Le9

Friday, 15 March 2024 10:29

Es continua

complessi coniugati

$$z(n)=\sum_{m=0}^{\infty}\alpha^m\left(A+\frac{1}{2}e^{j\theta_0(n-m)}+\frac{1}{2}e^{-j\theta_0(n-m)}\right)$$

$\frac{1}{2}B+\frac{1}{2}B^*=\frac{B+B^*}{2}=\mathrm{Re}[B]$

$$=A\underbrace{\sum_{m=0}^{\infty}\alpha^m}_{\frac{1}{1-\alpha}}+\frac{e^{j\theta_0 n}}{2}\sum_{m=0}^{\infty}\alpha^m e^{-j\theta_0 m}+\frac{e^{-j\theta_0 n}}{2}\sum_{m=0}^{\infty}\alpha^m e^{j\theta_0 m}$$

$\alpha^m e^{-j\theta_0 m}=(\alpha e^{-j\theta_0})^m \rightarrow \frac{1}{1-\alpha e^{-j\theta_0}}$

$\alpha^m e^{j\theta_0 m}=(\alpha e^{j\theta_0})^m \rightarrow \frac{1}{1-\alpha e^{j\theta_0}}$

$$=\frac{A}{1-\alpha}+\frac{e^{j\theta_0 n}}{2}\cdot\frac{1}{1-\alpha e^{-j\theta_0}}+\frac{e^{-j\theta_0 n}}{2}\cdot\frac{1}{1-\alpha e^{j\theta_0}}$$

$$=\frac{A}{1-\alpha}+\mathrm{Re}\left[\frac{e^{j\theta_0 n}}{1-\alpha e^{-j\theta_0}}\right]$$

$=B=|B|e^{j\varphi_B}$

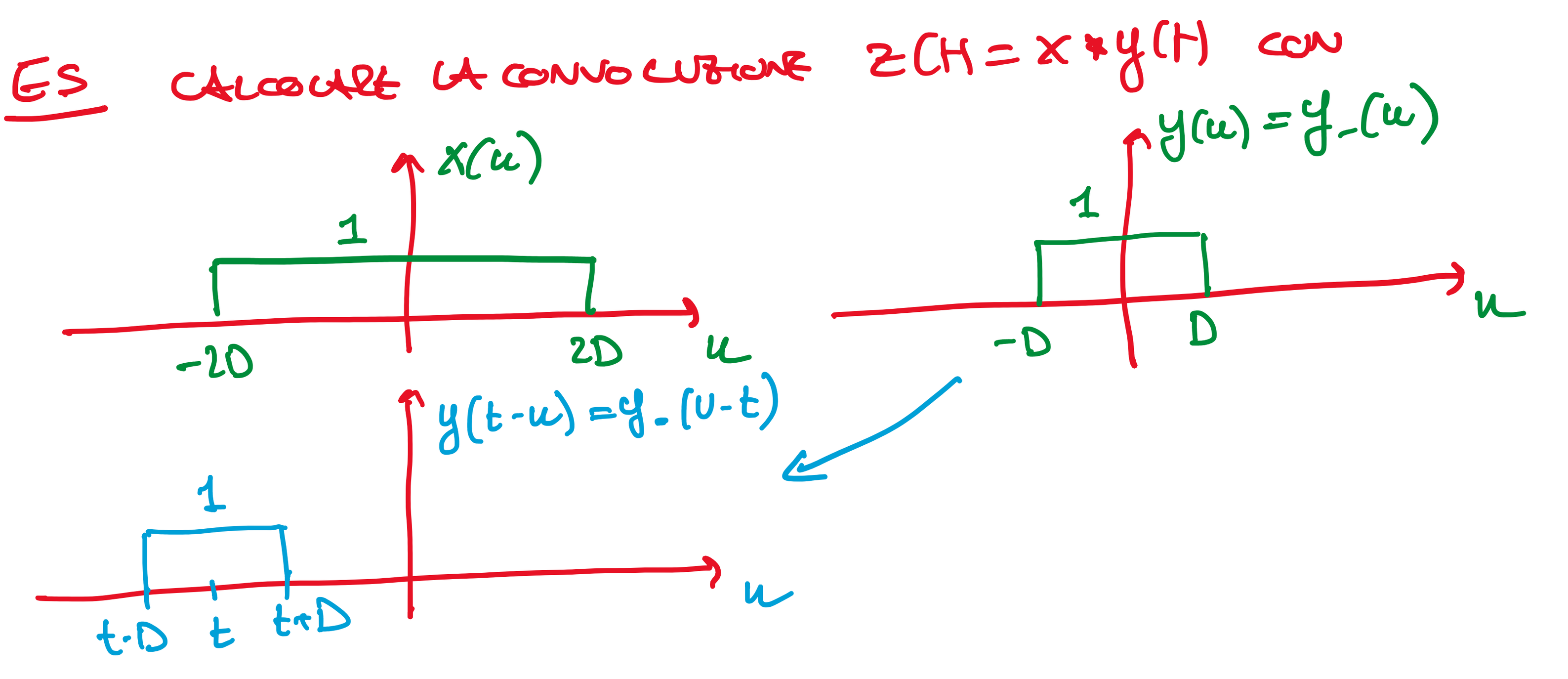
$$=\frac{A}{1-\alpha}+\mathrm{Re}\left[\frac{e^{j(\theta_0 n-\varphi_B)}}{|B|}\right]$$

$$z(n)=\frac{A}{1-\alpha}+\frac{\cos(\theta_0 n-\varphi_B)}{|B|}=x*y(n)$$

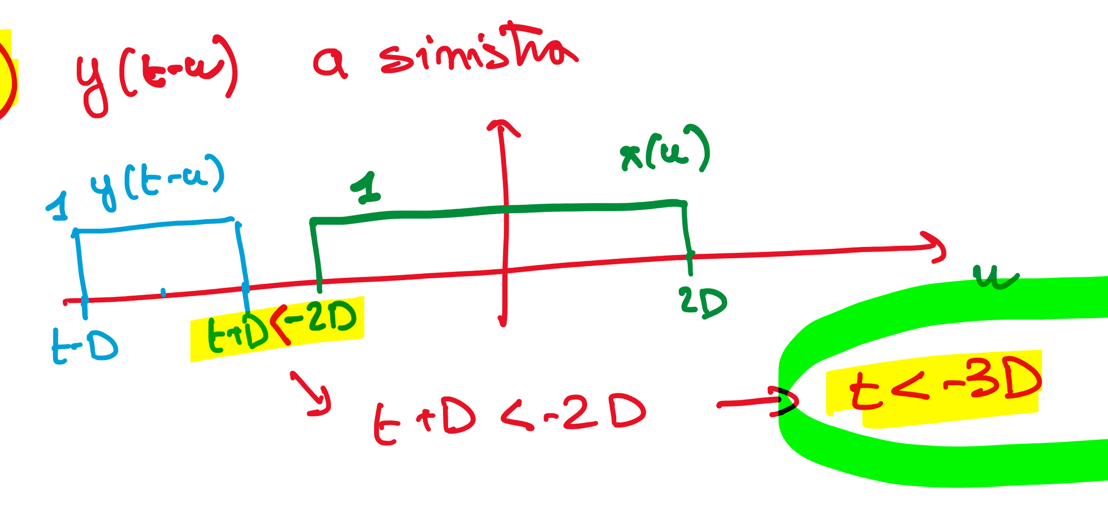
$x(n)=A+\cos(\theta_0 n)$

$y(n)=\alpha^n 1_0(n)$ $\quad\alpha$ reale, $|\alpha|<1$

Es Calcolare la convoluzione $z(t)=x*y(t)$ con $y(u)=y_-(u)$

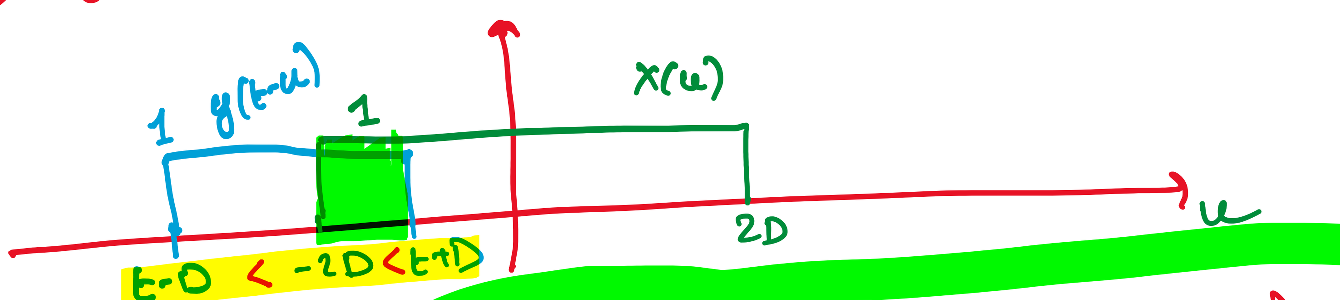


A) $y(t-u)$ a sinistra



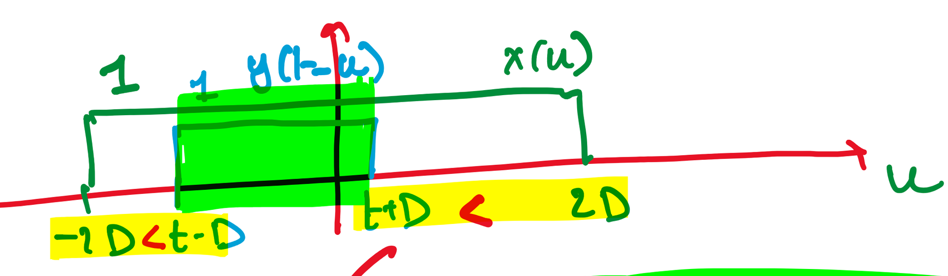
$t+D<-2D \rightarrow t<-3D \quad z(t)=0$

B) $y(t-u)$ entra da sinistra



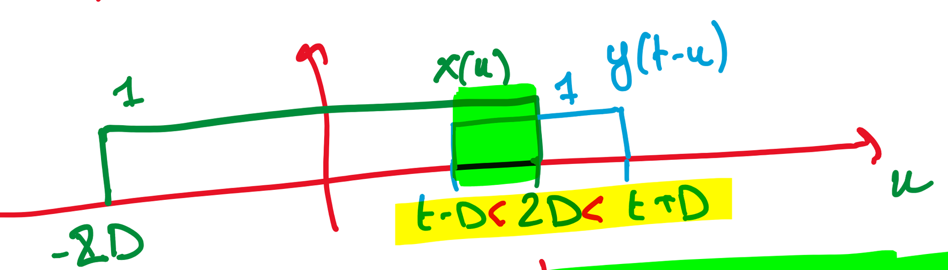
$t-D<-2D<t+D \rightarrow -3D<t<-D \quad z(t)=t+D-(-2D)=t+3D$

C) $y(t-u)$ dentro $x(u)$



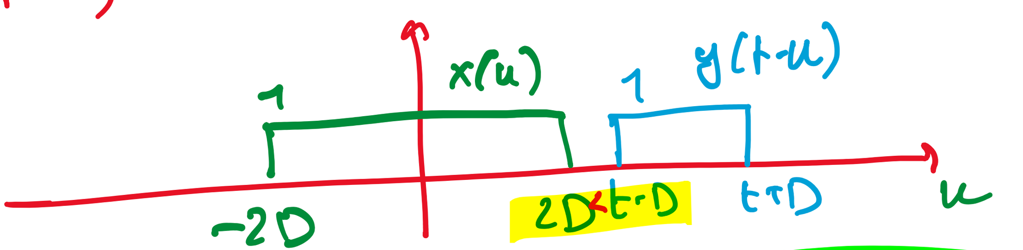
$-2D<t-D$, $t+D<2D \rightarrow -D<t<D \quad z(t)=2D$

D) $y(t-u)$ esce da destra



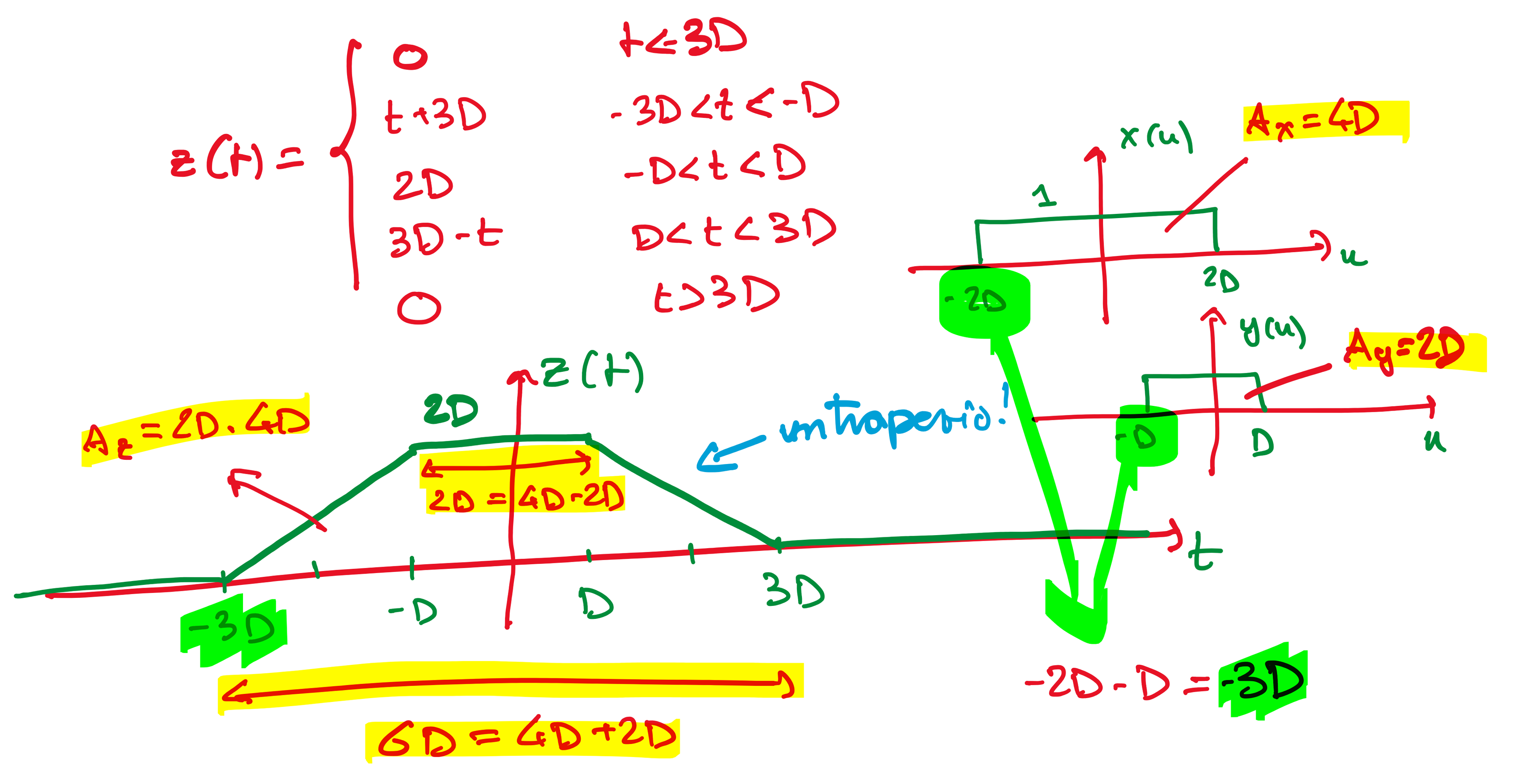
$t-D<2D<t+D \rightarrow D<t<3D \quad z(t)=2D-(t-D)=3D-t$

E) $y(t-u)$ a destra

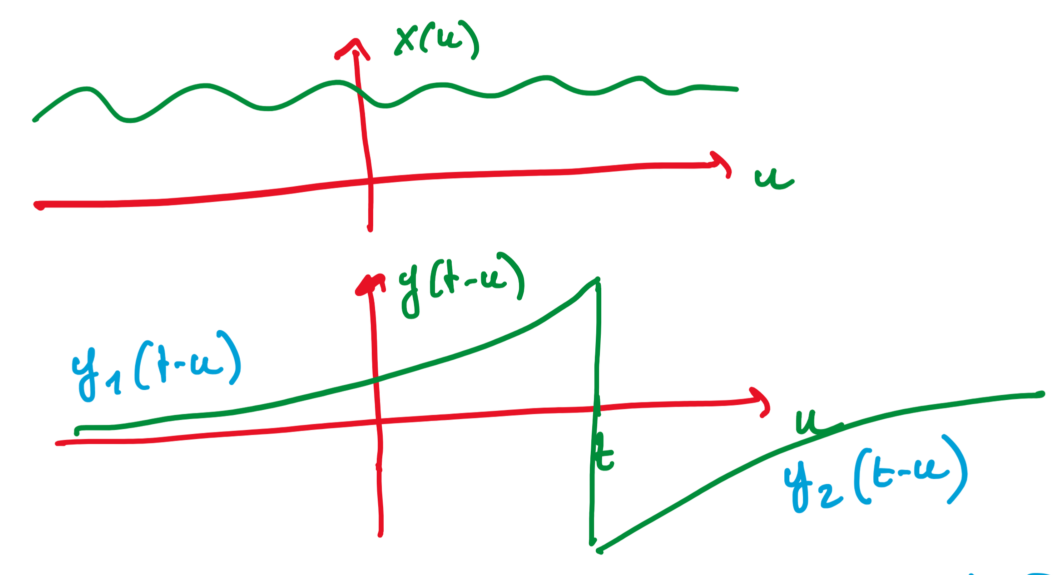


$2D<t-D \rightarrow t>3D \quad z(t)=0$

$$z(t)=\begin{cases}0 & t<-3D\\ t+3D & -3D<t<-D\\ 2D & -D<t<D\\ 3D-t & D<t<3D\\ 0 & t>3D\end{cases}$$

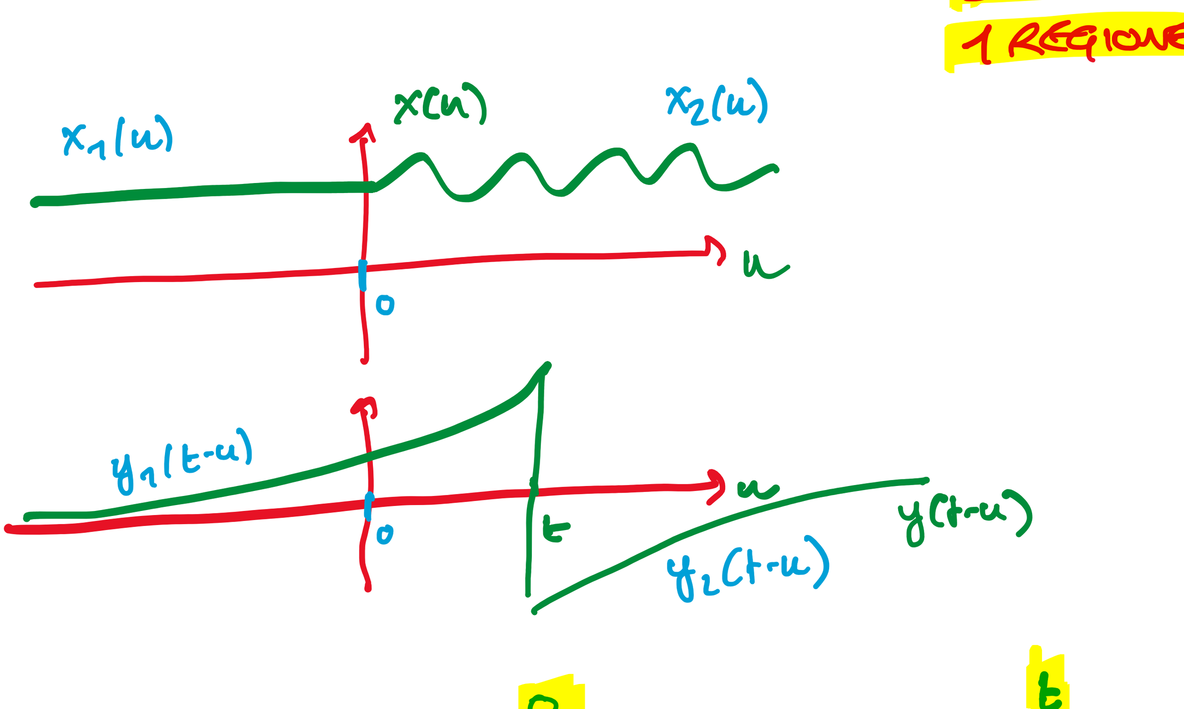


NOTA Regioni nell'integrale di convoluzione

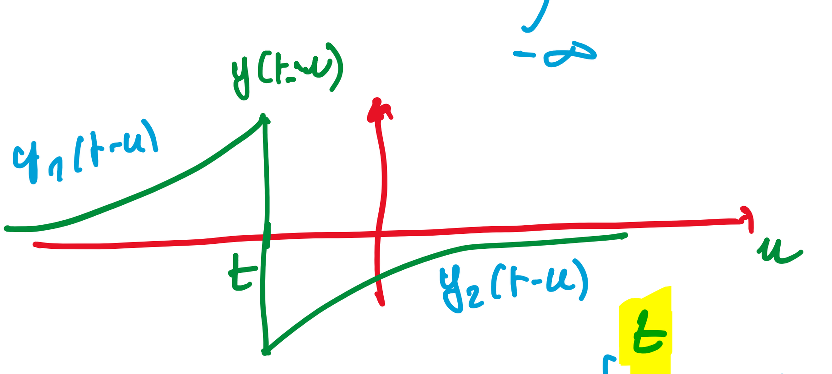


$$z(t)=\underbrace{\int_{-\infty}^{t}x(u)\,y_1(t-u)\,du}_{\text{expr 1}}+\underbrace{\int_{t}^{+\infty}x(u)\,y_2(t-u)\,du}_{\text{expr 2}}$$

2 integrali
1 regione



per $t>0 \rightarrow z(t)=\int_{-\infty}^{0}x_1(u)\,y_1(t-u)\,du+\int_{0}^{t}x_2(u)\,y_1(t-u)\,du+\int_{t}^{+\infty}x_2(u)\,y_2(t-u)\,du$



per $t<0 \rightarrow z(t)=\int_{-\infty}^{t}x_1(u)\,y_1(t-u)\,du+\int_{t}^{0}x_1(u)\,y_2(t-u)\,du+\int_{0}^{+\infty}x_2(u)\,y_2(t-u)\,du$

2 regioni
3 integrali