Le18

Friday, 19 April 2024 00:33

Es 35

AREA $s(t) = \text{sinc}^3(t) = \underbrace{\text{sinc}(t)}_{x(t)} \cdot \underbrace{\text{sinc}^2(t)}_{y(t)}$

$$A_s = S(j\omega)\big|_{\omega=0}$$

$$X(j\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)$$

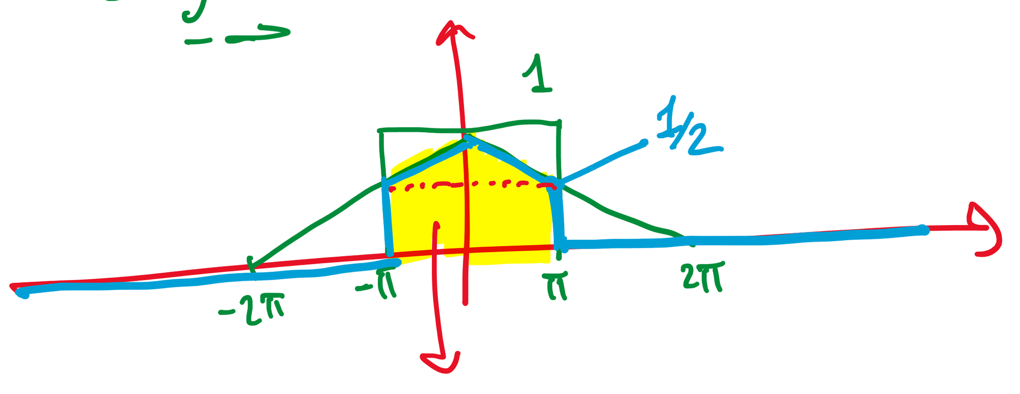
$$Y(j\omega) = \text{triangle}\left(\frac{\omega}{2\pi}\right)$$

$$S(j\omega) = \frac{1}{2\pi}\, X * Y(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(ju)\,Y(j(\omega-u))\,du$$

$$A_s = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(ju)\,Y(-ju)\,du$$

FUNZIONE PARI

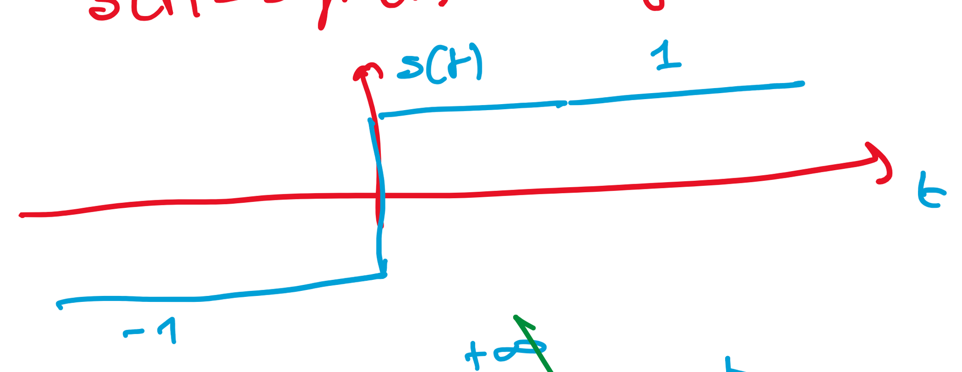
$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \text{rect}\left(\frac{\omega}{2\pi}\right)\text{triang}\left(\frac{+\omega}{2\pi}\right)d\omega$$



$$A_s = \frac{1}{2\pi}\cdot\frac{3}{4}\cdot 2\pi = \frac{3}{4}$$

Es 1h

$s(t) = \text{sign}(t)$ $S(j\omega) = ?$



$$S(j\omega) = \int_{-\infty}^{+\infty} s(t)\,e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{0} -1\, e^{-j\omega t}\,dt + \int_{0}^{+\infty} e^{-j\omega t}\,dt$$

$\frac{e^{-j\omega t}}{j\omega}$

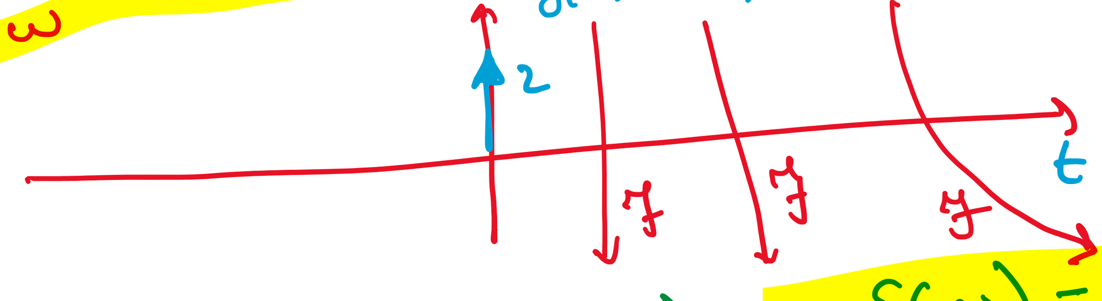
$$= -\left.\frac{e^{-j\omega t}}{-j\omega}\right]_{-\infty}^{0} + \left.\frac{e^{-j\omega t}}{-j\omega}\right]_{0}^{+\infty}$$

indeterminato

$$= \frac{1 - e^{+j\omega\infty}}{j\omega} + \frac{e^{-j\omega\infty} - 1}{-j\omega}$$

$$= \frac{2 - e^{j\omega\infty} - e^{-j\omega\infty}}{j\omega} = \frac{2 - 2\cos(\omega\infty)}{j\omega}$$

$y(t) = s'(t) = 2\delta(t)$

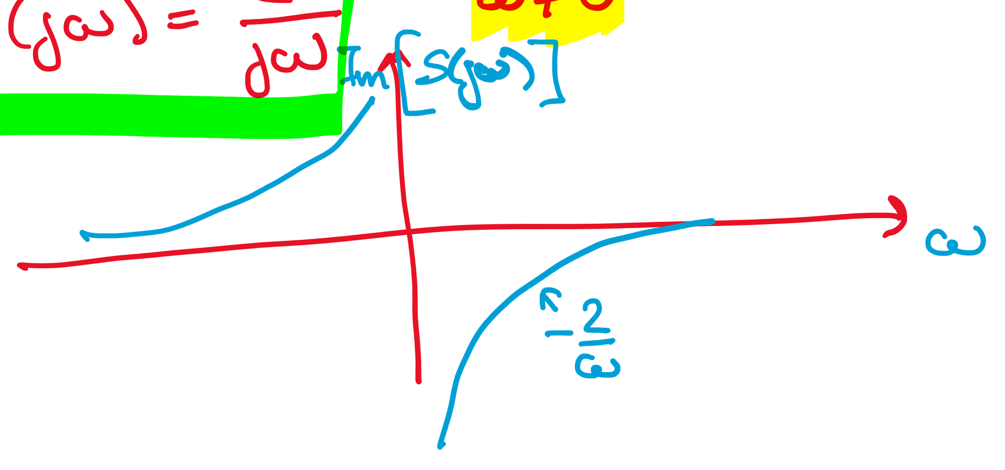


$\mathcal{F}$ $\mathcal{F}$ $\mathcal{F}$

$$Y(j\omega) = j\omega\, S(j\omega) = 2$$

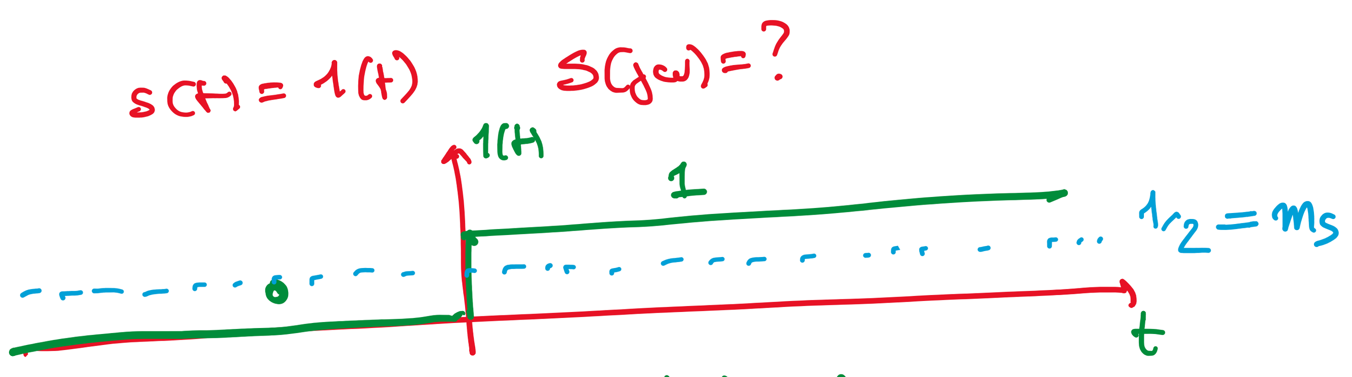
$$\boxed{S(j\omega) = \frac{2}{j\omega}}$$

$\omega \neq 0$

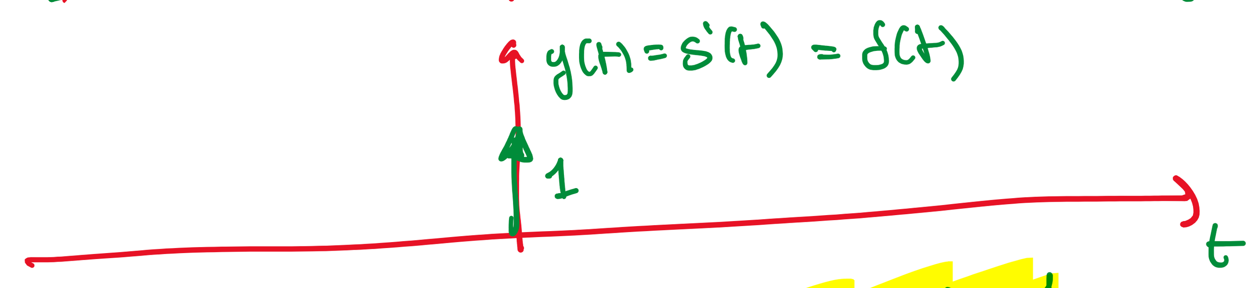


Es 1σ

$s(t) = 1(t)$ $S(j\omega) = ?$



$y(t) = s'(t) = \delta(t)$



$$Y(j\omega) = j\omega\, S(j\omega) = 1$$

$$S(j\omega) = \begin{cases} \frac{1}{j\omega} & \omega \neq 0 \\ ? & \omega = 0 \end{cases}$$

$$\text{sign}(t) \xrightarrow{\mathcal{F}} \frac{2}{j\omega}$$

$$1(t) = \frac{1}{2}\,\text{sign}(t) + \frac{1}{2} \xrightarrow{\mathcal{F}} \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{j\omega} + \frac{1}{2}\cdot 2\pi\cdot\delta(\omega)$$

m_s

$$\boxed{\begin{array}{l} \text{sign}(t) \xrightarrow{\mathcal{F}} \frac{2}{j\omega} \\ 1(t) \xrightarrow{\mathcal{F}} \frac{1}{j\omega} + \pi\delta(\omega) \end{array}}$$

RISULTATI NOTEVOLI

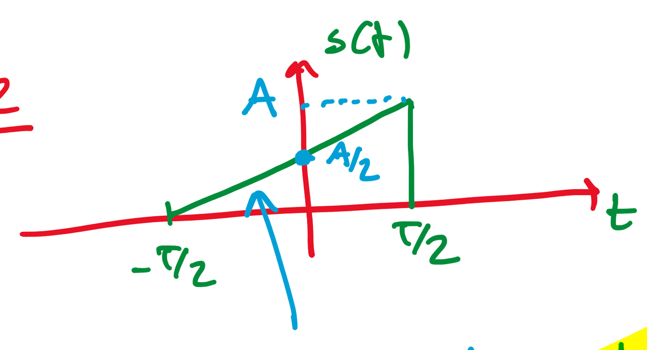
NOTA INVERSIONE REGOLA DI DERIVAZIONE

$$y(t) = s'(t) \xrightarrow{\mathcal{F}} \underline{Y(j\omega)} = j\omega\, S(j\omega)$$

SIA CONOSCIUTA

$$\boxed{S(j\omega) = \frac{Y(j\omega)}{j\omega} + m_s\cdot 2\pi\delta(\omega)}$$

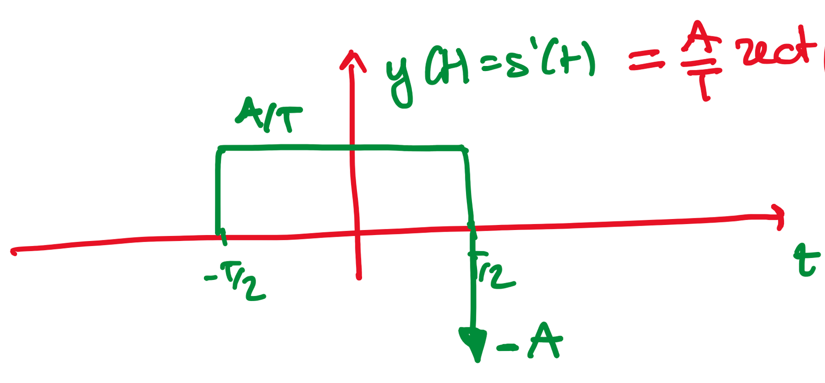
Es 2



$S(j\omega) = ?$

$$\frac{A}{2} + t\,\frac{A/2}{T/2} = \frac{A}{2} + \frac{tA}{T}$$

$$y(t) = s'(t) = \frac{A}{T}\,\text{rect}\left(\frac{t}{T}\right) - A\,\delta(t - T/2)$$



$\downarrow \mathcal{F}$

$$Y(j\omega) = \frac{A}{\cancel{T}}\cdot\cancel{T}\,\text{sinc}\left(\frac{\omega T}{2\pi}\right) - A\,e^{-j\omega T/2}$$

$$= j\omega\, S(j\omega)$$

$$\boxed{S(j\omega) = \frac{A}{j\omega}\left[\text{sinc}\left(\frac{\omega T}{2\pi}\right) - e^{-j\omega T/2}\right]}$$

Es 2 SOLUZIONE ALTERNATIVA

$$s(t) = A\,\text{rect}\left(\frac{t}{T}\right)\left(\frac{1}{2} + \frac{t}{T}\right)$$

$$= \underbrace{\frac{A}{2}\,\text{rect}\left(\frac{t}{T}\right)}_{x(t)} + t\cdot\underbrace{\frac{A}{T}\,\text{rect}\left(\frac{t}{T}\right)}_{y(t)}$$

$\mathcal{F}$ $\mathcal{F}$

$$X(j\omega) = \frac{A}{2}\cdot T\,\text{sinc}\left(\frac{\omega T}{2\pi}\right) \qquad Y(j\omega) = \frac{A}{\cancel{T}}\cdot\cancel{T}\,\text{sinc}\left(\frac{\omega T}{2\pi}\right)$$

$$s(t) = x(t) + t\,y(t)$$

$$S(j\omega) = X(j\omega) + j\,Y'(j\omega)$$

$$= \frac{AT}{2}\,\text{sinc}\left(\frac{\omega T}{2\pi}\right) + j\,\frac{A\cdot T}{2\pi}\cdot\text{sinc}'\left(\frac{\omega T}{2\pi}\right)$$