

Foglio 1

**Esercizio 1**

Scrivere in forma algebrica i seguenti numeri complessi:

a)  $z = (1 + i)^4$ , [R. -4.]

b)  $z = (1 - i)(2 + i)$ , [R.  $3 - i$ .]

c)  $3e^{\frac{5}{6}\pi i}$ , [R.  $-\frac{3\sqrt{3}}{2} + \frac{3}{2}i$ .]

d)  $z = \frac{\sqrt{2}i + \sqrt{3}}{\sqrt{2} - \sqrt{3}i}$ , [R.  $i$ .]

e)  $z = \frac{(1 + i)(3 - 2i)}{i}$ , [R.  $1 - 5i$ .]

f)  $z = (i)^{2022}$ , [R. -1]

**Esercizio 2**

Scrivere in forma trigonometrica i seguenti numeri complessi:

a)  $z = \sqrt{3} - i$ , [R.  $2 \left( \cos \left( -\frac{\pi}{6} \right) + i \sin \left( -\frac{\pi}{6} \right) \right)$ .]

b)  $z = (1 - i)^5$ , [R.  $4\sqrt{2} \left( \cos \left( \frac{3}{4}\pi \right) + i \sin \left( \frac{3}{4}\pi \right) \right)$ .]

c)  $z = \left( \frac{i - 1}{i + 1} \right)^3$ , [R.  $\cos \left( \frac{3}{2}\pi \right) + i \sin \left( \frac{3}{2}\pi \right)$ .]

d)  $z = \frac{4i}{\sqrt{3} + i}$ , [R.  $2 \left( \cos \left( \frac{\pi}{3} \right) + i \sin \left( \frac{\pi}{3} \right) \right)$ .]

e)  $z = (1 + i)^2(3 + \sqrt{3}i)$ , [R.  $4\sqrt{3} \left( \cos \left( \frac{2}{3}\pi \right) + i \sin \left( \frac{2}{3}\pi \right) \right)$ .]

f)  $z = \frac{(1 + i)}{(1 - i)(\sqrt{3} + i)}$ , [R.  $\frac{1}{2} \left( \cos \left( \frac{\pi}{3} \right) + i \sin \left( \frac{\pi}{3} \right) \right)$ .]

**Esercizio 3**

Trovare tutte le soluzioni complesse delle seguenti equazioni:

a)  $z^5 = \frac{\sqrt{3} - i}{\sqrt{3} + i}$ , [R.  $\cos \left( -\frac{\pi}{15} + \frac{2}{5}k\pi \right) + i \sin \left( -\frac{\pi}{15} + \frac{2}{5}k\pi \right)$ .]

$$\text{b) } z^3 = \frac{1-i}{i+1}, \quad [\text{R. } \cos\left(-\frac{\pi}{6} + \frac{2}{3}k\pi\right) + i \sin\left(-\frac{\pi}{6} + \frac{2}{3}k\pi\right).]$$

$$\text{c) } z^4 = 1, \quad [\text{R. } \cos\left(\frac{k\pi}{2}\right) + i \sin\left(\frac{k\pi}{2}\right).]$$

$$\text{d) } z^3 = \frac{(i-1)^4}{(i+1)^2}, \quad [\text{R. } \sqrt[3]{2} \left( \cos\left(\frac{\pi}{6} + \frac{2}{3}k\pi\right) + i \sin\left(\frac{\pi}{6} + \frac{2}{3}k\pi\right) \right).]$$

$$\text{e) } z^2 = -\frac{2i}{i-1}, \quad [\text{R. } \sqrt[4]{2} \left( \cos\left(\frac{3}{8}\pi + k\pi\right) + i \sin\left(\frac{3}{8}\pi + k\pi\right) \right).]$$

$$\text{f) } (\bar{z})^4 = \frac{(1+i)}{i}, \quad [\text{R. } \sqrt[8]{2} \left( \cos\left(\frac{\pi}{16} + \frac{k}{2}\pi\right) + i \sin\left(\frac{\pi}{16} + \frac{k}{2}\pi\right) \right).]$$

$$\text{g) } (\bar{z})^3 = -8i, \quad [\text{R. } 2 \left( \cos\left(\frac{\pi}{6} + \frac{2}{3}k\pi\right) + i \sin\left(\frac{\pi}{6} + \frac{2}{3}k\pi\right) \right).]$$

$$\text{h) } z^4 = 2(\bar{z})^2 \quad [\text{R. } 0, \sqrt{2} \left( \cos\left(\frac{k\pi}{3}\right) + i \sin\left(\frac{k\pi}{3}\right) \right).]$$

$$\text{i) } z^2 = (\sqrt{6} - \sqrt{2}i)|z|^3 \quad [\text{R. } 0, \frac{\sqrt{2}}{4} \left( \cos\left(-\frac{\pi}{12} + k\pi\right) + i \sin\left(-\frac{\pi}{12} + k\pi\right) \right).]$$

$$\text{j) } z^3 = \frac{4+4i}{|z|^2} \quad [\text{R. } 4\sqrt{2} \left( \cos\left(\frac{\pi}{12} + \frac{2}{3}k\pi\right) + i \sin\left(\frac{\pi}{12} + \frac{2}{3}k\pi\right) \right).]$$

$$\text{k) } z^4 = \frac{i}{(2\bar{z})^3} \quad [\text{R. } \frac{1}{\sqrt[7]{8}} \left( \cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) \right).]$$

$$\text{l) } z^4 = \frac{i(\bar{z})^3}{8} \quad [\text{R. } 0, \frac{1}{8} \left( \cos\left(\frac{\pi}{14} + \frac{2}{7}k\pi\right) + i \sin\left(\frac{\pi}{14} + \frac{2}{7}k\pi\right) \right).]$$