UNIVERSITÀ
DEGLI STUDI
DI PADOVA

Fisica 1

Lezione 42: Th. Gauss

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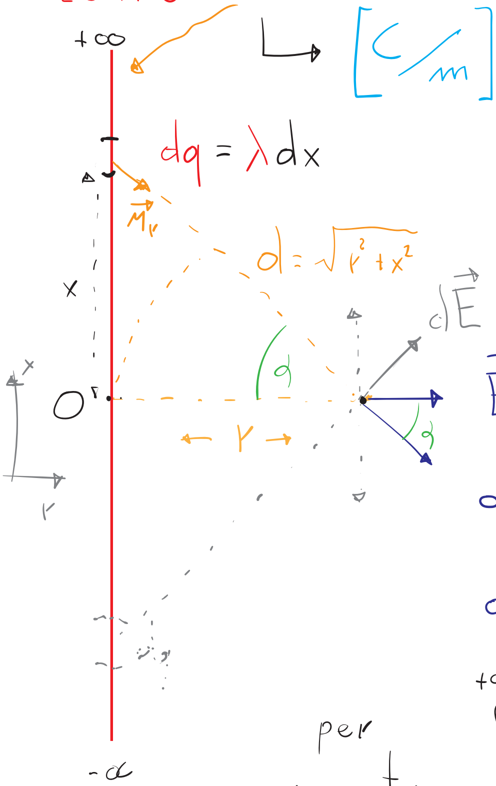
Esercizio $\vec{E}$ filo carico [stupido]

Filo carico

λ = densità lineare di carica

$\hookrightarrow [C/m]$



$\vec{E} = ?$

$$d\vec{E} = K\frac{dq}{d^2}\vec{u}_r$$

$$dE_{\parallel r} = d\vec{E}\cos\vartheta$$

per simmetria $\displaystyle\int_{x=-\infty}^{+\infty} dE_{\parallel x} = \phi$

$$|\vec{E}| \equiv E_{\parallel r} = \int_{x=-\infty}^{\infty} |dE|\cos\vartheta \, dx$$

$$\left.\begin{aligned} r &= d\cos\vartheta \\ \cos\vartheta &= r/d \end{aligned}\right\}$$

$$= \int_{-\infty}^{\infty} dE \, \frac{r}{d} \, dx$$

$$= \int_{-\infty}^{\infty} dE \, \frac{r}{\sqrt{r^2+x^2}} \, dx$$

$$= \int_{-\infty}^{\infty} K \underbrace{\frac{\lambda}{(r^2+x^2)} \frac{r}{\sqrt{r^2+x^2}} dx}_{dq}$$

$$= K\lambda r \int_{-\infty}^{\infty} \frac{1}{(r^2+x^2)^{3/2}} dx =$$

$x = r\,tg\vartheta$

$dx = r\sec^2\vartheta \, d\vartheta$

$$= K\lambda r \int_{-\pi/2}^{+\pi/2} \frac{r\sec^2\vartheta \, d\vartheta}{(r^2 + r^2 tg^2\vartheta)^{3/2}}$$

$$= K\lambda r \int_{-\pi/2}^{\pi/2} \frac{r}{r^3} \frac{\sec^2\vartheta}{(1+tg^2\vartheta)^{3/2}} = \frac{K\lambda}{r}\int_{-\pi/2}^{\pi/2} \frac{\sec^2\vartheta}{(1+tg^2\vartheta)^{3/2}} d\vartheta = \frac{K\lambda}{r}\int_{-\pi/2}^{\pi/2} \frac{\sec^2\vartheta}{\sec^3\vartheta} d\vartheta$$

$\hookrightarrow \sin^2\vartheta + \cos^2\vartheta = 1$

$\cdot 1/\cos^2$

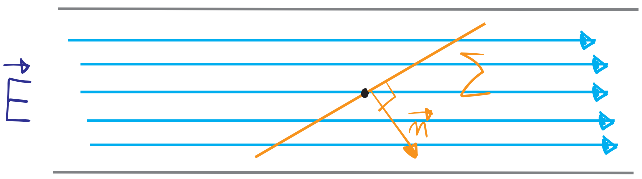
$tg^2\vartheta + 1 = \sec^2\vartheta$

$$= \frac{K\lambda}{r}\int_{-\pi/2}^{\pi/2} \frac{d\vartheta}{\sec\vartheta} = \frac{K\lambda}{r}\int_{-\pi/2}^{\pi/2} \cos\vartheta \, d\vartheta$$

$$= \frac{K\lambda}{r}\Big[\sin\vartheta\Big]_{-\frac{\pi}{2}}^{\pi/2} = \frac{2K\lambda}{r}$$

Th. di Gauss

Flusso attraverso una superficie orientata

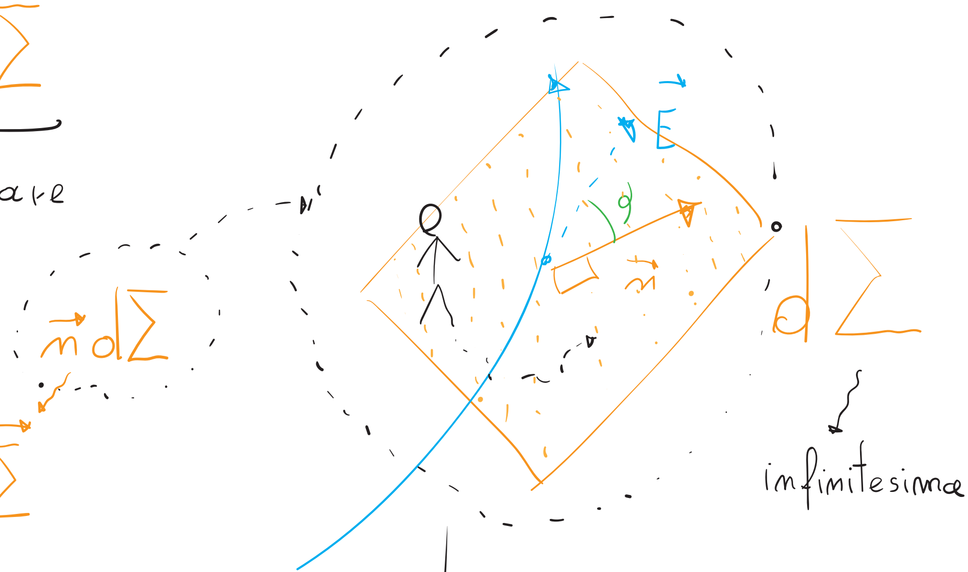


$\vec{n}$ normale alla superficie

Σ superficie

$\vec{n}\Sigma \rightarrow$ superficie orientata

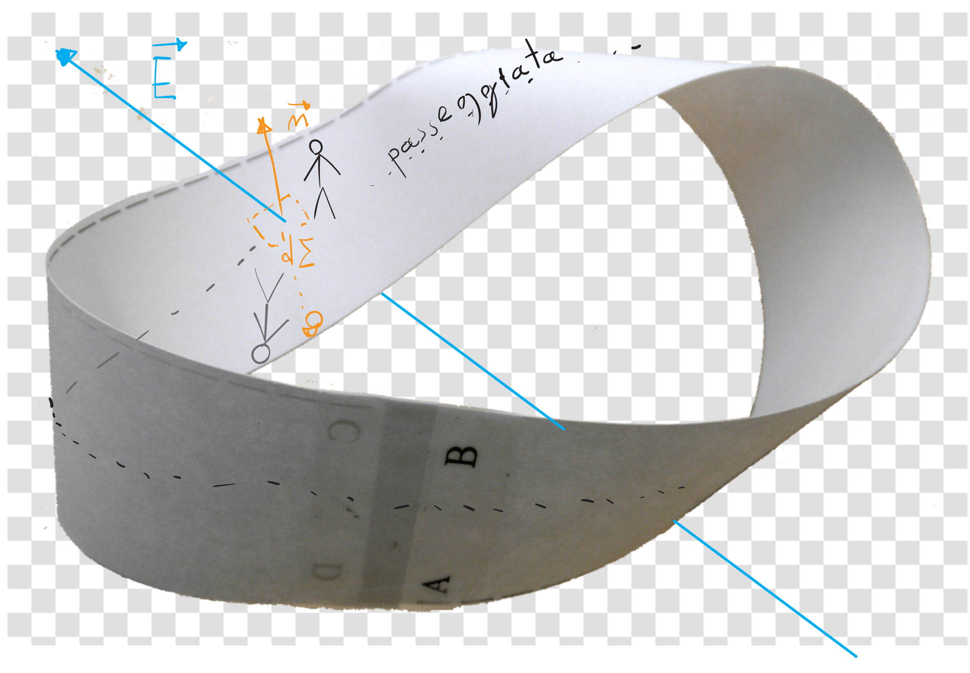
$$\underbrace{\phi_{\Sigma}(\vec{E})}_{\text{Flusso di } \vec{E} \text{ su } \Sigma} = \underbrace{\vec{E} \cdot \vec{n}\,\Sigma}_{\text{scalare}}$$



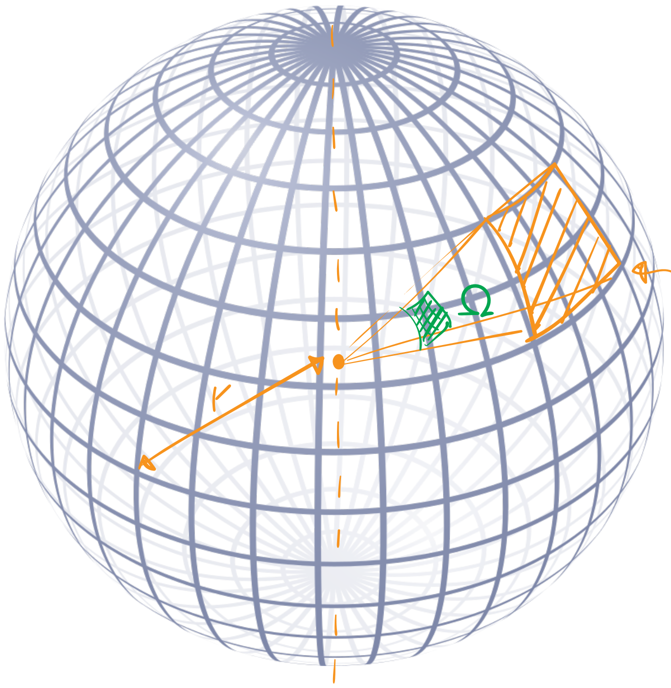
$$d\phi(\vec{E}) = \vec{E}\, d\vec{\Sigma}$$

$\vec{E}(x,y,z) \longrightarrow$ Se mi sposto su $d\Sigma$

↳ $\vec{E}(x,y,z) \simeq$ invariato



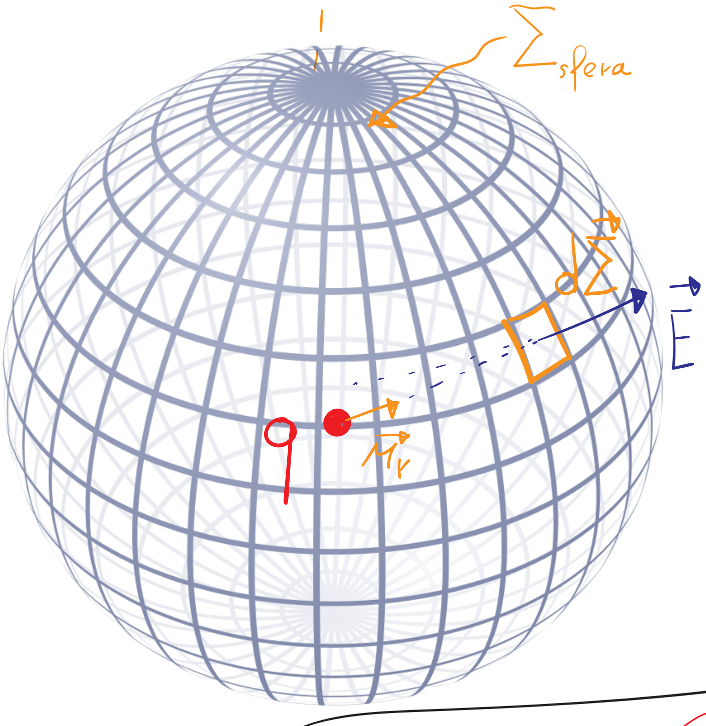
$\phi(\vec{E})$ superficie chiusa



Angolo solido [steradianti]

$$\Sigma = r^2 \Omega$$

$$\Sigma_{sfera} = 4\pi r^2 = \Omega r^2$$



$$\vec{E}(r) = K \frac{q}{r^2} \vec{u}_r$$

$$\phi_{Sfera}(\vec{E}(r)) = \iint_{\Sigma_{sfera}} \vec{E}(r) \, d\vec{\Sigma}$$

$$= \iint_{\Sigma} K \frac{q}{r^2} \vec{u}_r \cdot d\vec{\Sigma}$$

Superficie sfera

$$= K \frac{q}{r^2} \iint_{\Sigma} \overbrace{\vec{u}_r \cdot d\vec{\Sigma}}^{1} = K \frac{q}{r^2} \iint_{Sfera} d\Sigma = K \frac{q}{r^2} 4\pi r^2$$

Non dipende dalla geometria!

$$\phi_{Sfera}(\vec{E}(r)) = 4\pi K q = \boxed{\frac{q}{\varepsilon_0}} \quad , \quad K = \frac{1}{4\pi\varepsilon_0} \rightarrow \simeq 8.85 \cdot 10^{-12} \, [F/m]$$

costante dielettrica vuoto