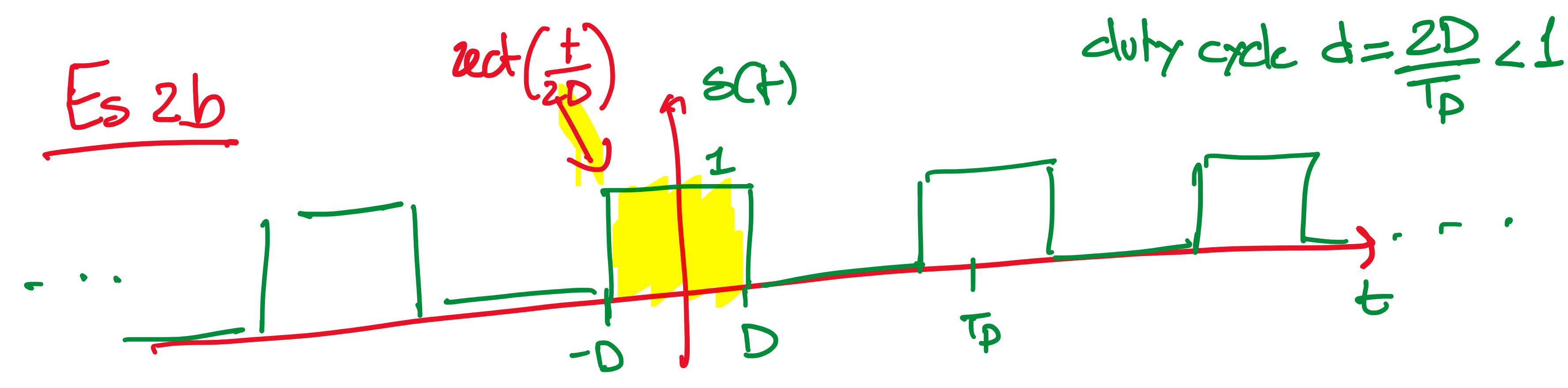
Le19

Thursday, 2 May 2024 08:41

Es 2b

duty cycle $d = \frac{2D}{T_P} < 1$



$S_K = ?$ $\quad \omega_0 = 2\pi / T_P$

$s(t) = \text{rep}_{T_P}\, u(t)$ $\qquad u(t) = \text{rect}\left(\frac{t}{2D}\right)$

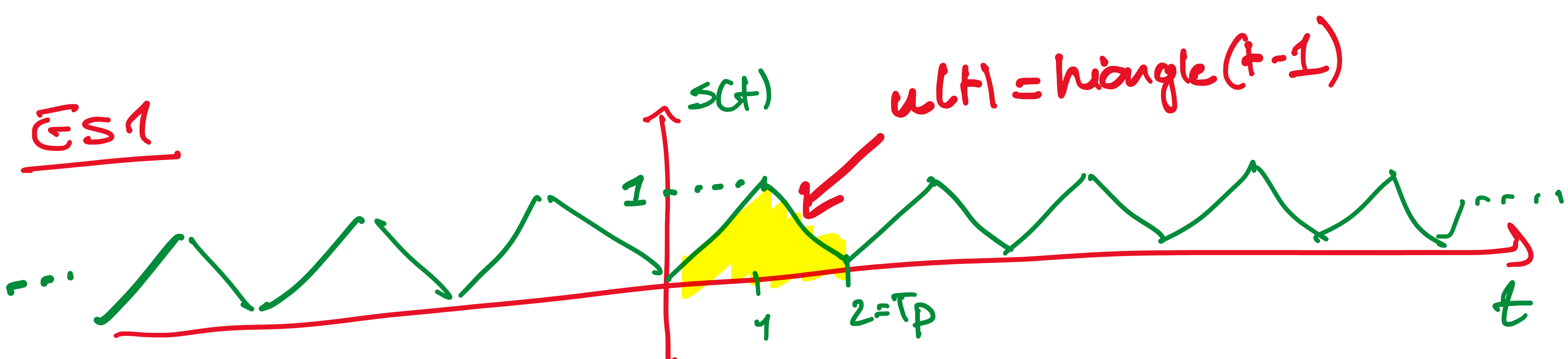
$\downarrow \mathcal{F}$ $\quad \downarrow \mathcal{F}$ $\qquad \downarrow \mathcal{F}$ $\quad \downarrow \mathcal{F}$

$S_K = \frac{1}{T_P}\, U(jK\omega_0)$ $\qquad U(j\omega) = 2D\, \text{sinc}\left(\frac{\omega}{2\pi} 2D\right)$

$= \frac{1}{T_P} \cdot 2D\, \text{sinc}\left(\frac{2D}{2\pi} \cdot K \frac{2\pi}{T_P}\right)$

$= d\, \text{sinc}(Kd)$

Es 1



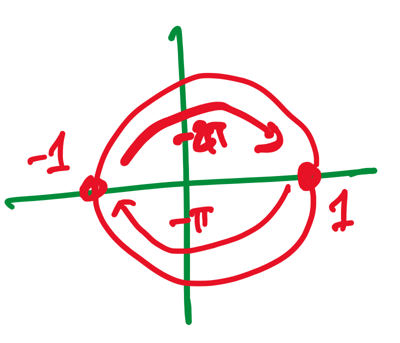
$S_K = ?$ $\quad \omega_0 = \frac{2\pi}{T_P} = \frac{2\pi}{2} = \pi$

$s(t) = \text{rep}_2\, u(t)$ $\qquad u(t) = \text{triangle}(t-1)$

$S_K = \frac{1}{2} U(jK\pi)$ $\qquad U(j\omega) = \text{sinc}^2\left(\frac{\omega}{2\pi}\right) e^{-j\omega \cdot 1}$

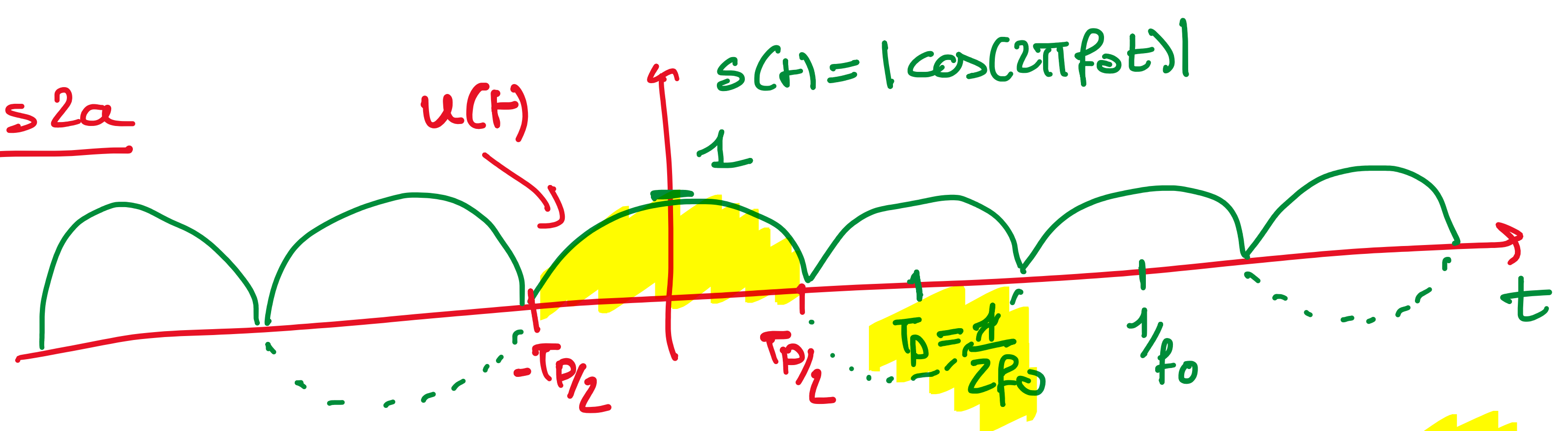
T_P $\quad \omega_0$

$= \frac{1}{2} \text{sinc}^2\left(\frac{1}{2\pi} \cdot K\pi\right) \underbrace{e^{-jK\pi}}_{(-1)^K}$



$= \frac{1}{2} \text{sinc}^2\left(\frac{K}{2}\right) (-1)^K$

Es 2a



$S_K = ?$ $\quad \omega_0 = \frac{2\pi}{T_P} = 4\pi f_0$

$s(t) = \text{rep}_{T_P}\, u(t)$ $\qquad u(t) = \underbrace{\cos(2\pi f_0 t)}_{x(t)} \cdot \underbrace{\text{rect}\left(\frac{t}{T_P}\right)}_{y(t)}$

$S_K = \frac{1}{T_P} U(jK\omega_0)$ $\qquad U(j\omega) = \frac{1}{2\pi} X * Y(j\omega)$

$X(j\omega) = \pi\delta(\omega - 2\pi f_0) + \pi\delta(\omega + 2\pi f_0)$

$= \pi\delta(\omega - \frac{1}{2}\omega_0) + \pi\delta(\omega + \frac{1}{2}\omega_0)$

$Y(j\omega) = T_P\, \text{sinc}\left(\frac{\omega}{2\pi} T_P\right)$

$= T_P\, \text{sinc}\left(\frac{\omega}{\omega_0}\right)$

$U(j\omega) = \frac{1}{2\cancel{\pi}}\left[\cancel{\pi}\delta(\omega - \frac{1}{2}\omega_0) + \cancel{\pi}\delta(\omega + \frac{1}{2}\omega_0)\right] * T_P\, \text{sinc}\left(\frac{\omega}{\omega_0}\right)$

$= \frac{T_P}{2} \text{sinc}\left(\frac{\omega - \frac{1}{2}\omega_0}{\omega_0}\right) + \frac{T_P}{2} \text{sinc}\left(\frac{\omega + \frac{1}{2}\omega_0}{\omega_0}\right)$

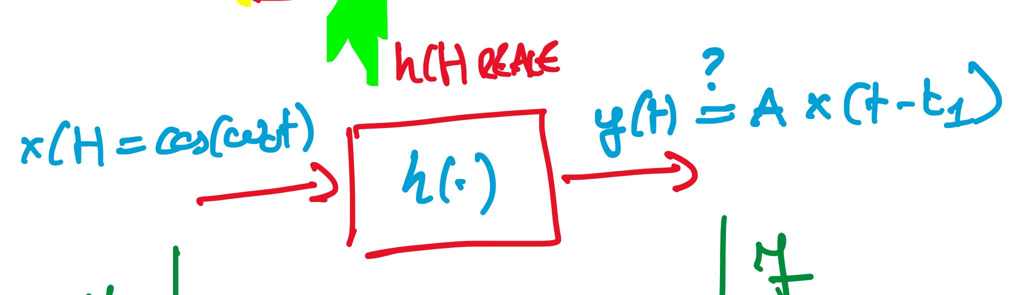
$= \frac{T_P}{2} \text{sinc}\left(\frac{\omega}{\omega_0} - \frac{1}{2}\right) + \frac{T_P}{2} \text{sinc}\left(\frac{\omega}{\omega_0} + \frac{1}{2}\right)$

$S_K = \frac{1}{T_P} U(jK\omega_0) = \frac{1}{\cancel{T_P}}\left[\frac{\cancel{T_P}}{2} \text{sinc}\left(\frac{K\omega_0}{\omega_0} - \frac{1}{2}\right) + \frac{\cancel{T_P}}{2} \text{sinc}\left(\frac{K\omega_0}{\omega_0} + \frac{1}{2}\right)\right]$

$= \frac{1}{2} \text{sinc}(K - \frac{1}{2}) + \frac{1}{2} \text{sinc}(K + \frac{1}{2})$

Es 1

UN FILTRO REALE DISTORCE $x(t) = \cos(\omega_0 t)$?



$\downarrow \mathcal{F}$ $\qquad \downarrow \mathcal{F}$

$X(j\omega) \longrightarrow \otimes \longrightarrow Y(j\omega) = X(j\omega) H(j\omega)$

$H(j\omega)$ HERMITIANA $H(-j\omega) = H^*(j\omega)$

$X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$

$Y(j\omega) = \pi H(j\omega_0)\delta(\omega - \omega_0) + \pi \underbrace{H(-j\omega_0)}_{H^*(j\omega_0)}\delta(\omega + \omega_0)$

$\downarrow \mathcal{F}^{-1}$

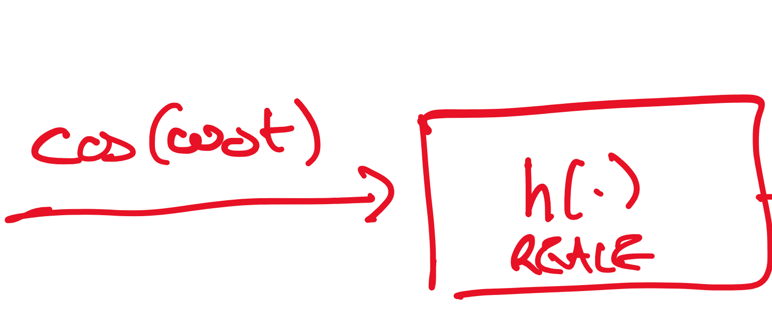
$y(t) = \cancel{\pi} H(j\omega_0) \cdot \frac{1}{2\cancel{\pi}} e^{j\omega_0 t} + \cancel{\pi} H^*(j\omega_0) \frac{1}{2\cancel{\pi}} e^{-j\omega_0 t}$

$= \frac{H(j\omega_0) e^{j\omega_0 t} + H^*(j\omega_0) e^{-j\omega_0 t}}{2}$

$= \text{Re}\left[H(j\omega_0) e^{j\omega_0 t}\right]$ $\qquad H(j\omega_0) = H_1 e^{j\varphi_1}$

$= \text{Re}\left[H_1 e^{j(\omega_0 t + \varphi_1)}\right]$

$= H_1 \cos(\omega_0 t + \varphi_1)$



$H_1 \cos(\omega_0 t + \varphi_1) = H_1 \cos\left(\omega_0\left(t + \frac{\varphi_1}{\omega_0}\right)\right)$

UN FILTRO REALE NON DISTORCE UNA SINUSOIDE