FALG_AA2122_2appello_temaA

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ES 1

$$W = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \in \mathbb{R}^4 \text{ t.c. } x_1 - 2x_3 = 0 \text{ e } x_1 - x_3 - x_4 = 0 \right\}$$

$$Z = \left\langle \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

a) Dimostrare che W è sottospazio vettoriale e calcolare base e dimensione di W e Z

$$\begin{cases} x_1 - 2x_3 = 0 \\ x_1 - x_3 - x_4 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 2x_3 \\ 2x_3 - x_3 = x_4 \end{cases} \Rightarrow \begin{cases} x_1 = 2x_3 \\ x_4 = x_3 \end{cases}$$

$$W = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

W ha dimensione 2 e una sua base è $B_W = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right\}$

Prendo $w_1 = \begin{pmatrix} 2b \\ a \\ b \\ b \end{pmatrix}$ e $w_2 = \begin{pmatrix} 2d \\ c \\ d \\ d \end{pmatrix} \in W$ e $\lambda_1, \lambda_2 \in \mathbb{R}$

$$\lambda_1 w_1 + \lambda_2 w_2 = \lambda_1 \begin{pmatrix} 2b \\ a \\ b \\ b \end{pmatrix} + \lambda_2 \begin{pmatrix} 2d \\ c \\ d \\ d \end{pmatrix} = \begin{pmatrix} 2(\lambda_1 b + \lambda_2 d) \\ \lambda_1 a + \lambda_2 c \\ \lambda_1 b + \lambda_2 d \\ \lambda_1 b + \lambda_2 d \end{pmatrix} =$$

$$= \begin{pmatrix} 2\beta \\ \alpha \\ \beta \\ \beta \end{pmatrix} \in W$$

e $0_{\mathbb{R}^4} \in W$
sì W è sottospazio vettoriale

$\lambda_1 a + \lambda_2 c = \alpha$
$\lambda_1 b + \lambda_2 d = \beta$

Considero $Z = \left\langle \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$ vedo che $\begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} 2 \\ -1 \\ 2 \\ 1 \end{pmatrix}$

Quindi la dimensione di Z è 2 e una base per Z è $B_Z = \left\{ \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 2 \\ 1 \end{pmatrix} \right\}$

Quindi la dimensione di Z è 2 e una base per Z è $B_Z = \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \right\}$

b) Calcolare una base e la dimensione di $W \cap Z$ e $W+Z$.
Sono in somma diretta?

Un generico vettore di W: $\begin{pmatrix} 2\alpha \\ \beta \\ \alpha \\ \alpha \end{pmatrix}$, un generico vettore di Z: $\begin{pmatrix} 2\gamma \\ \delta - \gamma \\ -\delta + 2\gamma \\ \gamma \end{pmatrix}$

$$\begin{cases} 2\alpha = 2\gamma \\ \beta = \delta - \gamma \\ \alpha = -\delta + 2\gamma \\ \alpha = \gamma \end{cases} \qquad \begin{cases} \alpha = \gamma \\ \delta = \gamma \\ \beta = 0 \end{cases} \qquad \begin{pmatrix} 2 \\ 0 \\ 1 \\ 1 \end{pmatrix} \in W \cap Z \qquad \dim(W \cap Z) = 1$$

$$B_{W \cap Z} = \left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right\}$$

Dato che l'intersezione non è l'insieme banale la somma di Z e W <u>non</u> è diretta

$$W + Z = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 2 \\ 1 \end{pmatrix} \right\rangle = \langle W \cup Z \rangle$$

c) Per quali valori di $h \in \mathbb{R}$ il vettore $\begin{pmatrix} h+1 \\ h \\ -h \\ 1 \end{pmatrix} \in W + Z$

$$\begin{pmatrix} 2 & -1 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ h+1 & h & -h & 1 \end{pmatrix} \to \begin{pmatrix} 2 & -1 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ h+1 & h & -h & 1 \end{pmatrix} \to \begin{pmatrix} 2 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ h+1 & h & -h & 1 \end{pmatrix} \to \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ h+1 & h & -h & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & h & -h & \frac{-h-1}{2}+1 \end{pmatrix} \to \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -h & -\frac{h+1}{2} \end{pmatrix} \to \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{-h+1}{2} \end{pmatrix}$$

Se il rango della matrice è 3 allora il vettore è comb. lineare dei vettori della base $W+Z$

$$\frac{-h+1}{\cancel{2}} = 0 \qquad \underline{h = 1}$$

$$\begin{pmatrix} 2 \\ 1 \\ \vdots \end{pmatrix} \in \left\langle \begin{pmatrix} 0 \\ 1 \\ \vdots \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ \vdots \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ \vdots \end{pmatrix} \right\rangle \quad \text{infatti } 1\begin{pmatrix} 2 \\ -1 \\ \vdots \end{pmatrix} + 3\begin{pmatrix} 0 \\ 1 \\ \vdots \end{pmatrix} - 1\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix}2\\1\\-1\\1\end{pmatrix} \in \left\langle \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \begin{pmatrix}0\\1\\-1\\0\end{pmatrix} \begin{pmatrix}2\\-1\\2\\1\end{pmatrix} \right\rangle \quad \text{infatti } 1\begin{pmatrix}2\\-1\\2\\1\end{pmatrix} + 3\begin{pmatrix}0\\1\\-1\\0\end{pmatrix} -1\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\1\\-1\\1\end{pmatrix}$$

d) Calcolare il complemento ortogonale di $W+Z$.

$$\begin{cases} (W+Z)^{\perp} + W+Z = \mathbb{R}^4 \quad \Rightarrow \text{ la dimensione di } (W+Z)^{\perp} \text{ sarà } \underline{1} \\ \text{e} \quad u \in (W+Z)^{\perp} \text{ t.c. } u \text{ ortogonale ai vettori di una base di } W+Z \end{cases}$$

$$u = \begin{pmatrix}a\\b\\c\\d\end{pmatrix} \qquad \begin{cases} u \cdot \begin{pmatrix}0\\1\\0\\0\end{pmatrix} = 0 \Rightarrow b=0 \\ u \cdot \begin{pmatrix}0\\1\\-1\\0\end{pmatrix} = 0 \Rightarrow b=c=0 \\ u \cdot \begin{pmatrix}2\\-1\\2\\1\end{pmatrix} = 0 \Rightarrow 2a-b+2c+d=0 \Rightarrow d=-2a \end{cases}$$

Quindi $u = \begin{pmatrix}1\\0\\0\\-2\end{pmatrix}$ $\qquad (W+Z)^{\perp} = \left\langle \begin{pmatrix}1\\0\\0\\-2\end{pmatrix} \right\rangle$

ES 2 $\quad \phi: \mathbb{R}^3 \to \mathbb{R}^3$ t.c. $v_1 = \begin{pmatrix}1\\2\\-3\end{pmatrix} \in \ker \phi$

$\mathrm{Im}(\phi) = \langle v_1 \rangle^{\perp}$

$v_2 = \begin{pmatrix}1\\1\\1\end{pmatrix}$ è autovettore di ϕ relativo all'autovalore $\lambda = 2$

a) Determinare $v_3 \in \mathbb{R}^3$ t.c. $B = \{v_1, v_2, v_3\}$ sia una base ortogonale di $\mathbb{R}^3$

Mostrare che $C = \{v_2, v_3\}$ è base di $\mathrm{Im}(\phi)$

Osserviamo che $v_1 \cdot v_2 = 0$ infatti $\begin{pmatrix}1\\2\\-3\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}=0$

Cerco $v_3 = \langle v_1, v_2 \rangle^\perp$

Quindi

$$v_3 = \begin{pmatrix}x\\y\\z\end{pmatrix} \text{ t.c. } \quad v_3\cdot\begin{pmatrix}1\\2\\-3\end{pmatrix}=0 \quad e \quad v_3\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}=0$$

$$\begin{cases} x+2y-3z=0 \\ x+y+z=0 \end{cases} \qquad 2y-3z-y-z=0 \qquad \begin{cases} y = 4z \\ x = 3z-8z = -5z \end{cases}$$

Da cui $v_3 = \begin{pmatrix}-5\\4\\1\end{pmatrix}$ e $B = \left\{\begin{pmatrix}1\\2\\-3\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}-5\\4\\1\end{pmatrix}\right\}$

$C = \left\{\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}-5\\4\\1\end{pmatrix}\right\}$ è una base di $\mathrm{Im}(\phi)$ infatti $\mathrm{Im}(\phi) = \langle v_1 \rangle^\perp$

$\Rightarrow \dim(\mathrm{Im}(\phi)) = 2$

e v_1 è ortogonale ai vettori di una base di $\mathrm{Im}(\phi)$

$\Rightarrow v_1 \cdot \begin{pmatrix}1\\1\\1\end{pmatrix} = 0$ e $v_1 \cdot \begin{pmatrix}-5\\4\\1\end{pmatrix} = 0$

$\Rightarrow C = \left\{\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}-5\\4\\1\end{pmatrix}\right\}$ è base di $\mathrm{Im}\phi$

b) $g: \mathrm{Im}\phi \longrightarrow \mathbb{R}^3$ $\qquad M_C^C(g)$ $\qquad \mathbb{R}^3 \xrightarrow{\phi} \mathrm{Im}\phi \xrightarrow{g} \mathrm{Im}\, g$

$\mathrm{Im}\phi = \langle v_2, v_3 \rangle$

$g(v_3) = \begin{pmatrix}3\\-6\\-3\end{pmatrix}$

$g(v) = \phi(v) \ \forall v \in \mathrm{Im}\phi$

Sappiamo che $\phi(v_2) = 2v_2 = \begin{pmatrix}2\\2\\2\end{pmatrix} = g(v_2)$

e $\phi(v_1) = \begin{pmatrix}0\\0\\0\end{pmatrix}$ $\qquad v_1 \in \ker\phi$

Possiamo quindi calcolare la matrice associata a ϕ $\qquad B = \left\{\begin{pmatrix}1\\2\\-3\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}-5\\4\\1\end{pmatrix}\right\}$

$$\begin{array}{ccc} v_1 & v_2 & v_3 \end{array}$$

$M^{\mathcal{E}}(\phi) = \begin{pmatrix}0 & 2 & 3 \\ \dots\end{pmatrix}$ $\qquad M^{\mathcal{E}}(\dots) = \begin{pmatrix}1 & 1 & -5 \\ \dots\end{pmatrix}$

$$A_{\mathcal{E}}^{\mathcal{E}}(\phi)=\begin{pmatrix}0&1&3\\0&2&-6\\0&2&-3\end{pmatrix} \qquad A_{B}^{\mathcal{E}}(Id_V)=\begin{pmatrix}1&1&-5\\2&1&4\\-3&1&1\end{pmatrix}$$

$v_1 \quad v_2 \quad v_3$

$$A_{\mathcal{E}}^{B}(Id_V)=A_{B}^{\mathcal{E}}(Id_V)^{-1}$$

$$\left(\begin{array}{ccc|ccc}1&1&-5&1&0&0\\2&1&4&0&1&0\\-3&1&1&0&0&1\end{array}\right)\rightarrow\left(\begin{array}{ccc|ccc}1&1&-5&1&0&0\\0&-1&14&-2&1&0\\0&-2&16&-3&0&1\end{array}\right)$$

$$\left(\begin{array}{ccc|ccc}1&0&9&-1&1&0\\0&1&-14&2&-1&0\\0&0&-12&1&-2&1\end{array}\right)$$

ES 2

$M_{2,2}(\mathbb{R})$

$$f: M_{2,2}(\mathbb{R})\rightarrow M_{2,2}(\mathbb{R})$$
$$\begin{pmatrix}a&b\\c&d\end{pmatrix}\rightarrow\begin{pmatrix}a&b-c\\0&d\end{pmatrix}$$

a) $\mathcal{E}=\left\{\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}0&0\\0&1\end{pmatrix}\right\}$ Determinare $A_{\mathcal{E}}^{\mathcal{E}}(f)$

Considero le immagini dei vettori di $\mathcal{E}$

$$f\begin{pmatrix}1&0\\0&0\end{pmatrix}=\begin{pmatrix}1&0\\0&0\end{pmatrix}\Rightarrow\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad f\begin{pmatrix}0&0\\1&0\end{pmatrix}=\begin{pmatrix}0&-1\\0&0\end{pmatrix}\Rightarrow\begin{pmatrix}0\\-1\\0\\0\end{pmatrix}$$

$$f\begin{pmatrix}0&1\\0&0\end{pmatrix}=\begin{pmatrix}0&1\\0&0\end{pmatrix}\Rightarrow\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \qquad f\begin{pmatrix}0&0\\0&1\end{pmatrix}=\begin{pmatrix}0&0\\0&1\end{pmatrix}\Rightarrow\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$A_{\mathcal{E}}^{\mathcal{E}}(f)=\begin{pmatrix}1&0&0&0\\0&1&-1&0\\0&0&0&0\\0&0&0&1\end{pmatrix}=A$$

$rkA=3 \Rightarrow \dim(Im(f))=3$
$\dim(\ker(f))=1$

b) $\ker(f):\left\{\begin{pmatrix}a&b\\c&d\end{pmatrix}\in M_{2,2}(\mathbb{R}) \text{ t.c. } A\begin{pmatrix}a\\b\\c\\d\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\}$

$$\begin{pmatrix}1&0&0&0\\0&1&-1&0\\0&0&0&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}a\\b\\c\\d\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\Rightarrow\begin{cases}a=0\\b=c\\d=0\end{cases}\Rightarrow \ker f=\left\langle\begin{pmatrix}0\\1\\1\\0\end{pmatrix}\right\rangle$$

coord. in base $\mathcal{E}$

$$=\left\langle\begin{pmatrix}0&1\\1&0\end{pmatrix}\right\rangle$$

$Im(f)=\left\langle\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\-1\\0\end{pmatrix}\begin{pmatrix}0\\0\\0\end{pmatrix}\right\rangle$ ma non sono vettori L.I infatti!

$$\mathrm{Im}(f) = \left\langle \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\begin{pmatrix}0\\-1\\0\\0\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \right\rangle$$

ma non sono vettori L.I infatti!

$$\begin{pmatrix}0\\-1\\0\\0\end{pmatrix} = -1\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$= \left\langle \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \right\rangle$$

coordinate in base $\mathcal{E}$ dei vettori di una base di Im(f)

$$= \left\langle \begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0&0\\0&1\end{pmatrix} \right\rangle$$

c) $f^{-1}(z)$ $\quad z = \begin{pmatrix}2&1\\0&1\end{pmatrix}$ $\begin{pmatrix}2&0\\1&0\end{pmatrix}$ tema B

cerco le coordinate $\begin{pmatrix}a\\b\\c\\d\end{pmatrix}$ della matrice $\begin{pmatrix}a&b\\c&d\end{pmatrix}$

t.c. $A\begin{pmatrix}a\\b\\c\\d\end{pmatrix} = \begin{pmatrix}2\\1\\0\\1\end{pmatrix}$

$$\begin{pmatrix}1&0&0&0\\0&1&-1&0\\0&0&0&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}a\\b\\c\\d\end{pmatrix} = \begin{pmatrix}2\\1\\0\\1\end{pmatrix} \Rightarrow \begin{cases}a=2\\b=c+1\\d=1\end{cases} \Rightarrow \underbrace{\begin{pmatrix}2\\1\\0\\1\end{pmatrix}}_{\text{sol particolare}} + \underbrace{\left\langle\begin{pmatrix}0\\1\\1\\0\end{pmatrix}\right\rangle}_{\ker(f)}$$

$f^{-1}(z)$ è una sottovarietà lineare, non un sottospazio vettoriale.

$f^{-1}(S(M_{2,2}(\mathbb{R})))$ dove $S(M_{2,2}(\mathbb{R}))$ sottospazio vettoriale delle matrici simmetriche

$$\begin{pmatrix}\alpha&\beta\\\beta&\gamma\end{pmatrix} \in S(M_{2,2}(\mathbb{R}))$$

$$S(M_{2,2}(\mathbb{R})) = \left\langle \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\begin{pmatrix}0\\1\\1\\0\end{pmatrix} \right\rangle$$

Ma $\begin{pmatrix}\alpha&\beta\\\beta&\gamma\end{pmatrix} \in \mathrm{Im}(f)$ sse $\underline{\beta = 0}$ sse matrice diagonale

$$\begin{pmatrix}1&0&0&0\\0&1&-1&0\\0&0&0&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}a\\b\\c\\d\end{pmatrix} = \begin{pmatrix}\alpha\\0\\0\\\gamma\end{pmatrix} \Rightarrow \begin{cases}a=\alpha\\b=c\\d=\gamma\end{cases} \Rightarrow \left\langle \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\begin{pmatrix}0\\1\\1\\0\end{pmatrix} \right\rangle$$

Quindi $f^{-1}(S(M_{2,2}(\mathbb{R}))) = S(M_{2,2}(\mathbb{R}))$ é spazio vettoriale

d) Determinare la matrice associata ad $f \circ f$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \xrightarrow{f} \begin{pmatrix} a & b-c \\ 0 & d \end{pmatrix} \xrightarrow{f} \begin{pmatrix} a & b-c \\ 0 & d \end{pmatrix}$$

quindi $f \circ f = f$

$\Downarrow$ $A_{\mathcal{E}}^{\mathcal{E}}(f)$ $\qquad$ $\Downarrow$ $\mathbb{I}_4 = A_{\mathcal{E}}^{\mathcal{E}}(Id_W)$

$$A(f \circ f) = \underbrace{A_{\mathcal{E}}^{\mathcal{E}}(Id_W)}_{\mathbb{I}_4} A_{\mathcal{E}}^{\mathcal{E}}(f) = A_{\mathcal{E}}^{\mathcal{E}}(f) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

ES 3

in $A^3(\mathbb{R})$

$$r = \begin{cases} 5x + y = 0 \\ x + z = -4 \end{cases} \qquad P_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

a) Eq. parametriche di r

$$\begin{cases} y = -5x \\ z = -x-4 \\ x = \alpha \end{cases} \qquad \begin{cases} x = \alpha \\ y = -5\alpha \\ z = -4-\alpha \end{cases} \qquad r: \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ -5 \\ -1 \end{pmatrix} \right\rangle$$

eq. parametriche $\qquad$ eq. vettoriali

Eq cartesiane di s passante per P_1, P_2

$$s = P_1 + \langle \overrightarrow{P_2 - P_1} \rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} \right\rangle \qquad \begin{cases} x = 0 \\ y = 1 - 2\beta \\ z = \beta \end{cases} \qquad \begin{cases} x = 0 \\ y + 2z = 1 \end{cases}$$

eq. vettoriali $\qquad$ eq. parametriche $\qquad$ eq. cartesiane

b) Eq. cartesiana di π $\qquad$ $P_1, P_2 \in \pi \iff s \in \pi$

$V_r \subseteq V_\pi$

$$\pi = \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}\right.}\underbrace{\left.\begin{pmatrix} 1 \\ -5 \\ -1 \end{pmatrix} \right\rangle} \qquad V_\pi = \left\langle \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -5 \\ -1 \end{pmatrix} \right\rangle$$

$$\pi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \left\langle \underbrace{\begin{pmatrix} -2 \\ 1 \end{pmatrix}}_{\text{passa per } P_1 \text{ e } P_2} \underbrace{\begin{pmatrix} -5 \\ -1 \end{pmatrix}}_{\parallel \text{ a } r} \right\rangle$$

ossia contiene s

$$V_\pi = \left\langle \begin{pmatrix} -2 \\ 1 \end{pmatrix} \begin{pmatrix} -5 \\ -1 \end{pmatrix} \right\rangle$$

Cerco $\pi: ax+by+cz+d=0$

con $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = V_\pi \perp$ e quindi $\begin{cases} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} = 0 \\ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \begin{pmatrix} 1 \\ -5 \\ -1 \end{pmatrix} = 0 \end{cases}$

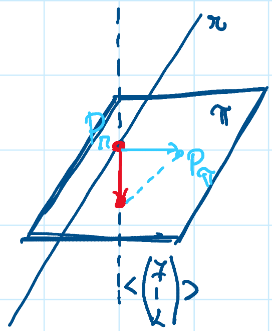
$$\begin{cases} -2b+c=0 \\ a-5b-c=0 \end{cases} \qquad \begin{cases} c=2b \\ a=7b \end{cases}$$

$$V_{\pi\perp} = \left\langle \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix} \right\rangle$$

$\pi: 7x+y+2z+d=0$ impongo il passaggio per $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow 1+d=0 \Rightarrow \underline{d=-1}$

$$\boxed{\pi: 7x+y+2z-1=0}$$

c) determinare la distanza tra r e π

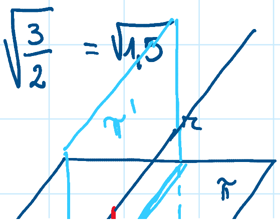


Prendo $P_r = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix} \in r$ e $P_\pi = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Calcolo $\overrightarrow{P_\pi - P_r} = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$ e considero la proiezione di questo vettore lungo l'ortogonale al piano $<\begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix}>$

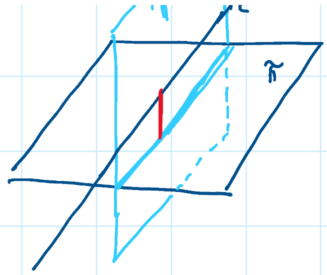
$$(P_\pi - P_r)_{V_{\pi\perp}} = \left(\begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix} \cdot \frac{\begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix}}{\left\| \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix} \right\|} \right) \frac{\begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix}}{\left\| \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix} \right\|} = \frac{9}{\sqrt{49+1+4}\sqrt{49+1+4}} \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix} = \frac{9}{54} \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix}$$

Ne calcolo la norma $\sqrt{\frac{49+1+4}{36}} = \sqrt{\frac{54}{36}} = \sqrt{\frac{6}{4}} = \sqrt{\frac{3}{2}} = \sqrt{1,5}$



d) Determinare l'equazione di π' t.c. $r \in \pi'$.

d) Determinare l'equazione di π' t.c. $r \in \pi'$,
$\pi \perp \pi'$



$r: \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ -5 \\ -1 \end{pmatrix} \right\rangle \qquad \pi = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -5 \\ -1 \end{pmatrix} \right\rangle$

$\pi' = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ -5 \\ -1 \end{pmatrix} \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix} \right\rangle$ e π' ha come vettore $\perp$

$\begin{cases} x - 5y - z = 0 \\ 7x + y + 2z = 0 \end{cases} \qquad \begin{cases} z = x - 5y \\ 7x + y + 2x - 10y = 0 \end{cases}$

$\begin{cases} z = x - 5y \\ \cancel{9}x - \cancel{9}y = 0 \end{cases} \qquad \begin{cases} z = -4y \\ x = y \end{cases} \qquad V_{\pi'\perp} = \left\langle \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} \right\rangle$

$\pi' = x + y - 4z + d = 0$ impongo il passaggio per $\begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$ $+16 + d = 0 \Rightarrow d = -16$

$\boxed{\pi' = x + y - 4z - 16 = 0}$