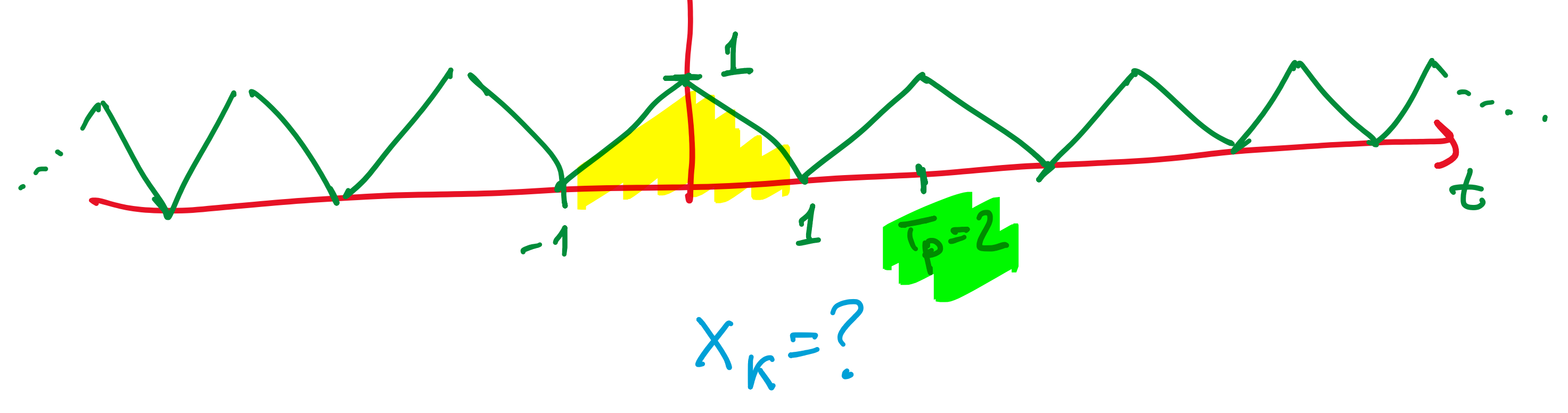
Le15

Friday, 12 April 2024 10:31

Es8

$x(t) = \text{rep}_2\, \text{triang}(t)$



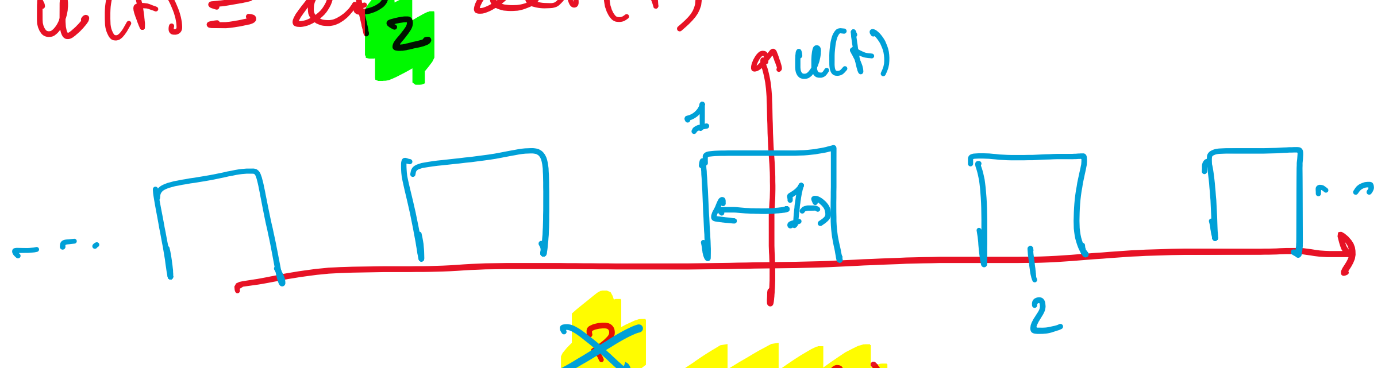
$X_K = ?$

x casa usare regola di derivazione

$\text{triang}(t) = \text{rect} * \text{rect}(t)$

$u(t) = \text{rep}_2\, \text{rect}(t)$ ONDA QUADRA $d=1/2$

$U_K = \frac{1}{2}\,\text{sinc}\left(\frac{K}{2}\right)$



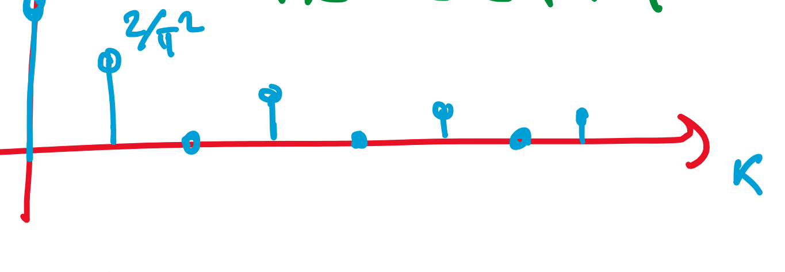
$x(t) = \text{rep}_2\, \text{triang}(t) \overset{\cancel{?}}{=} u * u(t)$

PERIODICA

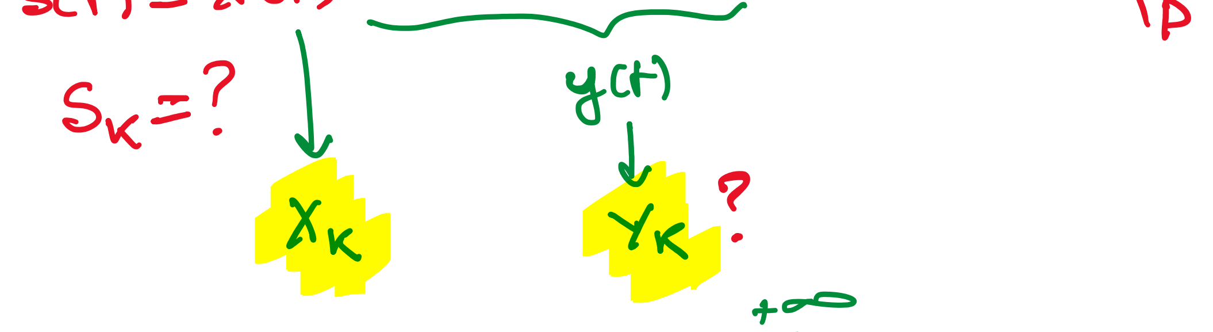
$X_K = T_p \cdot U_K \cdot U_K = \cancel{2} \cdot \frac{1}{\cancel{4}\,2}\,\text{sinc}^2\left(\frac{K}{2}\right)$

$= \frac{1}{2}\,\text{sinc}^2\left(\frac{K}{2}\right)$

$$= \begin{cases} 1/2 & K=0 \\ \frac{1}{2}\cdot\frac{\sin^2\left(\frac{K\pi}{2}\right)}{\left(\frac{K\pi}{2}\right)^2} & K\neq 0 \end{cases}$$



$$X_K = \begin{cases} 1/2 & K=0 \\ 0 & K\ \text{pari } K\neq 0\ \cancel{\text{dispari}} \\ \frac{2}{(K\pi)^2} & K\ \cancel{\text{pari } K\neq 0}\ \text{dispari} \end{cases}$$



REALE E PARI ✓

Es 1d

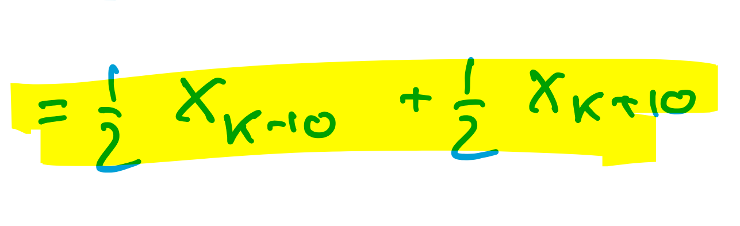
$s(t) = x(t)\, \underbrace{\cos(10\omega_0 t)}_{y(t)}$ periodico T_p $\qquad \omega_0 = \frac{2\pi}{T_p}$

$S_K = ?$ $\quad x(t) \to X_K \qquad y(t) \to Y_K\ ?$

$$S_K = X * Y_K = \sum_{m=-\infty}^{+\infty} X_m Y_{K-m}$$

$$y(t) = \cos(10\omega_0 t) = \frac{e^{j10\omega_0 t}}{2} + \frac{e^{-j10\omega_0 t}}{2}$$

$$Y_K = \frac{1}{2}\delta(K-10) + \frac{1}{2}\delta(K+10)$$



$$S_K = X_K * \left(\frac{1}{2}\delta(K-10) + \frac{1}{2}\delta(K+10)\right)$$

$$= \frac{1}{2} X_K * \delta_{K-10} + \frac{1}{2} X_K * \delta_{K+10}$$

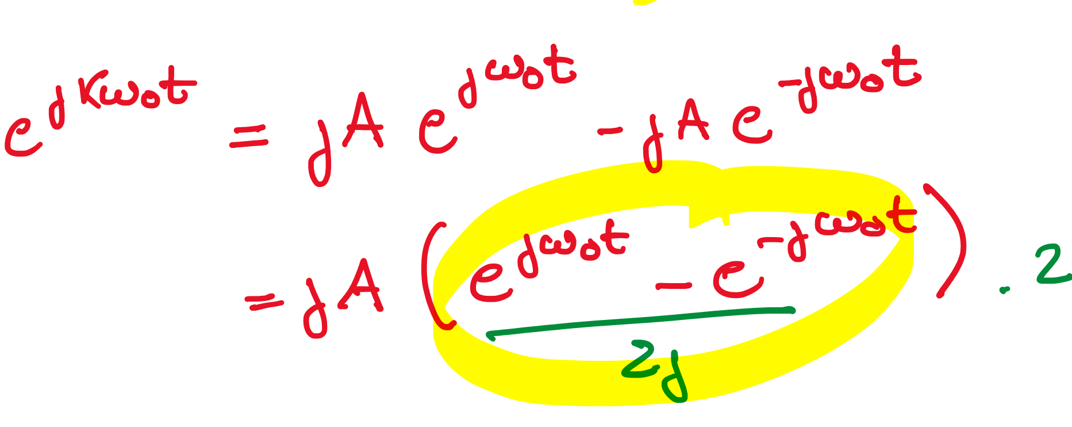
$$= \frac{1}{2} X_{K-10} + \frac{1}{2} X_{K+10}$$

Es 4

SIA DATO $s(t)$ TALE CHE

- REALE e DISPARI $\rightarrow$ S_K ~~HERMITIANI~~ IMMAGINARIO DISPARI
- PERIODICO $T_p=2$
- COEFF. FOURIER per $|K|>1$ SONO NULLI
  $S_K=0$ per $|K|>1$
- $P_s=1$

$s(t)=?$



$$P_s = \sum_K |S_K|^2 = 2A^2 = 1$$

$$A^2 = \frac{1}{2} \rightarrow A = \pm\frac{1}{\sqrt{2}}$$

$$s(t) = \sum_K S_K e^{jK\omega_0 t} = jA e^{j\omega_0 t} - jA e^{-j\omega_0 t}$$

$$= jA\left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right)\cdot 2j$$

$$= 2Aj^2 \sin(\omega_0 t)$$

$$= -2A\sin(\omega_0 t)$$

$$= \pm\sqrt{2}\sin(\omega_0 t)$$

$$\omega_0 = \frac{2\pi}{T_p} = \frac{2\pi}{2} = \pi$$

$$s(t) = \pm\sqrt{2}\sin(\pi t)$$

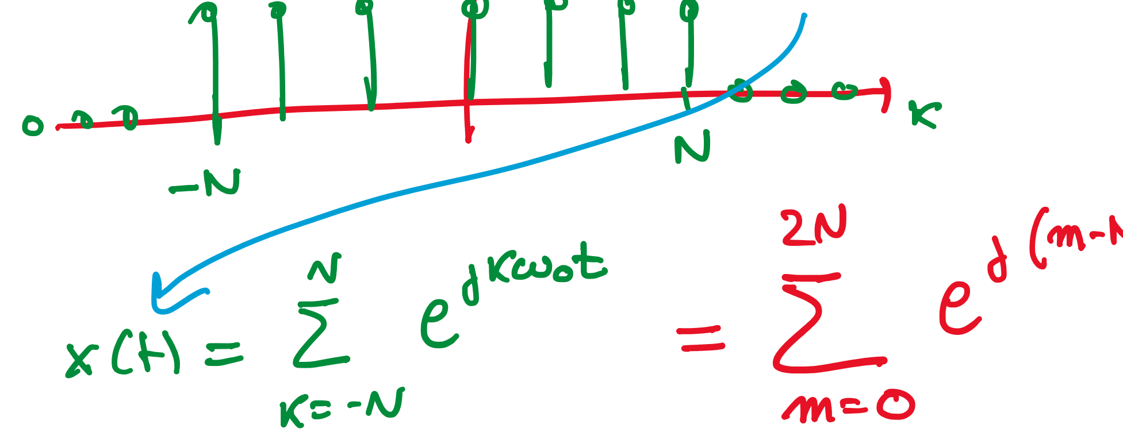
Es 5 SINC PERIODICO

$$s(t) = \frac{3}{5}\,\frac{\sin(\pi t)}{\sin(\pi t/5)}$$

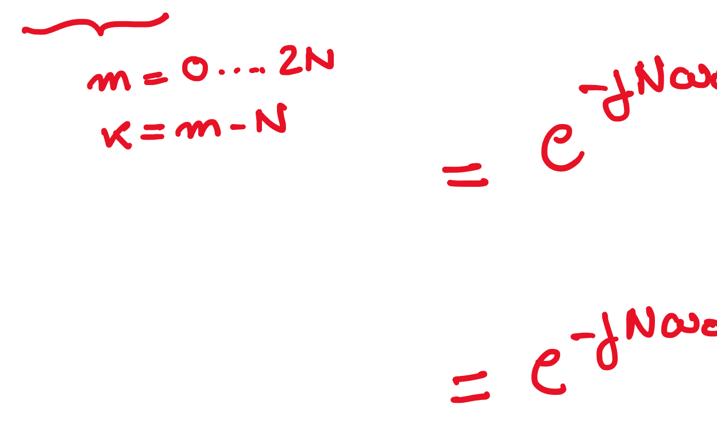
PERIODICO 2

PERIODICO 10

$S_K = ?$
$P_s = ?$



CLASSE DI SEGNALI



Reale e pari

$$x(t) = \sum_{\underbrace{K=-N}_{}}^{N} e^{jK\omega_0 t} = \sum_{m=0}^{2N} e^{j(m-N)\omega_0 t}$$

$m = 0 \dots 2N$
$K = m-N$

$$= e^{-jN\omega_0 t}\sum_{m=0}^{2N} e^{jm\omega_0 t} \qquad \alpha^m \quad \alpha = e^{j\omega_0 t}$$

$$= e^{-jN\omega_0 t}\cdot\frac{1-\alpha^{2N+1}}{1-\alpha}$$

$$= e^{-jN\omega_0 t}\cdot\frac{1-e^{j(2N+1)\omega_0 t}}{1-e^{j\omega_0 t}}$$

$$= \frac{e^{-jN\omega_0 t} - e^{j(N+1)\omega_0 t}}{1 - e^{j\omega_0 t}}\cdot\frac{e^{-j\frac{\omega_0}{2}t}}{e^{-j\omega_0/2\, t}}$$

$$x(t) = \frac{e^{-j(N+\frac{1}{2})\omega_0 t} - e^{j(N+\frac{1}{2})\omega_0 t}}{e^{-j\frac{\omega_0}{2}t} - e^{j\frac{\omega_0}{2}t}} = \frac{\cancel{2j}\,\sin\left(\left(N+\frac{1}{2}\right)\omega_0 t\right)}{\cancel{2j}\,\sin\left(\frac{\omega_0}{2}t\right)}$$

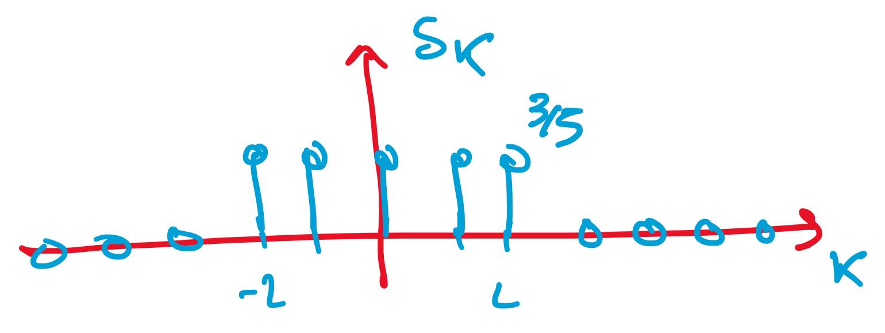
$$s(t) = \frac{3}{5}\,\frac{\sin(\pi t)}{\sin\left(\frac{\pi t}{5}\right)}$$

$\frac{\omega_0}{2} = \frac{\pi}{5} \rightarrow \omega_0 = \frac{2\pi}{5}$ DEN.

$T_p=5$

NUM. $5\cancel{\pi} = \left(N+\frac{1}{2}\right)\frac{2\cancel{\pi}}{\cancel{5}} = 2N+1$

$2N=4$
$N=2$



$$P_s = \sum_K |S_K|^2 = 5\cdot\left(\frac{3}{5}\right)^2 = \frac{9}{5}$$

$$m_s = S_0 = \frac{3}{5}$$