1. [5 points] Consider the DFA $A$ whose transition function is graphically represented below, where arcs with double direction represent two arcs in opposite directions.

(a) Provide the definition of equivalent pair of states for a DFA.
(b) Apply to $A$ the tabular algorithm for detecting pairs of equivalent states, reporting all the intermediate steps.
(c) Specify the minimal DFA equivalent to $A$.

2. [8 points] Consider the following languages, defined over the alphabet $\Sigma = \{a, b\}$

$$L_1 = \{w \mid w = a^p b a^q, p, q \geq 0, 1 \leq p + q\};$$
$$L_2 = \{w \mid w = a^p b a^q, p, q \geq 0, 1 \leq p - q\};$$
$$L_3 = \{w \mid w = a^p b a^q, p, q \geq 0, p = q^3 \text{ or } q = p^3\}.$$

For each of the above languages, state whether it belongs to the class REG. Provide a mathematical proof for all of your answers.

3. [5 points] Let $G$ be some CFG in CNF. Let $T$ be a parse tree for a string $w \in L(G)$. Using structural induction, prove that if the longest path in $T$ has $n$ arcs then $|w| \leq 2^{n-1}$

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4. [5 points] Consider the following language, defined over the alphabet \( \Sigma = \{a, b\} \)

\[
L = \{ w \mid w \in \Sigma^*, w = a^n b^{2n}, n \geq 1 \}.
\]

Define a Turing machine \( M \) such that \( L(M) = L \) and \( M \) stops for every possible input in \( \Sigma^* \). Graphically represent the transition function of \( M \) and provide an informal discussion of the computation associated with each state of \( M \).

5. [8 points] Let \( A \) be some fixed DFA with input alphabet \( \Sigma = \{0, 1\} \) such that \( L(A) \) is not finite and \( L(A) \neq \Sigma^* \). Define the following property of the RE languages over \( \Sigma \)

\[
\mathcal{P} = \{ L \mid L \in \text{RE}, L \subseteq \Sigma^*, L \cap L(A) = \emptyset \}.
\]

(a) Apply Rice’s theorem to prove that \( L_\mathcal{P} \) is not in REC.

(b) Prove that \( L_\mathcal{P} \) is in RE but not in REC, where \( \overline{\mathcal{P}} \) is the complement of \( \mathcal{P} \) with respect to all languages over \( \Sigma \) that are in RE.

(c) Prove that \( L_\mathcal{P} \) is not in RE.

6. [2 points] Let \( \mathcal{NP} \) be the class of languages that can be recognised in polynomial time by a nondeterministic TM. State whether the relation \( \mathcal{NP} \subseteq \text{REC} \) holds, and motivate your answer.