\mathcal{L}} Y(s) = s\, X(s) = s \cdot \frac{1}{s} = 1 \qquad \Gamma_y = \mathbb{C}$$

$$z(t) = x''(s) = \delta'(t) \xrightarrow{\mathcal{L}} s\, Y(s) = s$$

RISPOSTA IMPULSIVA DI UN BLOCCO DI DERIVAZIONE



$$\boxed{\frac{\partial^k \delta(t)}{\partial t^k} \xrightarrow{\mathcal{L}} s^k \qquad \Gamma = \mathbb{C}}$$

DERIVATA K-ESIMA = FILTRO CHE DA IN USCITA LA DERIVATA DI ORDINE K

NOTA USIAMO LA REGOLA DI INTEGRAZIONE

$$1(t) \xrightarrow{\mathcal{L}} \frac{1}{s}$$

$$t \cdot 1(t) = 1 * 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s^2}$$

$$\frac{t^2}{2} \cdot 1(t) = 1 * 1 * 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s^3}$$

$$\frac{t^3}{3!} \cdot 1(t) = 1 * 1 * 1 * 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s^4}$$

$$\frac{t^4}{4!}\, 1(t) = 1 * 1 * 1 * 1 * 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s^5}$$

$$\frac{t^k\, 1(t)}{k!} \xrightarrow{\mathcal{L}} \frac{1}{s^{k+1}} \qquad \{Re[s] > 0\}$$

regola di modulazione

$$\frac{t^k e^{s_0 t}\, 1(t)}{k!} \xrightarrow{\mathcal{L}} \frac{1}{(s - s_0)^{k+1}} \qquad Re[s] > Re[s_0]$$