ii) The error $E$ is, by definition:

\[ f(x_0) = f(x) + f'(x)(x-x_0) + O(E^2) \]

Taylor expand around $x$: \[ f(x) = f(x_0) + f'(x_0)(x-x_0) + O(E^2) \]

\[ E = f(x) - f(x_0) \]

At the next step we get $x'$ and \[ E' = x' - x_0 \]

Expand:

\[ f(x_0) = f(x') + f'(x')(x'-x_0) + O(E'^2) \]

Remember that $f(x_0) = 0$. Take the first expansion:
\[ f(x) + f'(x)(x-x_0) + O(x^2) = 0 \]
\[ \frac{1}{2} f''(x)(x-x_0)^2 \]
\[ f(x) + f'(x)(x-x_0) + \frac{1}{2} f''(x)(x-x_0)^2 = 0 \]

Now divide by \( f'(x) \):
\[ \frac{f(x)}{f'(x)} + (x-x_0) + \frac{1}{2} \frac{f''(x)}{f'(x)} (x-x_0)^2 = 0 \]

\[ \Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} + \frac{1}{2} \frac{f''(x_0)}{f'(x_0)} (x_0-x_0)^2 = x_0 + \frac{1}{2} \frac{f''(x_0)}{f'(x_0)} E^2 \]

\[ \text{Hence} \]
\[ \frac{E}{E_0} = 1 + \frac{1}{2} \frac{f''(x)}{f'(x)} E^2 \]

1) Newton's method converges quadratically. This is very fast:

\[ \begin{align*}
  x_1 &= x_0 + E_0 \\
  E_1 &= -\frac{1}{2} \frac{f''(x_0)}{f'(x_0)} E_0^2 \\
  E_2 &\approx -c E_1^2 = c E_0^4 \\
  E_3 &\approx -c E_2^2 = c^2 E_0^8 \\
  &\vdots
\end{align*} \]

\[ \Rightarrow |E_n| \approx \frac{(cE_0)^{2^n}}{c} \]

Exponential of an exponential.

iii) Set \( x_0 = x + E_0 \), \( x_1 = x_0 + cE_0^2 \). Hence:

\[ x - x_0 + E_0 (1 - cE_0^2) = 0 \quad \text{and} \quad x - x_1 + E_0 (1 - cE_0) = 0 \]

If \( E_0 \) is small we set \( E_0 (1 - cE_0) \approx E_0 \) and \( |x - x_1 | \approx E_0 \)