1. **[4 points]** Consider the regular expression \( R = (1 + \epsilon)(00^*1)^* \). Convert \( R \) into an equivalent \( \epsilon \)-NFA using the construction provided in the textbook, and report the intermediate steps.

2. **[7 points]** Consider the following languages, defined over the alphabet \( \{a, b\} \):

\[
L_1 = \{ba^nbb \mid n \geq 0, \text{n even}\};
\]
\[
L_2 = \{ba^nab \mid n \geq 0, \text{n even}\};
\]
\[
L_3 = L_2L_2.
\]

For each of the above languages, state whether it belongs to \( \text{REG} \) or else \( \text{CFL} \setminus \text{REG} \), and provide a mathematical proof for all of your answers.

3. **[6 points]** Consider the CFG \( G \) implicitly defined by the following productions:

\[
S \rightarrow BAB \mid BBB
\]
\[
A \rightarrow aB
\]
\[
B \rightarrow bA \mid \epsilon
\]

Perform on \( G \) the following transformations that have been specified in the textbook, in the given order.

(a) Eliminate the \( \epsilon \)-productions.
(b) Eliminate the unary productions.
(c) Eliminate the useless symbols.
(d) Produce a CFG \( G' \) in Chomsky normal form such that \( L(G') = L(G) \setminus \{\epsilon\} \).

Discuss each intermediate step, reporting the obtained CFGs.

(please see next page)
4. [8 points] Assess whether the following statements are true or false, providing motivations for all of your answers.

(a) Let $L$ be a language in CFL and let $L'$ be a finite language. Then the language $L \setminus L'$ is always in CFL.

(b) Let $L$ be a language in CFL and let $L'$ be an infinite language. Then the language $L \setminus L'$ is always in CFL.

(c) Let $L$ be a language in $\text{REC} \setminus \text{CFL}$. For every natural number $n$, there exists a string $w \in L$ such that $|w| \geq n$.

(d) If an NP-complete problem is in $\mathcal{P}$, then $\mathcal{P} = \mathcal{NP}$.

5. [5 points] Define the language $L_{ne}$ which we have studied in class. Prove that $L_{ne}$ does not belong to the class REC, using the proof presented in the textbook.

6. [3 points] Consider the following property of the RE languages defined over the alphabet $\Sigma = \{0, 1\}$:

$$\mathcal{P} = \{L \mid L \in \text{CFL}\}$$

and define $L_{\mathcal{P}} = \{\text{enc}(M) \mid L(M) \in \mathcal{P}\}$. Use Rice’s theorem to show that $L_{\mathcal{P}}$ is not in REC.