martedì 14 novembre 2023 10:53

$\mathcal{U} = \left\langle \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\rangle$

$\mathcal{V} = \left\langle \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\rangle$ t.c. $\mathcal{U}$ e $\mathcal{V}$ sono SUPPLEMENTARI cioè $\mathcal{U} \oplus \mathcal{V} = Mat_{2\times 2}(\mathbb{R})$

③ Decomporre in tutti i modi possibili $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ come $v_1 + v_2$ con $v_1 \in \mathcal{U}$ e $v_2 \in \mathcal{V}$

$\mathcal{U} \oplus \mathcal{V} \overset{\text{caratterizzazione}}{\iff} \mathcal{U} \cap \mathcal{V} = \{\vec{0}\}$

$\overset{\text{def}}{\iff}$ ogni vettore v di $\mathcal{U} \oplus \mathcal{V}$ si decompone in modo unico come $v = v_1 + v_2$ con $v_1 \in \mathcal{U}$ e $v_2 \in \mathcal{V}$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \underbrace{a \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}_{\text{generico vettore di } \mathcal{U}} + \underbrace{d \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}_{\text{generico vettore di } \mathcal{V}}$$

$$= \begin{pmatrix} a & b \\ c & -a \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & d \end{pmatrix}$$

$$= \begin{pmatrix} a & b \\ c & -a+d \end{pmatrix}$$

$$\iff \begin{matrix} a = 1 & b = 2 \\ c = 3 & -a+d = 4 \end{matrix} \iff \begin{matrix} a = 1 & b = 2 \\ c = 3 & d = 5 \end{matrix}$$

④ Dato $W = \left\langle \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix} \right\rangle$

determinare una base e la dim di $\mathcal{U} + W$

e $\mathcal{U} \cap W$.

Sappiamo già che $\mathcal{U}$ ha dim 3

Verifichiamo se $\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ e $\begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}$ sono LI.

METODO 1

$$\lambda_1 \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} + \lambda_3 \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

A casa faccio i conti e trovo che quest'uguaglianza è vera $\forall \lambda_3$ $\Rightarrow$ sono LD.

$$\begin{pmatrix} \lambda_1 + 4\lambda_3 & 2\lambda_1 + \lambda_2 + 6\lambda_3 \\ \lambda_1 + \lambda_2 + 2\lambda_3 & \lambda_2 - 2\lambda_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{cases} \lambda_1 + 4\lambda_3 = 0 \\ 2\lambda_1 + \lambda_2 + 6\lambda_3 = 0 \\ \lambda_1 + \lambda_2 + 2\lambda_3 = 0 \\ \lambda_2 - 2\lambda_3 = 0 \end{cases}$$

$\lambda_1 = -4\lambda_3$

$\lambda_2 = 2\lambda_3$

$2(-4\lambda_3) + 2\lambda_3 + 6\lambda_3 = 0$

$-8\lambda_3 + 8\lambda_3 = 0$

$0 = 0$

$(-4\lambda_3) + 2\lambda_3 + 2\lambda_3 = 0$

$-4\lambda_3 + 4\lambda_3 = 0$

$0 = 0$

$\lambda_3 \in \mathbb{R}$, $\quad \lambda_1 = -4\lambda_3 \quad \lambda_2 = 2\lambda_3$

Per $\lambda_3 = 1 \quad \lambda_1 = -4$ e $\lambda_2 = 2$

$$-4\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} + 2\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} + 1\begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad *$$

$\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}$ sono LD.

$$W = \left\langle \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \cancel{\begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}} \right\rangle$$

$$* \Rightarrow \quad \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix} = 4\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} - 2\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\in \left\langle \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\rangle$$

$$W = \left\langle \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\rangle$$

Verifico che $\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ sono LI

$$\lambda_1 \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\Rightarrow \lambda_1 = \lambda_2 = 0$

Dunque $\left\{ \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\}$ sono generatori LI

e dunque formano una base e $\dim W = 2$

METODO 2 $\quad v_1, v_2, v_3$ sono LI $\Leftrightarrow$

$$\begin{pmatrix} — v_1 — \\ \ldots \end{pmatrix} = ?$$

$$\mathrm{rk}\begin{pmatrix} — & v_1 & — \\ — & v_2 & — \\ — & v_3 & — \end{pmatrix} = 3$$

$$\boxed{\mathrm{rk}\begin{pmatrix} — & v_1 & — \\ — & v_2 & — \\ — & v_3 & — \end{pmatrix} < 3 \iff v_1, v_2, v_3 \text{ sono LD}}$$

$$\mathrm{rk}\begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 4 & 6 & 2 & -2 \end{pmatrix} \xrightarrow[R_3 - 4R_1]{} \mathrm{rk}\begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & -2 & -2 \end{pmatrix}$$

$$\xrightarrow[R_3 + 2R_2]{} \mathrm{rk}\begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = 2$$

$\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ sono LI

$W = \left\langle \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\rangle$ $\qquad$ $U = \{ A \mid \mathrm{tr}(A) = 0 \}$

$\dim W = 2$ $\qquad$ $\dim U = 3$

Calcoliamo $U \cap W$

$$w \in W \iff w = a\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} a & 2a+b \\ a+b & b \end{pmatrix}$$

$$w \in U \cap W \iff w \in W \text{ e } w \in U$$

$$\iff w = \begin{pmatrix} a & 2a+b \\ a+b & b \end{pmatrix} \text{ e } \mathrm{tr}(w) = 0$$

$$\iff w = \begin{pmatrix} a & 2a+b \\ \end{pmatrix} \text{ e } a+b = 0$$

$$\iff w = \begin{pmatrix} a & 2a+b \\ a+b & b \end{pmatrix} \text{ e } a+b=0$$

$$\iff w = \begin{pmatrix} a & 2a+b \\ a+b & b \end{pmatrix} \text{ e } b=-a$$

$$\iff w = \begin{pmatrix} a & 2a-a \\ a-a & -a \end{pmatrix}$$

$$\iff w = \begin{pmatrix} a & a \\ 0 & -a \end{pmatrix}$$

$$\iff w \in \left\langle \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \right\rangle$$

$$\mathcal{U} \cap W = \left\langle \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \right\rangle \text{ e } \dim \mathcal{U} \cap W = 1$$

Oss $\quad W = \langle w_1, w_2 \rangle$

$\mathcal{U} = \langle u_1, u_2, u_3 \rangle$

$$v \in W \cap \mathcal{U} \iff v = \lambda_1 w_1 + \lambda_2 w_2 = \varepsilon_1 u_1 + \varepsilon_2 u_2 + \varepsilon_3 u_3$$

$$\iff \lambda_1 w_1 + \lambda_2 w_2 = \varepsilon_1 u_1 + \varepsilon_2 u_2 + \varepsilon_3 u_3$$

$$\underbrace{\dim \mathcal{U}+W}_{\Downarrow\ 4} = \underbrace{\dim \mathcal{U}}_{3} + \underbrace{\dim W}_{2} - \underbrace{\dim \mathcal{U} \cap W}_{1}$$

$\mathcal{U} + W \subseteq M_2(\mathbb{R}) \qquad \dim \mathcal{U}+W = 4$

$\mathcal{U} + W \subseteq Mat_{2\times 2}(\mathbb{R})$, $\dim \mathcal{U}+W = 4$

$\dim Mat_{2\times 2}(\mathbb{R}) = 4$

$\Longrightarrow \quad \mathcal{U} + W = Mat_{2\times 2}(\mathbb{R})$

Una base di $\mathcal{U}+W$ è $\begin{pmatrix}1&0\\0&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&0\\0&1\end{pmatrix}$

Es: Sia $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ un'app lineare.
definita da $f(x,y,z) = (x+y, x+y, z)$

① Scrivere la matrice associata ad f rispetto alle basi canoniche sia del dominio che del codominio

f è un endomorfismo perché dominio di f = codominio di f ed f è lineare.

$\mathcal{E} = \{ e_1 = (1\,0\,0),\ e_2 = (0\,1\,0),\ e_3 = (0\,0\,1) \}$

$$M_{\mathcal{E}}^{\mathcal{E}}(f) = \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\qquad\qquad\quad f(e_1) \quad f(e_2) \quad f(e_3)$$

$(x+y, x+y, z)$

$x = 1$
$y = z = 0$

$y = 1$
$x = z = 0$

$z = 1$
$x = y = 0$

$x = 1$ $\quad$ $y = 1$ $\quad$ $z = 1$

$y = z = 0$ $\quad$ $x = z = 0$ $\quad$ $x = y = 0$

Oss Se $\mathcal{U} \subseteq V$ e $\dim \mathcal{U} = \dim V$

$\Rightarrow \mathcal{U} = V$

$$f(x\,y\,z) = (x+y,\ x+y,\ z)$$

$$f(1\,0\,0) = (1,\ 1,\ 0)$$

$$f(0\,1\,0) = (1,\ 1,\ 0)$$

$$f(1\,1\,1) = (2\ 2\ 1)$$

(2) Mostrare che $\mathcal{B} = \{ b_1 = (1\,1\,-1),\ b_2 = (1\,1\,0),\ b_3 = (1\,-1\,0) \}$ è una base di $\mathbb{R}^3$

$\mathcal{B} = \{b_1, b_2, b_3\}$ è base $\Longleftrightarrow$ b_1, b_2, b_3 sono generatori LI

$\dim \mathbb{R}^3 = 3$, $\mathcal{B}$ è composto da 3 vettori.

$\mathcal{B}$ è base $\Longleftrightarrow$ b_1, b_2, b_3 sono generatori

$\Longleftrightarrow$ b_1, b_2, b_3 sono LI

Basta verificare che b_1, b_2, b_3 sono LI.

METODO 1 $\lambda_1 b_1 + \lambda_2 b_2 + \lambda_3 b_3 = \vec{0}$

Devo verificare se $\lambda_1 = \lambda_2 = \lambda_3 = 0$

METODO 2

Devo verificare che $rk \begin{pmatrix} - b_1 - \\ - b_2 - \\ - b_3 - \end{pmatrix} = 3$

METODO 3 (SOLO PER MATRICI QUADRATE)

Devo verificare che $\det \begin{pmatrix} - b_1 - \\ - b_2 - \\ - b_3 - \end{pmatrix} \neq 0$

$A \in Mat_{n \times n}(\mathbb{R})$

$\det(A) \neq 0 \iff \exists A^{-1} \iff rk(A) = n$

$\iff$ le righe di A (o le colonne di A) sono LI

$\det(A) = 0 \iff \nexists A^{-1} \iff rk(A) < n$

$\iff$ le righe di A (o le colonne di A) sono LD

$$\det \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix} = (-1)^{1+3}(-1) \det \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$+ \cancel{(-1)^{2+3}\, 0} \det \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$+ \cancel{(-1)^{3+3}\, 0} \det \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$= -\left[1(-1) - 1\cdot 1\right] = -(-1-1) = 2 \neq 0$

$\Rightarrow \{(1\,1\,-1), (1\,1\,0), (1\,-1\,0)\}$ sono LI

③ Scrivere la matrice associata ad f rispetto alla base canonica del dominio e la base $\mathcal{B}$ nel codominio

$$\begin{tikzcd}
 & & \mathbb{R}^3\ \mathcal{B} \\
\mathcal{e}\ \mathbb{R}^3 \arrow[rru, "f", "M_e^{\mathcal{B}}(f)"'] \arrow[rr, "f", "M_e^e(f)"'] & & \mathbb{R}^3\ \mathcal{e} \arrow[u, "M_e^{\mathcal{B}}(Id)"']
\end{tikzcd}$$

METODO 1

$$M_e^{\mathcal{B}}(f) = \begin{pmatrix} (f(e_1))_{\mathcal{B}} & (f(e_2))_{\mathcal{B}} & (f(e_3))_{\mathcal{B}} \end{pmatrix}$$

$\mathcal{e} = \{\underset{(100)}{e_1}, \underset{(010)}{e_2}, \underset{(001)}{e_3}\}$

$\mathcal{B} = \{b_1, b_2, b_3\}$

$$\begin{aligned}
f(e_1) = f(100) = (110)_e &= 1(100) + 1(010) + 0(001) \\
&= 1\,e_1 + 1\,e_2 + 0\,e_3 \\
= (x_1 x_2 x_3)_{\mathcal{B}} &= x_1 b_1 + x_2 b_2 + x_3 b_3 \\
&= 0\,b_1 + 1\,b_2 + 0\,b_3 \\
&= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{\mathcal{B}}
\end{aligned}$$

$b_2 = (110)_e$

$$= \begin{pmatrix} \\ 1 \\ 0 \end{pmatrix}_{\mathcal{B}}$$

$$f(e_2) = f(0\,1\,0) = \underbrace{(1\,1\,0)}_{b_2} = x_1 b_1 + x_2 b_2 + x_3 b_3 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathcal{B}}$$

$$= 0\, b_1 + 1\, b_2 + 0\, b_3$$

$$= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{\mathcal{B}}$$

$$f(e_3) = f(0\,0\,1) = (0\,0\,1) = x_1 b_1 + x_2 b_2 + x_3 b_3 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathcal{B}}$$

$$= x_1 (1\,1\,{-1}) + x_2 (1\,1\,0) + x_3 (1\,{-1}\,0)$$

$$\begin{cases} 0 = x_1 + x_2 + x_3 \\ 0 = x_1 + x_2 - x_3 \\ 1 = -x_1 \end{cases} \qquad \begin{aligned} x_1 &= -1 \\ x_3 &= 0 \\ x_2 &= -x_1 = 1 \end{aligned}$$

$$f(e_3) = -1\, b_1 + 1\, b_2 + 0\, b_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}_{\mathcal{B}}$$

$$M^{\mathcal{B}}_{e}(f) = \begin{array}{c} \left(\begin{array}{c|c|c} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right) \\ \begin{array}{ccc} (f(e_1))_{\mathcal{B}} & (f(e_2))_{\mathcal{B}} & (f(e_3))_{\mathcal{B}} \end{array} \end{array}$$

<u>METODO 2</u>

$$\boxed{M^{\mathcal{B}}_{e}(f) = M^{\mathcal{B}}_{e}(Id)\, M^{e}_{e}(f)}$$

$$\boxed{Mat(f, e, \mathcal{B}) = Mat(Id, e, \mathcal{B}) \quad Mat(f, e, e)}$$

$$\operatorname{Mat}(f, \mathcal{e}, \mathcal{B}) = \operatorname{Mat}(Id, \mathcal{e}, \mathcal{B}) \operatorname{Mat}(f, \mathcal{e}, \mathcal{e})$$

$$M_{\mathcal{e}}^{\mathcal{B}}(Id) = \operatorname{Mat}(Id, \mathcal{e}, \mathcal{B}) \qquad \mathcal{e} = \{e_1, e_2, e_3\}$$

$$\mathcal{B} = \{b_1, b_2, b_3\}$$

$$= \begin{array}{ccc} \left(\begin{array}{c|c|c} \; & \; & \; \\ \; & \; & \; \\ \; & \; & \; \end{array} \right) \\ (e_1)_{\mathcal{B}} \quad (e_2)_{\mathcal{B}} \quad (e_3)_{\mathcal{B}} \end{array}$$

$$e_1 = (1\,0\,0) \qquad e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{\mathcal{e}} = 1\, e_1 + 0\, e_2 + 0\, e_3$$

$$e_1 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathcal{B}} = x_1 b_1 + x_2 b_2 + x_3 b_3$$

$$\overbrace{(1\,0\,0)}^{e_1} = x_1 \overbrace{(1\,1\,-1)}^{b_1} + x_2 \overbrace{(1\,1\,0)}^{b_2} + x_3 \overbrace{(1\,-1\,0)}^{b_3}$$

$$\begin{cases} 1 = x_1 + x_2 + x_3 \\ 0 = x_1 + x_2 - x_3 \\ 0 = -x_1 \end{cases} \qquad \begin{array}{l} x_1 = 0 \\ x_2 = x_3 \\ 1 = x_2 + x_2 \\ 1 = 2x_2 \\ x_2 = \frac{1}{2} \end{array}$$

$$(1\,0\,0) = \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}_{\mathcal{B}}$$

$$= 0\, b_1 + \frac{1}{2} b_2 + \frac{1}{2} b_3$$

$$e_2 = (0\,1\,0) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathcal{B}} = x_1 b_1 + x_2 b_2 + x_3 b_3$$

$$= x_1\,(1\,1\,{-1}) + x_2\,(1\,1\,0) + x_3\,(1\,{-1}\,0)$$

$$\begin{cases} 0 = x_1 + x_2 + x_3 \\ 1 = x_1 + x_2 - x_3 \\ 0 = -x_1 \end{cases}$$

$x_1 = 0$

$x_2 = -x_3$

$1 = x_2 + x_2$

$1/2 = x_2$

$$e_2 = \begin{pmatrix} 0 \\ 1/2 \\ -1/2 \end{pmatrix}_{\mathcal{B}}$$

$$e_3 = (0\,0\,1) = x_1 b_1 + x_2 b_2 + x_3 b_3 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathcal{B}}$$

$$= x_1\,(1\,1\,{-1}) + x_2\,(1\,1\,0) + x_3\,(1\,{-1}\,0)$$

$$\begin{cases} 0 = x_1 + x_2 + x_3 \\ 0 = x_1 + x_2 - x_3 \\ 1 = -x_1 \end{cases} \longrightarrow \begin{aligned} x_1 &= -1 \\ x_3 &= 0 \\ x_2 &= -x_1 = 1 \end{aligned}$$

$$e_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}_{\mathcal{B}} = -b_1 + b_2 + 0\,b_3$$

$$M_e^{\mathcal{B}}(Id) = \left(\begin{array}{c|c|c} 0 & 0 & -1 \\ 1/2 & 1/2 & 1 \\ 1/2 & -1/2 & 0 \\ (e_1)_{\mathcal{B}} & (e_2)_{\mathcal{B}} & (e_3)_{\mathcal{B}} \end{array} \right)$$

$M^{\mathcal{B}}(\ )\quad M^{\mathcal{B}}(D)\ M^{e}(\ )$

$$M_e^{\mathcal{B}}(f) = M_e^{\mathcal{B}}(Id)\; M_e^{e}(f)$$

$$M_e^{\mathcal{B}}(f) = \begin{pmatrix} 0 & 0 & -1 \\ 1/2 & 1/2 & 1 \\ 1/2 & -1/2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$