Le25

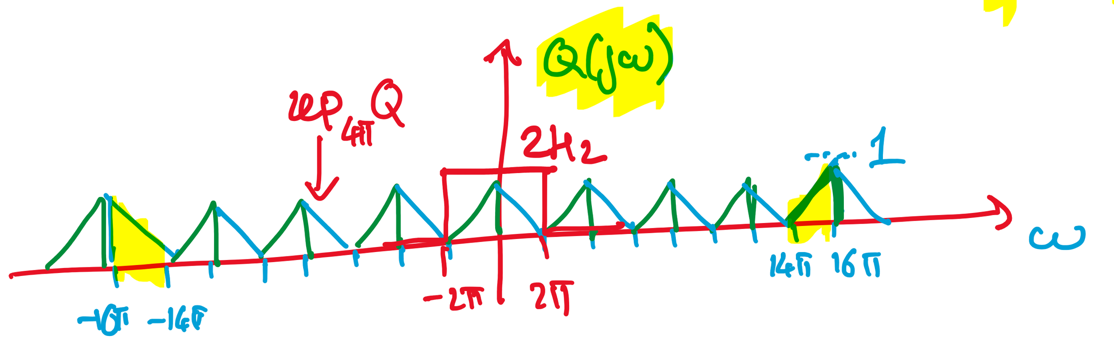
Friday, 17 May 2024 10:30

ES 3

$x(t) \to \otimes \to \boxed{h_1} \xrightarrow{q(t)} \boxed{\downarrow} \to \boxed{\uparrow h_2} \to y(t)$

$\cos(14\pi t)$ $\quad T = 1/2$

$H_2(j\omega) = \frac{1}{2}\,\mathrm{rect}\left(\frac{\omega}{4\pi}\right)$



$Q(j\omega) \to \boxed{\mathrm{rep}_{4\pi}} \to \otimes \to Y(j\omega) = \mathrm{triang}\left(\frac{\omega}{2\pi}\right)$

$2\pi/T$

$\frac{H_2(j\omega)}{T} = 2H_2(j\omega) = \mathrm{rect}\left(\frac{\omega}{4\pi}\right)$

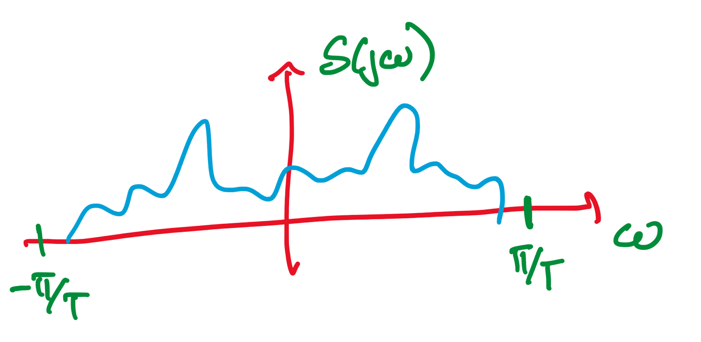
$$\mathrm{rep}_{4\pi}\, Q(j\omega) = \mathrm{rep}_{4\pi}\, \mathrm{triang}\left(\frac{\omega}{2\pi}\right)$$

$$Y(j\omega) = \mathrm{triang}\left(\frac{\omega}{2\pi}\right) \xrightarrow{\mathcal{F}^{-1}} y(t) = \mathrm{sinc}^2(t)$$

ES2

SCHEMA CAMPIONAMENTO / RICOSTRUZIONE PER

$s(t) = \mathrm{sinc}^2(t)\, e^{j\frac{19}{2}\pi t}$

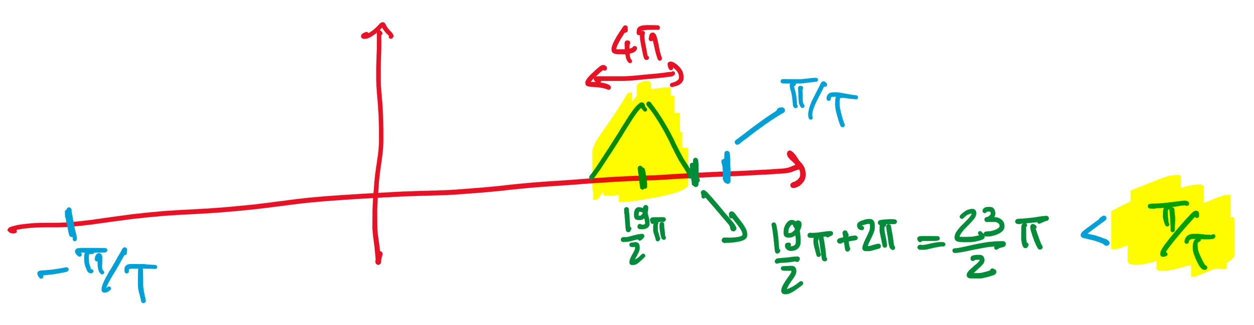


$s(t) \to \boxed{\downarrow} \xrightarrow{s(nT)} \boxed{\uparrow h} \to s(t)$

$T \qquad h = \mathrm{sinc}\left(\frac{t}{T}\right)$

$$s(t) = \sum_n s(nT)\, \mathrm{sinc}\left(\frac{t-nT}{T}\right)$$

$$s(t) = \mathrm{sinc}^2(t)\, e^{j\frac{19}{2}\pi t} \longrightarrow S(j\omega) = \mathrm{triang}\left(\frac{\omega - \frac{19}{2}\pi}{2\pi}\right)$$



$\frac{19}{2}\pi + 2\pi = \frac{23}{2}\pi < \frac{\pi}{T}$

$T < \frac{\pi}{\frac{23\pi}{2}} = \frac{2}{23}$

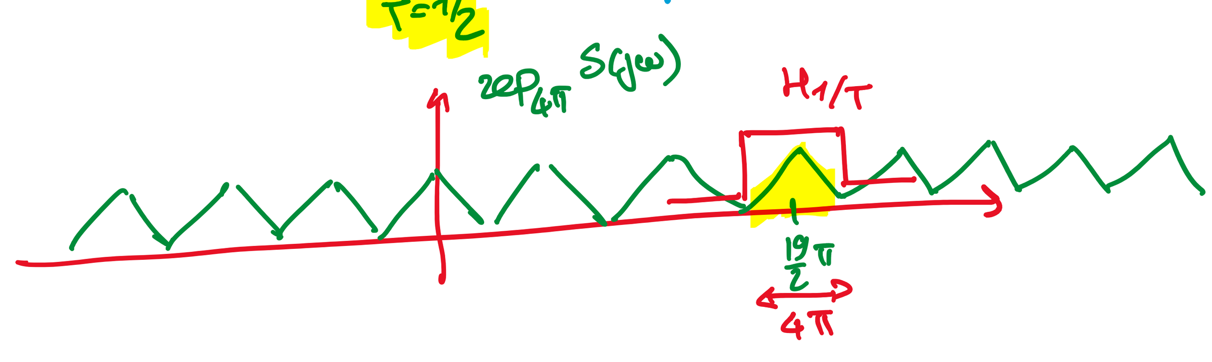
$T = \frac{2}{23}$

$S(j\omega) \to \boxed{\mathrm{rep}_{2\pi/T}} \to \otimes \to S(j\omega)$

$2\pi/T = 4\pi$

$T = 1/2$

$\frac{H_1(j\omega)}{T} = \mathrm{rect}\left(\frac{\omega - \frac{19}{2}\pi}{4\pi}\right)$



$H_1(j\omega) = T\,\mathrm{rect}\left(\frac{\omega - 19/2\,\pi}{4\pi}\right)$ $\quad T = \frac{1}{2}$

$= T\,\mathrm{rect}\left(\frac{T}{2\pi}(\omega - 19/2\,\pi)\right) \xrightarrow{\mathcal{F}^{-1}} \mathrm{sinc}\left(\frac{t}{T}\right) e^{j\frac{19}{2}\pi t} = h_1(t)$

$$s(t) = \sum_{n=-\infty}^{+\infty} s(nT)\, \mathrm{sinc}\left(\frac{t-nT}{T}\right) e^{j\frac{19}{2}\pi(t-nT)}$$

$$= e^{j\frac{19}{2}\pi} \cdot \sum_{n=-\infty}^{+\infty} s(nT)\, e^{-j\frac{19}{2}\pi nT} \cdot \mathrm{sinc}\left(\frac{t-nT}{T}\right)$$

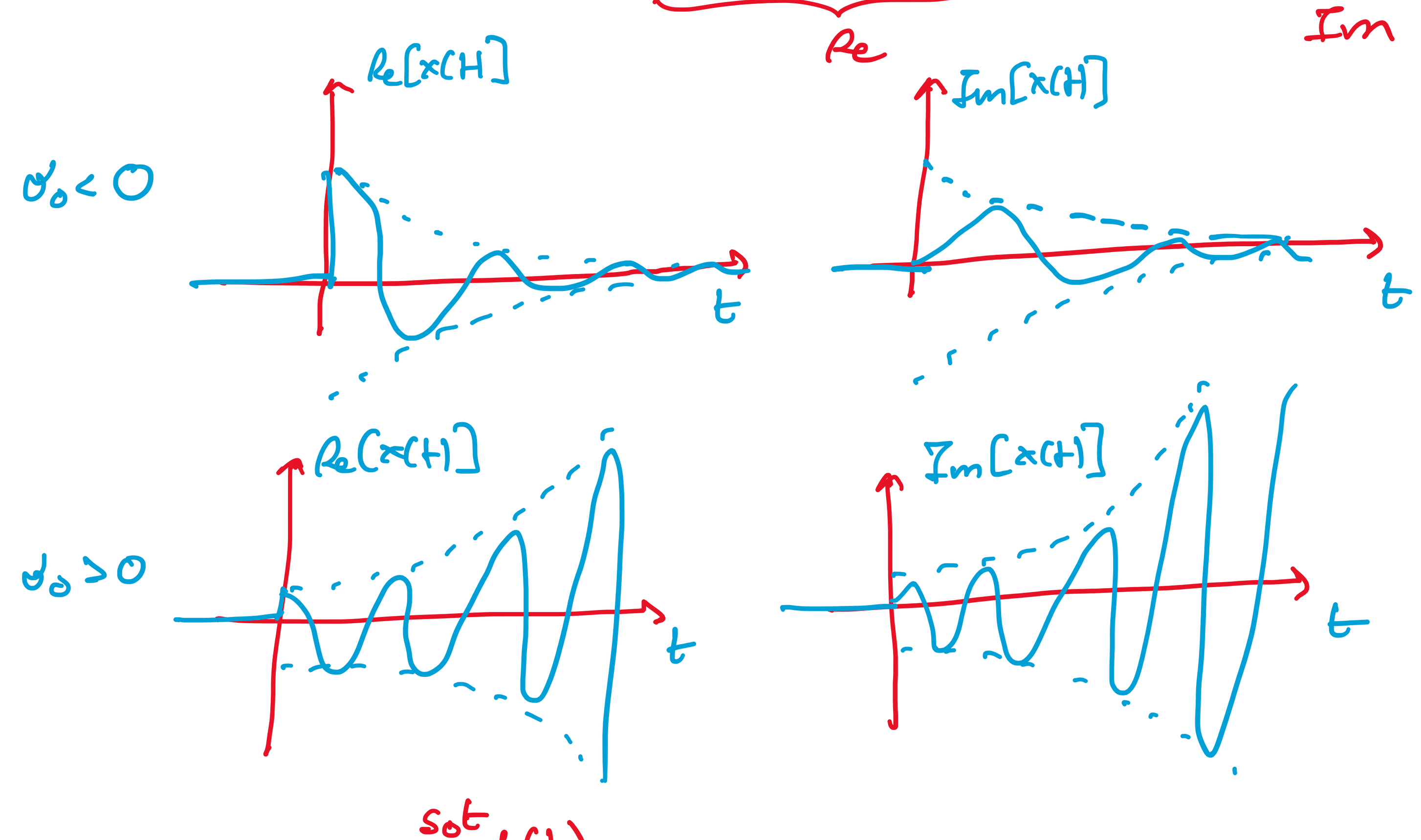
Es1a

$x(t) = e^{s_0 t}\, 1(t)$ $\qquad s_0 \in \mathbb{C}$

$X(s) = ?$ $\qquad s_0 = \sigma_0 + j\omega_0$

$$x(t) = e^{\sigma_0 t} e^{j\omega_0 t}\, 1(t)$$

$$= \underbrace{e^{\sigma_0 t}\cos(\omega_0 t)\, 1(t)}_{Re} + j\,\underbrace{e^{\sigma_0 t}\sin(\omega_0 t)\, 1(t)}_{Im}$$



$x(t) = e^{s_0 t}\, 1(t)$

$$X(s) = \int_{\cancel{-\infty}\ 0}^{+\infty} \cancel{x(t)}\, e^{s_0 t}\, e^{-st}\, dt = \int_0^{+\infty} e^{(s_0 - s)t}\, dt$$

$$= \left.\frac{e^{(s_0-s)t}}{s_0 - s}\right]_0^{+\infty}$$

$$= \frac{1}{s_0 - s}\left(\underbrace{\lim_{t\to+\infty} e^{(s_0-s)t}}\ - 1\right)$$

$\lim_{t\to+\infty} e^{Re[s_0-s]t}\, e^{j\,Im[s_0-s]t}$

$\exists$ ed è uguale a 0

se e solo se $Re[s_0 - s] < 0$

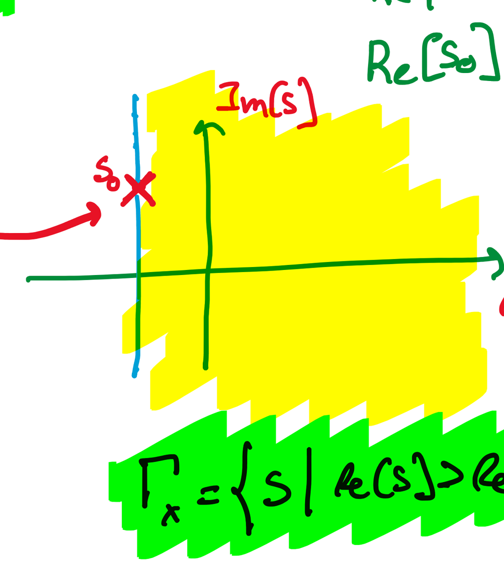
$X(s) = \frac{-1}{s_0 - s} = \frac{1}{s - s_0}$ PER $Re[s_0 - s] < 0$

ROC

REGIONE DI CONVERGENZA

$Re[s_0] < Re[s]$

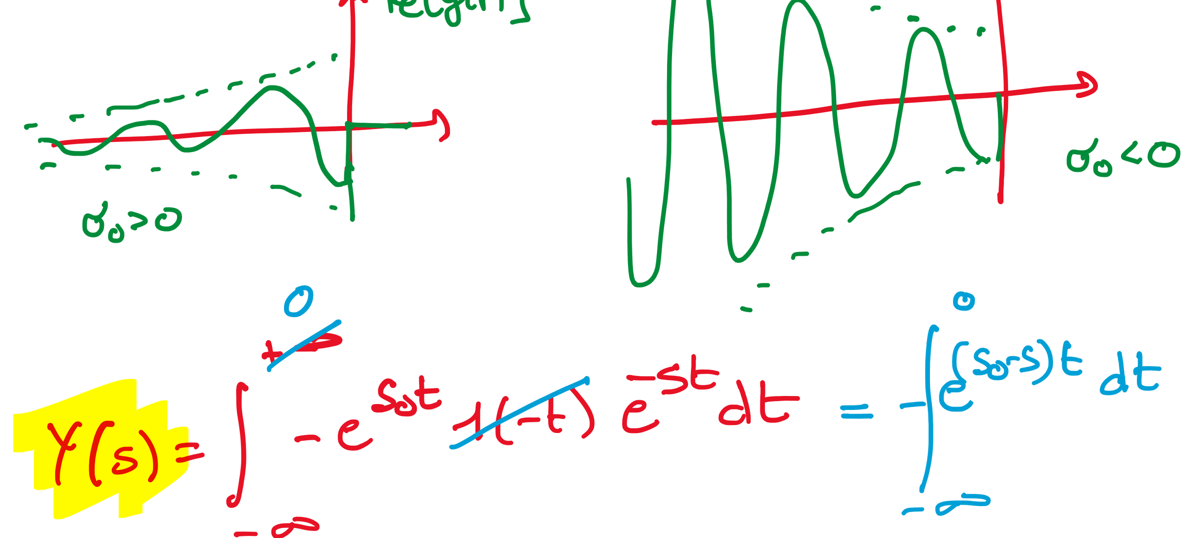
POLO = PUNTO IN CUI L'ESPR. ANALITICA DIVERGE



$\Gamma_x = \{ s \mid Re(s) > Re(s_0) \}$

Es1b

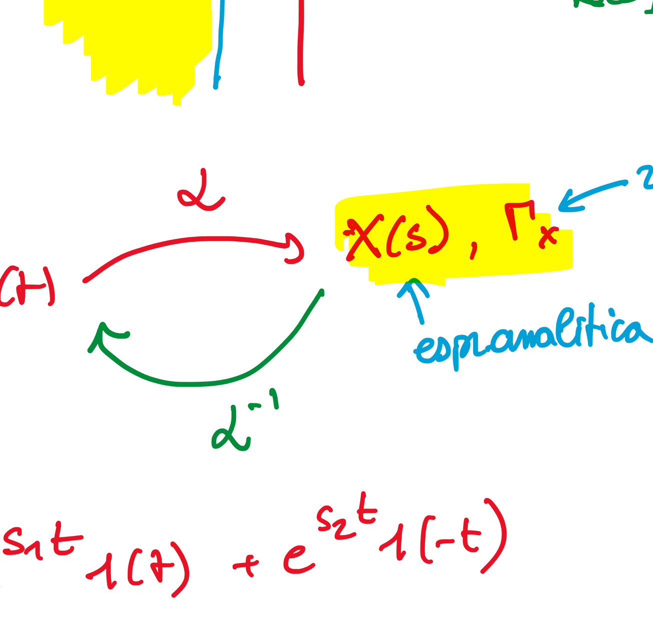
$y(t) = -e^{s_0 t}\, 1(-t)$ $\qquad s_0 = \sigma_0 + j\omega_0$



$$Y(s) = \int_{-\infty}^{\cancel{+\infty}\ 0} -e^{s_0 t}\, \cancel{1(-t)}\, e^{-st}\, dt = -\int_{-\infty}^{0} e^{(s_0-s)t}\, dt$$

$$= \left.\frac{-e^{(s_0-s)t}}{s_0-s}\right]_{-\infty}^{0} = \frac{-1}{s_0-s} = \frac{1}{s-s_0}$$

$Re[s_0 - s] > 0$



$\Gamma_y = \{ s \mid Re[s] < Re[s_0] \}$

TRASF. LAPLACE

$x(t) \xrightarrow{\mathcal{L}} X(s),\ \Gamma_x$ ← regione di convergenza

$X(s) \xrightarrow{\mathcal{L}^{-1}} x(t)$

espr. analitica

Es1c

$x(t) = e^{s_1 t}\, 1(t) + e^{s_2 t}\, 1(-t)$

$$X(s) = \frac{1}{s - s_1} - \frac{1}{s - s_2}$$

$Re(s) > Re(s_1)$ $\qquad Re(s) < Re(s_2)$

$Re[s_1] < Re[s] < Re[s_2]$