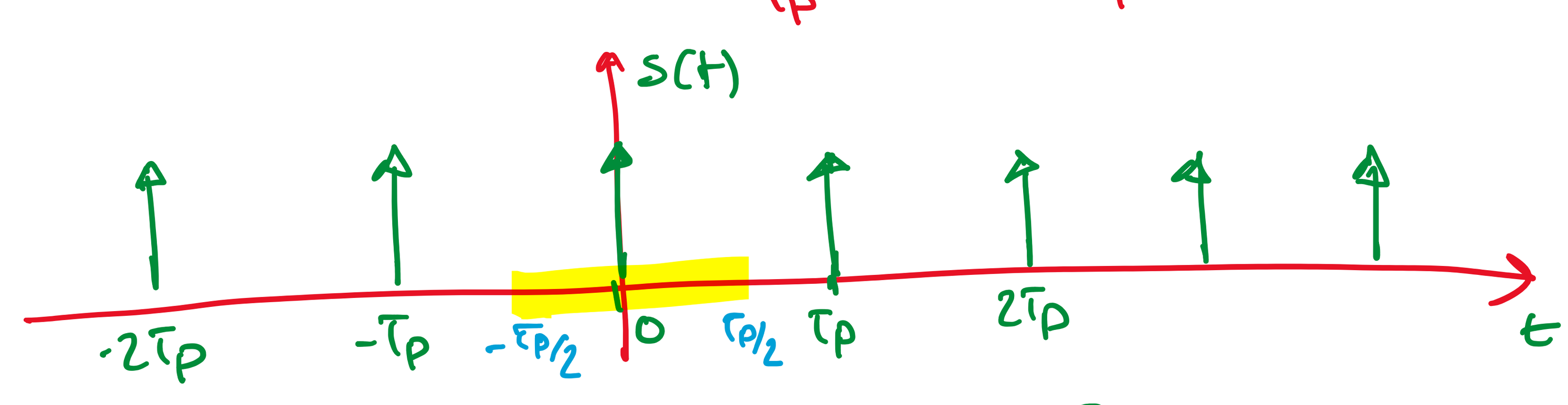
Le13

Thursday, 4 April 2024 08:51

Es 1a

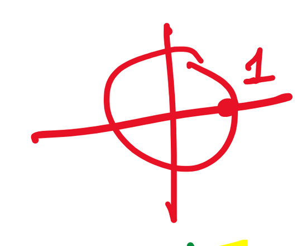
$s(t) = comb_{T_p}(t) = rep_{T_p}\,\delta(t)$



$S_k = ?$ serie di Fourier = ?

$S_k = \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} \delta(t)\, e^{-j k\omega_0 t}\,dt \qquad \omega_0 = 2\pi/T_p$

$= \frac{1}{T_p} e^{-jk\omega_0 \cdot 0} = \frac{1}{T_p}$



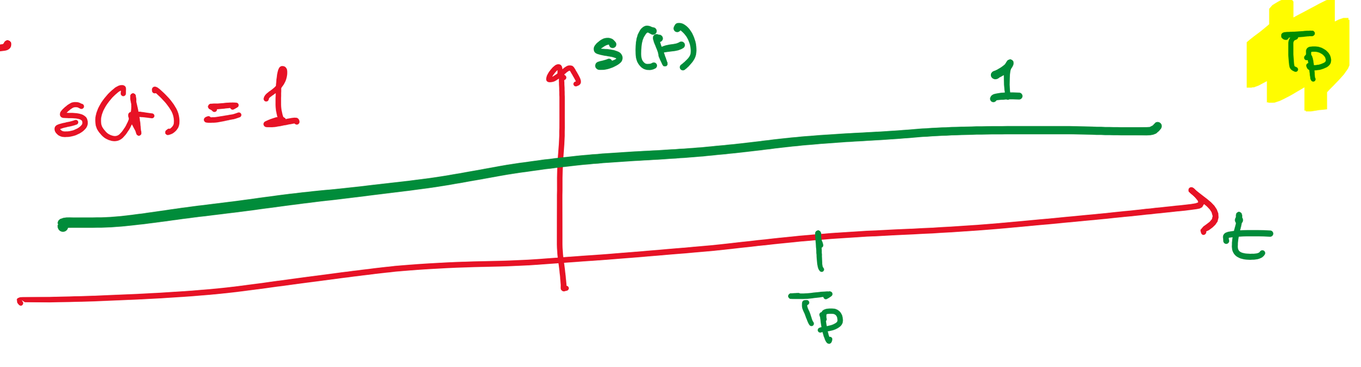
delta periodico

$s(t) = rep_{T_p}\,\delta(t) \xrightarrow{\mathcal{F}} S_k = \frac{1}{T_p}$ segnale costante

$s(t) = \sum_{k=-\infty}^{+\infty} S_k e^{jk\omega_0 t} = \frac{1}{T_p}\sum_{k=-\infty}^{+\infty} e^{j2\pi \frac{kt}{T_p}}$ serie di Fourier

Es 1b

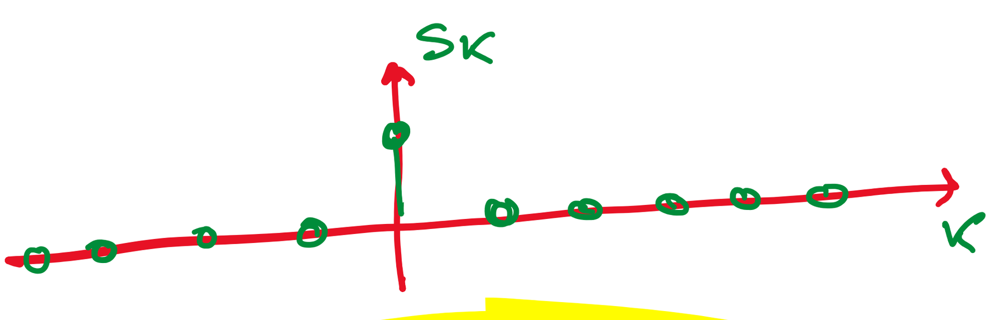
$s(t) = 1$



$S_k = \frac{1}{T_p}\int_0^{T_p} 1 \cdot e^{-jk\omega_0 t}\,dt \qquad \omega_0 = 2\pi/T_p$

$= \begin{cases} 1 & k=0 \\ 0 & k\neq 0 \end{cases}$

$k \neq 0$: $= \frac{1}{T_p}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_0^{T_p} = \frac{e^{-jk\frac{2\pi}{T_p}T_p} - 1}{-jk\omega_0 T_p} = \frac{1-1}{-jk\omega_0 T_p} = 0$



$s(t) = 1 \xrightarrow{\mathcal{F}} S_k = \delta(k)$

segnale costante — delta discreto

DUALITÀ DELTA - COSTANTE

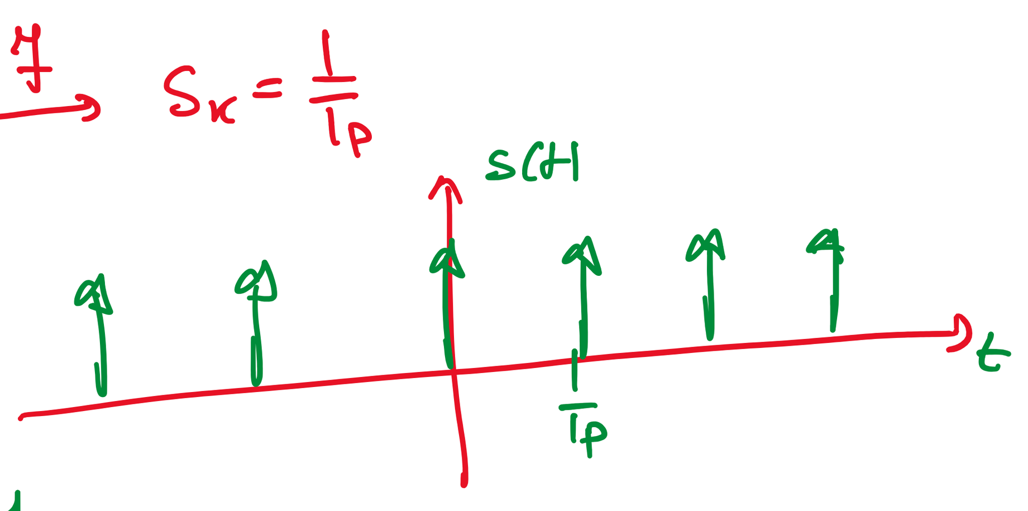
delta $\xleftrightarrow{\mathcal{F}}$ segnale costante

$s(t) = \sum_{k=-\infty}^{+\infty} S_k e^{+jk\omega_0 t} = 1$

VALOR MEDIO E POTENZA

A) $s(t) = rep_{T_p}\,\delta(t) \xrightarrow{\mathcal{F}} S_k = \frac{1}{T_p}$

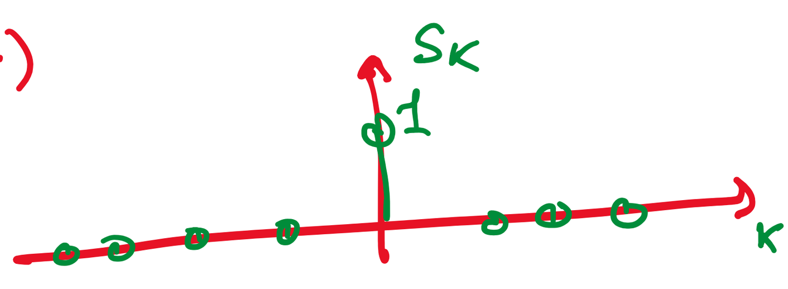
$m_s = S_0 = \frac{1}{T_p}$



$P_s = \sum_k |S_k|^2 = \sum_{k=-\infty}^{+\infty} \frac{1}{T_p^2} = \infty$

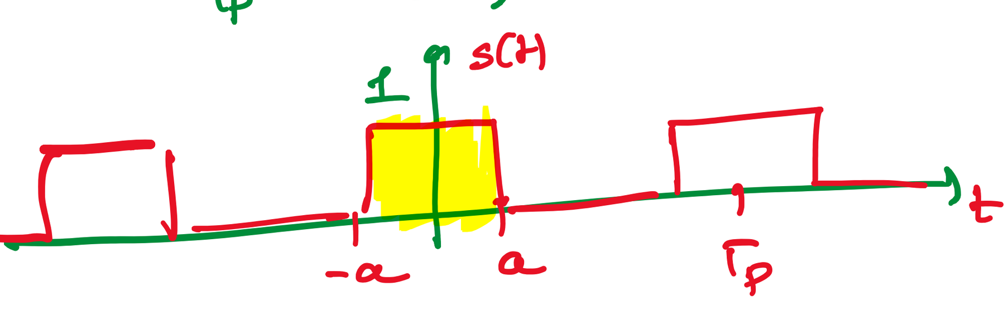
B) $s(t) = 1 \longrightarrow S_k = \delta(k)$

$m_s = S_0 = 1$



$P_s = \sum_k |S_k|^2 = 1$

C) $s(t) = rep_{T_p}\, rect\left(\frac{t}{2a}\right) \longrightarrow S_k = d\, sinc(kd) \qquad d = \frac{2a}{T_p}$



$m_s = S_0 = d$

$P_s = \sum_{k=-\infty}^{+\infty} |S_k|^2 = \sum_{k=-\infty}^{+\infty} d^2 sinc^2(kd) = ?$

$= d$ ma calcolandolo dal dominio del tempo

Es 1c

$s(t) = \cos(\omega_0 t + \varphi_0) \qquad \omega_0 = 2\pi/T_p$

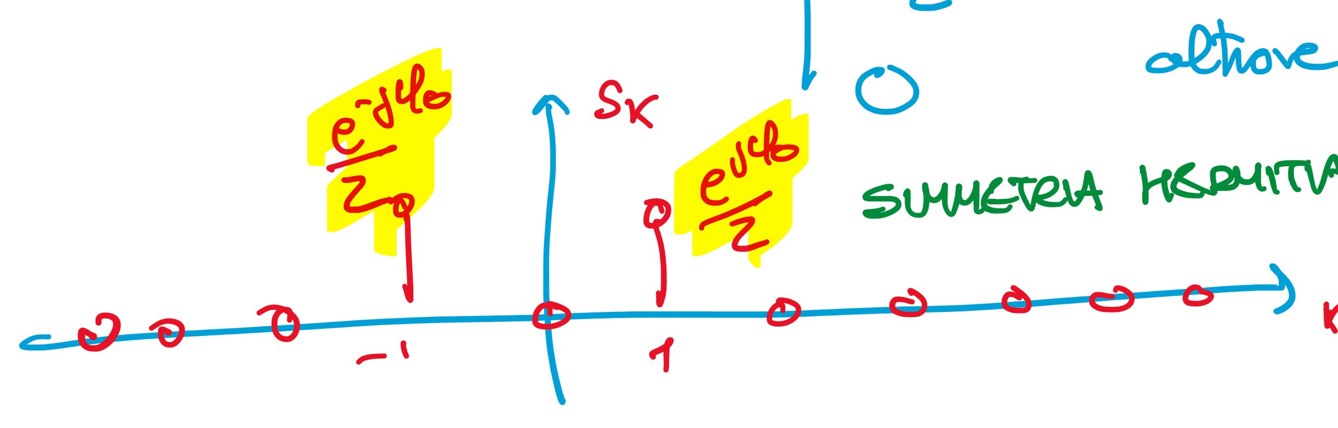
$S_k = ?$

$m_s = ? \quad P_s = ?$

$s(t) = \underbrace{\frac{e^{j\varphi_0}}{2}}_{S_1} e^{j\omega_0 t} + \underbrace{\frac{e^{-j\varphi_0}}{2}}_{S_{-1}} e^{-j\omega_0 t}$ ($k=1$, $k=-1$)

PER ISPEZIONE $= \sum_k S_k e^{jk\omega_0 t}$

$S_k = \begin{cases} \frac{e^{j\varphi_0}}{2} & k=1 \\ \frac{e^{-j\varphi_0}}{2} & k=-1 \\ 0 & \text{altrove} \end{cases}$



SIMMETRIA HERMITIANA $S_{-k} = S_k^*$

$m_s = S_0 = 0$

$P_s = \sum_k |S_k|^2 = \left|\frac{e^{j\varphi_0}}{2}\right|^2 + \left|\frac{e^{-j\varphi_0}}{2}\right|^2$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Es

$s(t) = 3 - \sin(2t) + 4\cos(2t) + 2\cos(6t - \pi/4)$

($0 \cdot \omega_0$; $\omega_0 = 2$; $3\omega_0 = 6$)

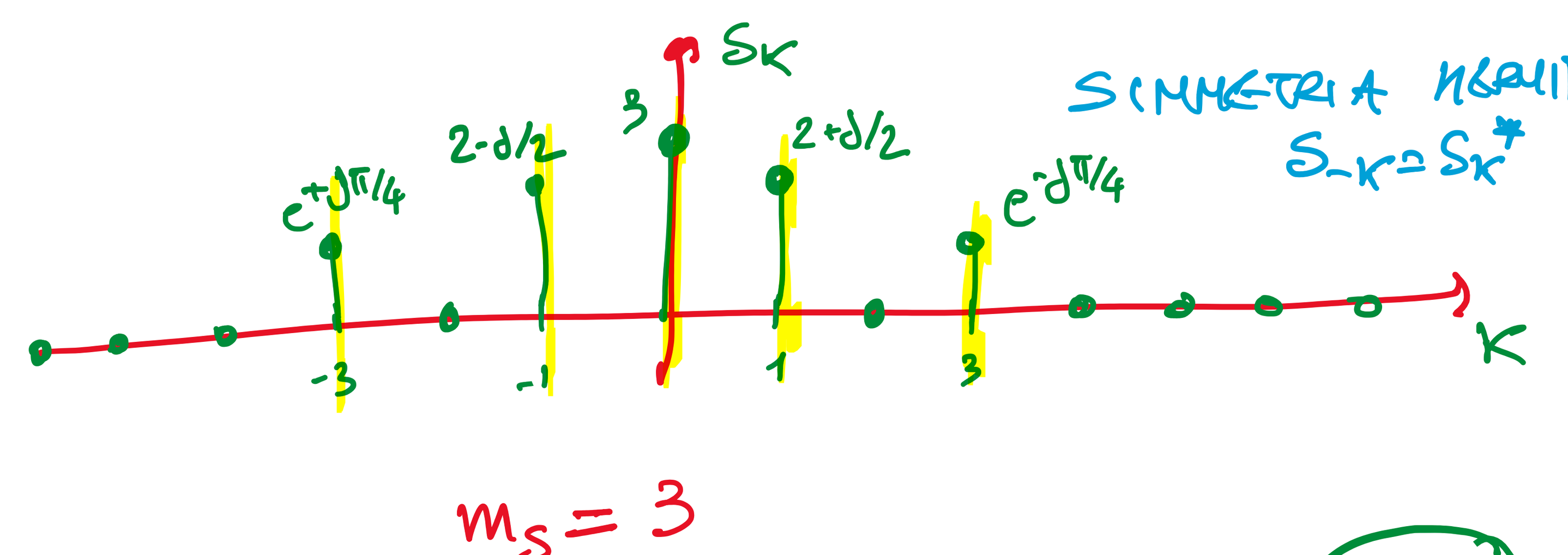
$S_k = ? \quad m_s = ? \quad P_s = ?$

$s(t) = 3 + j\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2} + 4\,\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$ (4 / 2 → · 2)

$+ 2\,\frac{e^{j3\omega_0 t}e^{-j\pi/4} + e^{-j3\omega_0 t}e^{j\pi/4}}{2}$

$= 3 + \left(\frac{j}{2} + 2\right)e^{j\omega_0 t} + \left(-\frac{j}{2} + 2\right)e^{-j\omega_0 t}$

$+ e^{-j\pi/4}e^{j3\omega_0 t} + e^{j\pi/4}e^{-j3\omega_0 t}$



SIMMETRIA HERMITIANA $S_{-k} = S_k^*$

$m_s = 3$

$P_s = \underbrace{3^2}_{9} + 2\underbrace{\left|2 + \frac{j}{2}\right|^2}_{4 + 1/4} + 2\underbrace{\left|e^{-j\pi/4}\right|^2}_{1}$

$= 9 + 8 + \frac{1}{2} + 2 = \frac{39}{2}$