Le20

Friday, 3 May 2024 08:51

RISULTATO FONDAMENTALE

$x(t) = \cos(\omega_0 t + \varphi_0)$ / $\sin$ $\longrightarrow$ FILTRO $\boxed{h(\cdot)}$ REALE $\longrightarrow$ $y(t) = \underbrace{|H(j\omega_0)|}_{\text{VALORE ASSOLUTO}} \cos(\omega_0 t + \underbrace{\angle H(j\omega_0)}_{\text{FASE}} + \varphi_0)$ / $\sin$

MULT. X COSTANTE — TRASLAZIONE

UN FILTRO REALE NON DISTORCE LE SINUSOIDI

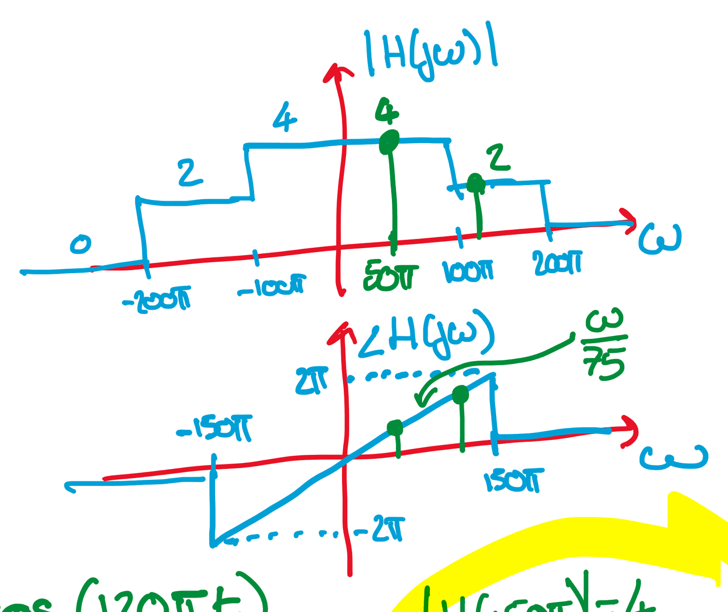
$x(t) = e^{j\omega_0 t}$ $\longrightarrow$ FILTRO $\boxed{h(\cdot)}$ $\longrightarrow$ $y(t) = H(j\omega_0)\, e^{j\omega_0 t} = |H(j\omega_0)|\, e^{j(\omega_0 t + \angle H(j\omega_0))}$

UN FILTRO NON DISTORCE GLI ESP. COMPLESSI A FASE LINEARE

Es2

$x(t) \longrightarrow \boxed{h(\cdot)} \longrightarrow y(t)$

REALE



$x(t) = \cos(50\pi t) + 5\cos(120\pi t)$

$y(t) = ?$

IL FILTRO DISTORCE IL SEGNALE?

$|H(j50\pi)| = 4$

$|H(j120\pi)| = 2$

$\angle H(j50\pi) = \frac{50\pi}{75}$

$\angle H(j120\pi) = \frac{120\pi}{75}$

$y(t) = 4\cos\left(50\pi t + \frac{50\pi}{75}\right) + 2 \cdot 5\cos\left(120\pi t + \frac{120\pi}{75}\right)$

$= 4\cos\left(50\pi \underbrace{\left(t + \tfrac{1}{75}\right)}_{\text{TRASL.}}\right) + 2 \cdot 5\cos\left(120\pi \underbrace{\left(t + \tfrac{1}{75}\right)}_{\text{TRASL.}}\right)$

COSTANTI MOLTIPLICATIVE DIFFERENTI

$\Downarrow$

IL SEGNALE VIENE DISTORTO

Es3 LA DECONVOLUZIONE

$x(t) \longrightarrow$ FILTRO $\boxed{h(\cdot)} \longrightarrow y(t)$

$x(t) = \text{triangle}(t/3)$

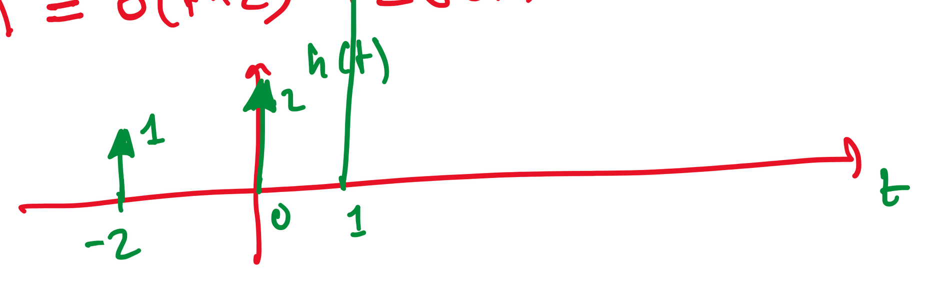
$y(t) = \text{triangle}\left(\frac{t+2}{3}\right) + 2\,\text{triangle}\left(\frac{t}{3}\right) + 4\,\text{triangle}\left(\frac{t-1}{3}\right)$

1) $H(j\omega) = ?$ $\longrightarrow$ $Y(j\omega) = X(j\omega)H(j\omega)$ $\longrightarrow$ $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$
2) $h(t) = ?$
3) BIBO STABILE?
4) $z(t) = h * 1(t) = ?$

$$H(j\omega) = \frac{\cancel{3\,\text{sinc}^2\left(\frac{\omega 3}{2\pi}\right)} \cdot e^{j2\omega} + 2 \cdot \cancel{3\,\text{sinc}^2\left(\frac{\omega 3}{2\pi}\right)} + 4 \cdot \cancel{3\,\text{sinc}^2\left(\frac{\omega 3}{2\pi}\right)}\, e^{-j\omega}}{\cancel{3\,\text{sinc}^2\left(\frac{\omega 3}{2\pi}\right)}}$$

$= e^{j2\omega} + 2 + 4e^{-j\omega}$

$h(t) = \delta(t+2) + 2\delta(t) + 4\delta(t-1)$



$\int |h(t)|\,dt = \int h(t)\,dt = 1 + 2 + 4 = 7 < \infty$

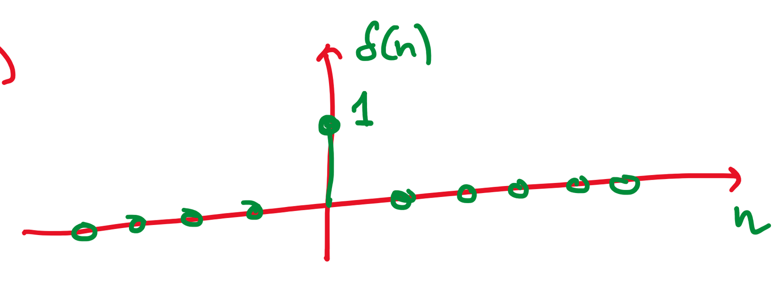
BIBO STABILE

$z(t) = $ X CASA

Es1a

$s(n) = \delta(n)$

$S(e^{j\theta}) = ?$

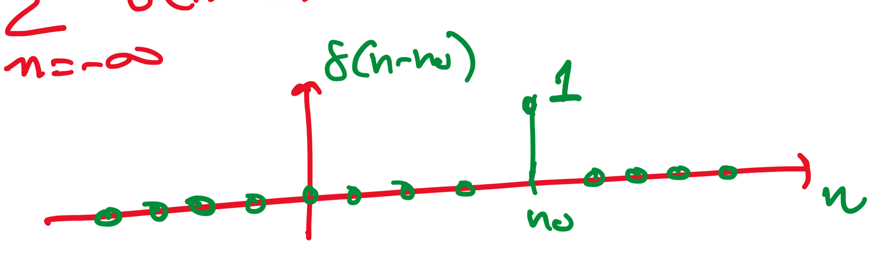


$S(e^{j\theta}) = \sum_{n=-\infty}^{+\infty} \delta(n)\, e^{-j\theta n} = e^{-j\theta n}\Big|_{n=0} = 1$

Es1b

$s(n) = \delta(n - n_0)$

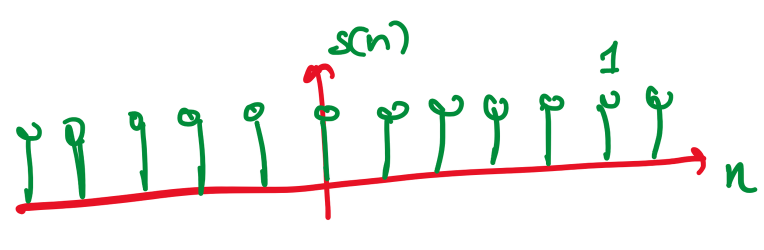
$S(e^{j\theta}) = \sum_{n=-\infty}^{+\infty} \delta(n-n_0)\, e^{-j\theta n} = e^{-j\theta n}\Big|_{n=n_0} = e^{-j\theta n_0}$



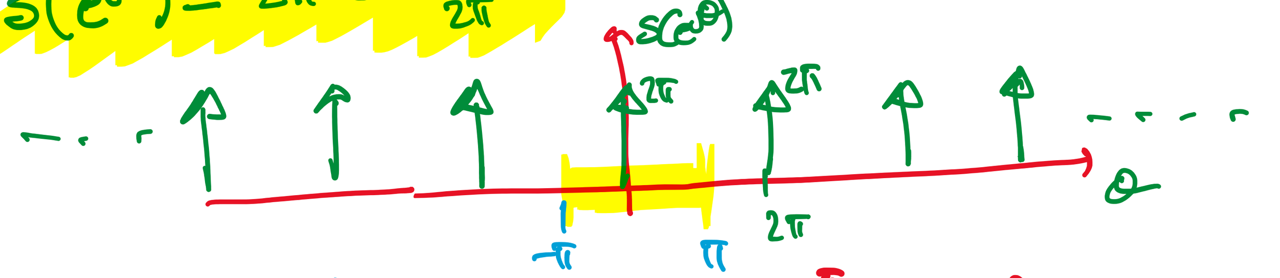
Es1c

$s(n) = 1$

$S(e^{j\theta}) = ?$



$S(e^{j\theta}) = 2\pi\, \text{comb}_{2\pi}(\theta) = 2\pi\, \text{rep}_{2\pi}\, \delta(\theta)$



$s(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \underbrace{\cancel{S(e^{j\theta})}}_{2\pi\delta(\theta)}\, e^{j\theta n}\, d\theta = \int_{-\pi}^{\pi} \delta(\theta)\, e^{j\theta n}\, d\theta = 1$ ✓

$$\boxed{\begin{aligned} \delta(n) &\xrightarrow{\mathcal{F}} 1 \\ 1 &\xrightarrow{\mathcal{F}} 2\pi\, \text{rep}_{2\pi}\, \delta(\theta) \end{aligned}}$$

DELTA E SEGNALE COSTANTE SONO DUALI

$$\boxed{\begin{aligned} \delta(n-n_0) &\xrightarrow{\mathcal{F}} e^{-j\theta n_0} \\ e^{j\theta_0 n} &\xrightarrow{\mathcal{F}} 2\pi\, \text{rep}_{2\pi}\, \delta(\theta-\theta_0) \end{aligned}}$$

REGOLA DI TRASL./MODULAZ.

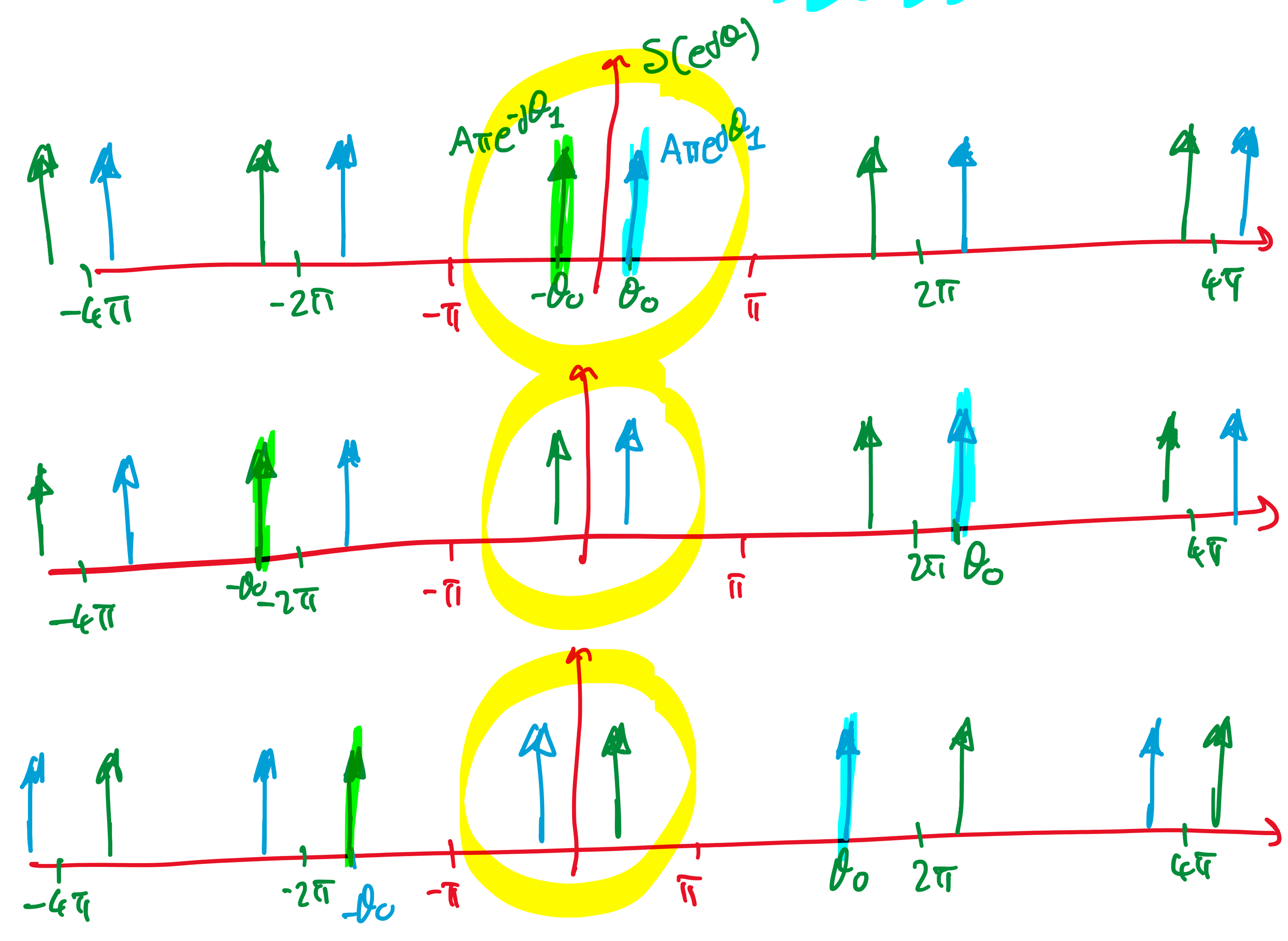
ESP. COMPLESSI E DELTA TRASLATI SONO DUALI

Es1d

$s(n) = A\cos(\theta_0 n + \theta_1) = \frac{Ae^{j\theta_1}}{2} \cdot e^{j\theta_0 n} + \frac{Ae^{-j\theta_1}}{2}\, e^{-j\theta_0 n}$

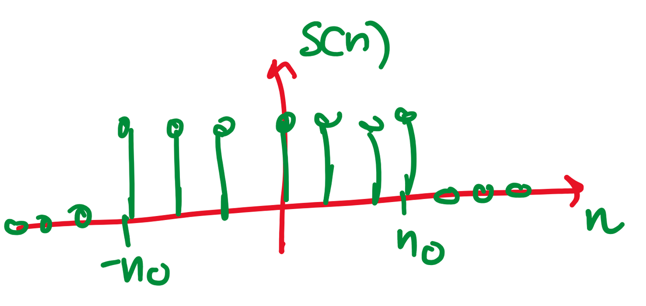
$S(e^{j\theta}) = ?$

$S(e^{j\theta}) = \frac{Ae^{j\theta_1}}{2}\, 2\pi\, \text{rep}_{2\pi}\, \delta(\theta-\theta_0) + \frac{Ae^{-j\theta_1}}{2}\, 2\pi\, \text{rep}_{2\pi}\, \delta(\theta+\theta_0)$



X CASA

$s(n) = \text{rect}\left(\frac{n}{2n_0+1}\right)$



$S(e^{j\theta}) = \frac{\sin(\theta(n_0 + 1/2))}{\sin(\theta/2)}$