Geo 1 - mod A - Lezione 3 - 4/10/2023

Note Title

Dimostro che se $p(x) \in \mathbb{R}[x]$ pol. a coeff. reali nell' indet. x

se $\underset{a+ib}{\beta} \in \mathbb{C}$ e $p(\beta)=0 \Longrightarrow p(\bar\beta)=0$

$p(\cancel{\bar{\bar\beta}})=0 \Longleftarrow p(\bar\beta)=0$

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_0 \qquad a_i \in \mathbb{R}$$

$n \geq 1$

Sia $\beta \in \mathbb{C}$ t.c.

zero

$$0 = p(\beta) = a_n\beta^n + a_{n-1}\beta^{n-1} + \dots + a_0\cancel{x^0} = \sum_{j=0}^{n} a_j\beta^j$$

coniugato

$$\overline{0} = \overline{a_n\beta^n + \dots + a_0} = \overline{\sum a_j\beta^j}$$

$$0 = \overline{a_n}(\overline{\beta})^n + \overline{a_{n-1}}(\overline{\beta})^{n-1} + \dots + \overline{a_0} \qquad a_i = \overline{a_i} \in \mathbb{R}$$

$$= a_n(\overline{\beta})^n + a_{n-1}(\overline{\beta})^{n-1} + \dots + a_0$$

$$= p(\overline{\beta})$$

Conclusione $\beta \in \mathbb{C}$ è zero di $p(x) \in \mathbb{R}[x]$ se e solo se $\overline{\beta}$ è zero di $p(x)$

$$(x-\beta)(x-\overline{\beta}) = x^2 - \overline{\beta}x - \beta x + \beta\overline{\beta}$$

se $\beta \notin \mathbb{R}$

$$= x^2 - (\underbrace{\beta+\overline{\beta}}_{2\operatorname{Re}\beta})x + \underbrace{\beta\overline{\beta}}_{|\beta|^2} \in \mathbb{R}[x]$$

il pol. ha $\Delta < 0$ (verificarlo!)

<u>Conseguenza</u>: se $p(x) \in \mathbb{R}[x]$ ha grado dispari $\Rightarrow$ ha almeno uno zero reale (in $\mathbb{R}$)

Esercizio Calcolare modulo e argomento del numero

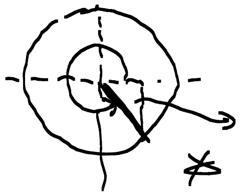
$$\frac{(1+i)^7}{(1-\sqrt{3}i)^4}$$

$|1+i| = \sqrt{2}$ $\qquad 1+i = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

$= \sqrt{2}\, e^{i\frac{\pi}{4}}$

$$(1+i)^7 = (\sqrt{2})^7\, e^{7\cdot\frac{\pi}{4}i}$$

$|1-\sqrt{3}i| = \sqrt{1+3} = 2$ $\qquad 1-\sqrt{3}i = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 2\, e^{i\frac{5\pi}{3}}$



$-\frac{\pi}{3} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$$(1-\sqrt{3}i)^4 = 2^4 \cdot e^{\frac{20\pi}{3}i} = \overset{(\sqrt{2})^8}{2^4} \cdot e^{\frac{2\pi}{3}i}$$

$\frac{20}{3}\pi = 2k\pi + ?$ $\qquad \frac{6\pi}{3} = 2\pi$

$= \underbrace{\boxed{\frac{18\pi}{3}}}_{3\cdot 2\pi} + \frac{2\pi}{3}$

$e^{2\pi i} = 1$

$e^{4\pi i} = 1$

$e^{6\pi i} = 1$

$e^{\frac{20}{3}\pi i} = \cancel{e^{6\pi i}} \cdot e^{\frac{2}{3}\pi i}$

$$\frac{(1+i)^7}{(1-\sqrt{3}i)^4} = \frac{(\sqrt{2})^7\, e^{i\frac{7}{4}\pi}}{2^4 \cdot e^{i\cdot\frac{2}{3}\pi}} = \frac{1}{\sqrt{2}} \cdot e^{i\left(\frac{7}{4}-\frac{2}{3}\right)\pi}$$

$$= \frac{1}{\sqrt{2}}\, e^{i\left(\frac{13}{12}\pi\right)}$$

risposta è:
- $\rho = \frac{1}{\sqrt{2}}$ modulo
- argom. $\theta = \frac{13}{12}\pi$

Scrivere le radici quadrate / cubiche / quarte / quinte... di 1 e disegnarle nel piano AG.

usando le formule di de Moivre.

$z_0 = \rho_0 e^{i\theta_0}$ $\qquad z^n = z_0$ $\qquad z = \rho e^{i\theta}$

$\rho = \sqrt[n]{\rho_0}$, $\theta = \frac{\theta_0}{n} + \frac{2k\pi}{n}$ $\qquad k = 0, 1, \dots, n-1$, poi si ripetono

$n=2$ $\quad z_0 = 1 = 1 \cdot e^{0i}$ $\quad \theta_0 = 0$ $\quad \rho_0 = 1$

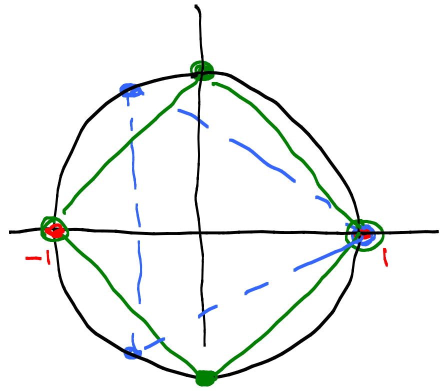
radici quadrate di 1 sono $\sqrt[2]{1} \cdot e^{i\frac{\theta_0}{2}}$, $\sqrt[2]{1}\, e^{i\left(\frac{\theta_0}{2}+\pi\right)}$

$$\sqrt[2]{1} \cdot e^{i\frac{\theta_0}{2}} = 1, \qquad \sqrt[2]{1}\, e^{i\left(\frac{\theta_0}{2}+\pi\right)} = -1$$

cubiche di 1

$\rho = \sqrt[3]{1} = 1$

$\theta = 0$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$



3 radici cubiche di 1

sono 1, $e^{i\frac{2\pi}{3}}$, $e^{i\frac{4}{3}\pi}$

$$e^{i\frac{2\pi}{3}} = -\frac{1}{2}+\frac{\sqrt{3}}{2}i, \qquad e^{i\frac{4}{3}\pi} = -\frac{1}{2}-\frac{\sqrt{3}}{2}i$$

4 radici quarte di 1

sono $\rho = \sqrt[4]{1} = 1$ $\quad \theta = 0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi$

$1, i, -1, -i$ sono vertici di un quadrato

e così via.

Esercizio: calcolare radici quadrate / cubiche / quarte ... di -1 e disegnarle