UNIVERSITÀ DEGLI STUDI DI PADOVA

Fisica 1

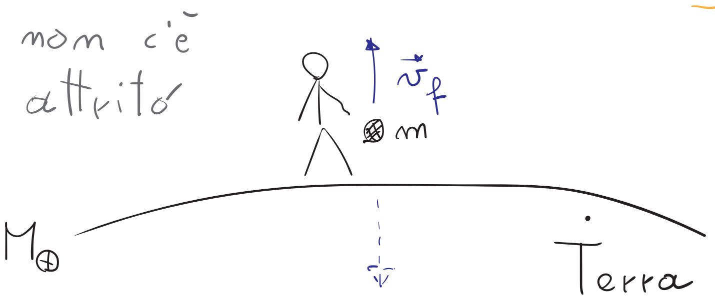
Lezione 38 : Keplero, esercizi

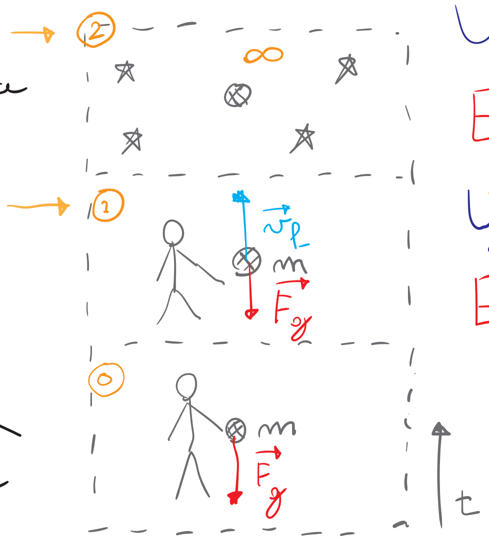
Prof. Giubilato

Gravitazione

$\vec{v}_f$ t.c. il sasso non ricada a terra

non c'è attrito





$U_{g,\infty} = \emptyset$

$E_K = \emptyset$

$U_g = -G\frac{M_\oplus}{R} m$

$E_K = \frac{1}{2} m v_f^2$

$U_g = -G\frac{M_\oplus}{R} m$

$E_K = \emptyset$

condizioni finale

$$U_g + E_K = cost$$

$$-G\frac{M_\oplus m}{R_\oplus} + \frac{1}{2} m v_f^2 = \emptyset \Rightarrow v_f^2 = \frac{2GM_\oplus}{R_\oplus}$$

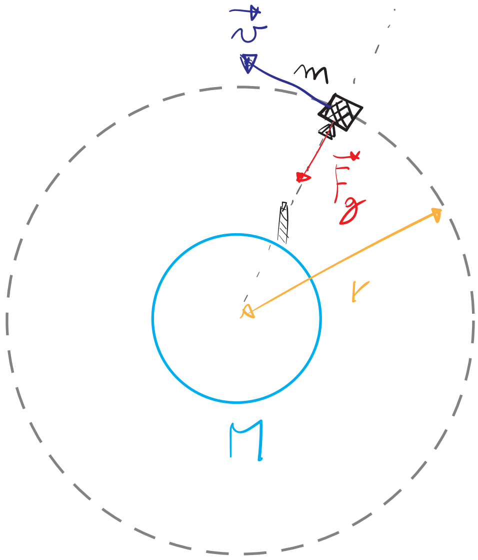
$$v_f = \sqrt{\frac{2GM_\oplus}{R_\oplus}} \simeq \sqrt{2}\sqrt{\frac{40\cdot 10^{13}}{6.4\cdot 10^{6}}} \simeq 11{,}183 \ [m/s]$$

1 m, 1 Kg

$$W_{lancio} = \vec{F}\cdot\vec{s} = \frac{1}{2} m v_f^2$$

$$|F| = \frac{v_f^2}{2}\frac{(1\,Kg)}{(1\,m)} \simeq 60\ MN$$

Orbita geostazionaria

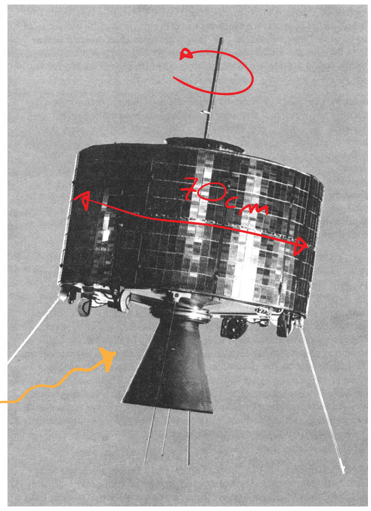


acc. centripeta

$$\vec{F}_g = m a_c = m \frac{v^2}{r}$$

$$\Downarrow$$

$$v = \sqrt{\frac{GM}{r}} \quad \text{orbita ?}$$



$$\omega_{sat.} \equiv \omega_{Terra} = \frac{2\pi}{T_{Terra}}$$

$T_{Terra} = 24 \cdot 60 \cdot 60$

$= 24 \cdot 3600$

$= 86{,}400\,s$

$$v/r \Rightarrow v = \frac{2\pi r}{T}$$

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}} \rightarrow \frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$r^3 = \frac{GMT^2}{4\pi^2} \rightarrow r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$\simeq 43 \cdot 10^6\,m$ rispetto al centro terra

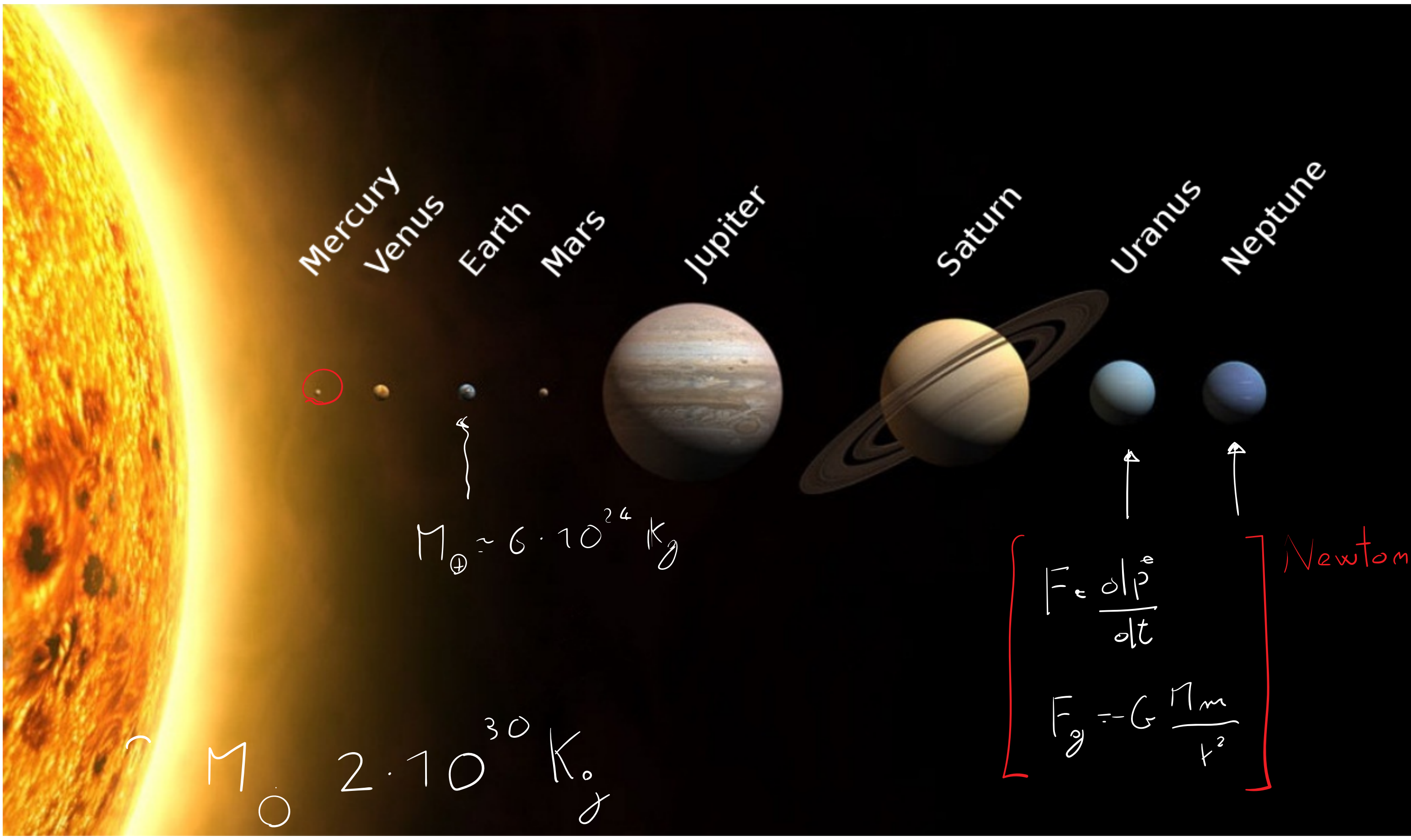
quota

$$h = r - R_{\oplus} \simeq 43 \cdot 10^6 - 6.4 \cdot 10^6 \simeq 36{,}000\,Km$$

400,000 Km

$\simeq$ 40,000 Km Geostazionaria

$\simeq$ 400 Km Low Earth Orbit (LEO)



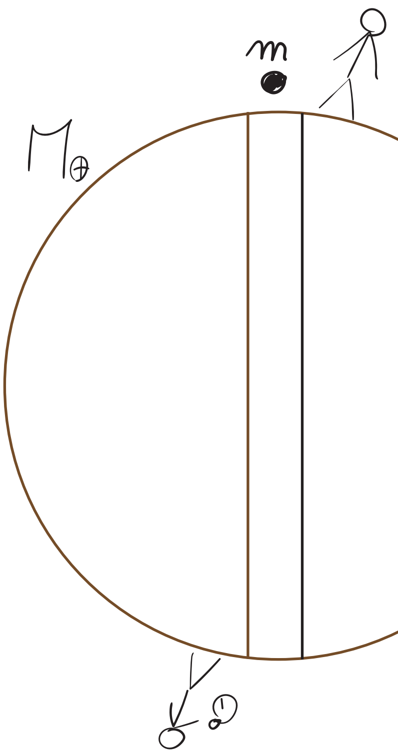
Sistemi legati (chiusi)

$\hookrightarrow$ $E_K + U_g < \phi$ Chiuso → pianeti (→ orbite ellittiche)

$= \phi$ Critico → velocità fuga (→ parabola)

$> \phi$ Aperto → Voiager (→ iperbole)

Esempio



$T = ?$

new zeland

$$F(r) = -G\frac{M(r)\,m}{r^2}$$

$$= -\frac{G}{r^2}\,\frac{4}{3}\pi r^3 \rho_\oplus\, m$$

$$= -\frac{4}{3} G \rho_\oplus \pi\, r\, m$$

$$= -\frac{4}{3}\pi\, G \frac{M_\oplus}{\frac{4}{3}\pi R_\oplus^3}\, r\, m$$

$$= -G\frac{M_\oplus}{R_\oplus^2}\,\frac{m}{R_\oplus}\, r$$

$$\frac{d^2}{dt^2} r = a = \frac{F}{m} = -\frac{g_0}{R_\oplus}\, r$$

$$K \cdot r \Rightarrow K = \frac{g_0}{R_\oplus} m$$

Moto armonico: $T = \frac{2\pi}{\omega}$

$\hookrightarrow \omega = \sqrt{\frac{K}{m}}$

$$= 2\pi\sqrt{R/g} =$$

$$\simeq 5077\,s$$

$$\simeq 85\ \text{min}$$