1. **[4 points]** Consider the regular expression $r = (0+1)^*\emptyset(\epsilon+01)$. Convert $r$ into an equivalent $\epsilon$-NFA using the construction we have presented in class. **Important:** do not simplify the regular expression before applying the construction, use $r$ as is.

2. **[9 points]** Consider the following languages, defined over the alphabet $\Sigma = \{a, b, c\}$
   
   $L_1 = \{w \mid w = XuYvZ, X, Y, Z \in \Sigma, u, v \in \Sigma^*, X = Z, |u| = |v|\}$;
   
   $L_2 = \{w \mid w = XuYvZ, X, Y, Z \in \Sigma, u, v \in \Sigma^*, X = Y = Z\}$;
   
   $L_3 = \{w \mid w = XuYvZ, X, Y, Z \in \Sigma, u, v \in \Sigma^*, X = Y = Z, |u| = |v|\}$.

   State whether the above are regular languages, and provide a mathematical proof of your answers.

3. **[6 points]** Assess whether the following statements are true or false, providing motivations for all of your answers.

   (a) If $L_1, L_2 \in \text{CFL}$ then $L_1 \cap L_2 \in \text{REC}$;

   (b) If $L_1 \in \text{REG}$ and $L_2 \in \text{CFL} \setminus \text{REG}$ then $L_1 \cdot L_2$ is never in REG;

   (c) If $L_1 \cdot L_2 \in \text{REG}$ then $L_1, L_2 \in \text{REG}$.

   (please see next page)
4. [6 points] Consider the language \( L = \{a^n b^m \mid n, m \geq 0\} \) and the context-free grammar \( G = (\{S, A\}, \{a, b\}, P, S) \), where \( P \) contains the following rules

\[
S \rightarrow aS \mid A \\
A \rightarrow bA \mid \varepsilon
\]

Using mutual induction, we want to construct a mathematical proof that \( L(G) = L \). In order to do this, for each variable \( X \in \{S, A\} \) define a property \( \mathcal{P}_X \) over strings \( x \in \{a, b\}^\ast \) such that \( \mathcal{P}_X(x) \) holds if and only if there is a derivation in \( G \) that starts with \( X \) and produces \( x \). **Important**: develop your proof only for property \( \mathcal{P}_S \).

5. [8 points] Define the notion of property of the languages generated by TMs and state Rice’s theorem. Provide the proof of Rice’s theorem that we have developed in class.