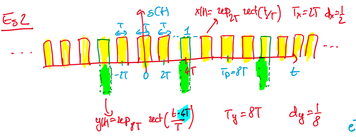
Le16

Monday, 15 April 2024 10:28

Es 2



$s(t) = x(t) - y(t) \rightarrow Y_k = \frac{1}{8}\,\text{sinc}\left(\frac{k}{8}\right) e^{-j\frac{\pi}{8}k} e^{-j\frac{\pi}{8}k}$

$X_m = \frac{1}{2}\,\text{sinc}\left(\frac{m}{2}\right)$ $\quad \omega_x = \frac{2\pi}{2T}$ $\quad \omega_y = \frac{2\pi}{8T} = \frac{\pi}{4T}$

S_k: $\omega_s = \omega_x = \omega_y = \frac{\pi}{4T}$

$$X_k = \begin{cases} \frac{1}{2}\,\text{sinc}\left(\frac{k}{2}\right) & k = 4m \leftarrow m = \frac{k}{4} \\ 0 & k \neq 4m \end{cases}$$

$$S_k = \begin{cases} X_k - Y_k = \frac{1}{2}\,\text{sinc}\left(\frac{k}{2}\right) - \frac{1}{8}\,\text{sinc}\left(\frac{k}{8}\right) & k = 4m \\ = \frac{3}{8}\,\text{sinc}\left(\frac{k}{2}\right) & \\ -Y_k = +\frac{1}{8}\,\text{sinc}\left(\frac{k}{8}\right)(-1)^{k+1} & k \neq 4m \end{cases}$$

$$S_k = \frac{1}{8}\,\text{sinc}\left(\frac{k}{8}\right) \cdot \begin{cases} 3 & k = 4m \\ (-1)^{k+1} & k \neq 4m \end{cases}$$

$$S_k = \frac{1}{8}\,\text{sinc}\left(\frac{k}{8}\right)\left(1 + 2\cos(k\pi)\right)$$

Es 1a

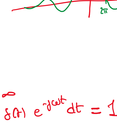
$s(t) = 1(t)\, e^{-at}$ $\quad a > 0$

$S(j\omega) = ?$

$$S(j\omega) = \int_{-\infty}^{+\infty} s(t)\, e^{-j\omega t}\, dt = \int_0^{+\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\Bigg]_0^{\infty} = \frac{0 - 1}{-(a+j\omega)}$$

$$= \frac{1}{a+j\omega}$$



PROVARE IN MATLAB!

Es 1c

$s(t) = \text{rect}(t)$

$S(j\omega) = ?$

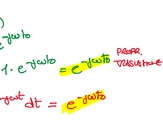
$$S(j\omega) = \int_{-\infty}^{+\infty} s(t)\, e^{-j\omega t}\, dt = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j\omega t}\, dt$$

$$= \frac{e^{-j\omega t}}{-j\omega}\Bigg]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{-e^{-j\frac{\omega}{2}} + e^{j\frac{\omega}{2}}}{+j\omega}$$

$$= \frac{2j\sin\left(\frac{\omega}{2}\right)}{j\omega} = \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$

$$\text{rect}(t) \xrightarrow{\mathcal{FT}} \text{sinc}\left(\frac{\omega}{2\pi}\right)$$



Es 1d

$s(t) = \delta(t)$

$S(j\omega) = ?$

$$S(j\omega) = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j\omega t}\, dt = 1$$

$$\delta(t) \xrightarrow{\mathcal{FT}} 1$$

Es

$s(t) = 1$

$S(j\omega) = ?$

$$S(j\omega) = \int_{-\infty}^{+\infty} 1 \cdot e^{-j\omega t}\, dt = ?$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} 2\pi\,\delta(\omega)\, e^{j\omega t}\, d\omega = \frac{1}{2\pi} 2\pi = 1$$

$\Rightarrow$ ANTITRASF.

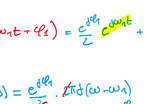
$$1 \xrightarrow{\mathcal{FT}} 2\pi\,\delta(\omega)$$

regola dualità segn. costante $\leftrightarrow$ delta

Es 1e

$s(t) = \delta(t - t_0)$

$S(j\omega) = ?$



$x(t) = \delta(t) \xrightarrow{\mathcal{FT}} 1 = X(j\omega)$

$s(t) = x(t - t_0) \xrightarrow{\mathcal{FT}} X(j\omega)\, e^{-j\omega t_0}$

$S(j\omega) = 1 \cdot e^{-j\omega t_0} = e^{-j\omega t_0}$ PURE DELAY

$$S(j\omega) = \int_{-\infty}^{+\infty} \delta(t - t_0)\, e^{-j\omega t}\, dt = e^{-j\omega t_0}$$

Es 1f

$s(t) = e^{j\omega_1 t} \cdot 1$

$S(j\omega) = ?$

$$1 \xrightarrow{\mathcal{FT}} 2\pi\,\delta(\omega)$$

$$s(t) = 1 \cdot e^{j\omega_1 t} \xrightarrow{\mathcal{FT}} 2\pi\,\delta(\omega - \omega_1) = S(j\omega)$$

ASSUMI DI MANIPOLARE!

CONTINUO E ANTITRASFORMO

$$s(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \underbrace{2\pi\,\delta(\omega - \omega_1)}_{S(j\omega)}\, e^{j\omega t}\, d\omega = e^{j\omega_1 t}$$

$$\delta(t - t_0) \xrightarrow{\mathcal{FT}} e^{-j\omega t_0}$$

$$e^{j\omega_1 t} \xrightarrow{\mathcal{FT}} 2\pi\,\delta(\omega - \omega_1)$$

DUALITÀ: EFFETTO / SEGNALE ESPONENZIALE

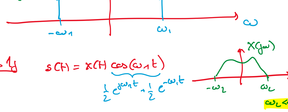
Es 1g

$s(t) = \cos(\omega_1 t + \varphi_1) = \frac{e^{j\varphi_1}}{2} e^{j\omega_1 t} + \frac{e^{-j\varphi_1}}{2} e^{-j\omega_1 t}$

$S(j\omega) = ?$

$$S(j\omega) = \frac{e^{j\varphi_1}}{2} \cdot 2\pi\,\delta(\omega - \omega_1) + \frac{e^{-j\varphi_1}}{2} 2\pi\,\delta(\omega + \omega_1)$$

SPETTRO RICOSTITUITA (PICCHI SIMMETRICI DOPPIO)



Es 1i

$s(t) = x(t)\cos(\omega_1 t)$

$\frac{1}{2}e^{j\omega_1 t} + \frac{1}{2}e^{-j\omega_1 t}$

$$s(t) = \frac{1}{2}x(t)\, e^{j\omega_1 t} + \frac{1}{2}x(t)\, e^{-j\omega_1 t}$$

$$S(j\omega) = \frac{1}{2}X(j(\omega - \omega_1)) + \frac{1}{2}X(j(\omega + \omega_1))$$