PROBLEM 3

The circuit reported in the figure below is a single-ended output mirrored OTA. Assuming for simplicity the validity of the long-channel MOSFET equations:

1) Identify the high-impedance and the low-impedance nodes of the circuit;
2) derive an approximate expression of the OTA differential-mode transfer function $A_{DM}(s) = \frac{V_o}{V_{ID}}$, $V_o = V_2 - V_1$, considering only poles associated to high-impedance nodes and neglecting the zeroes;
3) determine the bias current $I_{Q2} = I_{Q1}$ of $M_{1,2}$ so that the unity gain frequency of the OTA is $f_u = 20 \text{ MHz}$ (consider the parasitic capacitances of the OTA negligible with respect to $C_1$);
4) determine the size W/L of all transistors so that the low-frequency gain of the OTA is $|A_{DC}| = 40 \text{ dB}$;
5) compute the parasitic capacitance of the output node.

DATA: $V_{CC} = 1.2 \text{ V}$, $I_{REF} = 10 \mu A$, $C_1 = 5 \text{ pF}$, $L_2 = 180 \text{ mm}$, $L_3 = 865 \text{ mm}$, $L_4 = L_2$, $W_7 = W_9$, $W_8 = M\cdot W_4$, $W_{10} = M\cdot W_4$, $M = 4$, $\gamma_{m1} = 10$, $\gamma_{m2} = \gamma_{m3} = \gamma_{m4} = 5$, where $\gamma_{m4} = g_m/L_4$ is the transconductance efficiency of device $M_4$.

SOLUTION

1) Identify the high-impedance and the low-impedance nodes of the circuit

![Circuit Diagram]

Low-impedance nodes:

$N_1 \to V_{Q1}$  $N_2 \to V_{Q2}$  $N_3 \to 1/\gamma_{m3}$  $N_4 \to \text{ac ground}$

High-impedance node:

$O \to R_o = R_{g1} || R_{10}$

2) Derive an approximate expression of the OTA differential-mode transfer function $A_{DM}(s) = \frac{V_o}{V_{ID}}$, $V_o = V_2 - V_1$, considering only poles associated to high-impedance nodes and neglecting the zeroes

The OTA features a single high-impedance node, so its approximate differential-mode voltage gain takes the form:

$$A_{DM}(s) = \left. \frac{V_o}{V_{ID}} \right|_{s = 0} = \frac{A_v}{1 + s/\omega_p}$$
with \( A_p = A_{ds} \) and the pole frequency that can be estimated associating the pole to the high-impedance node 0:

\[
\omega_p = \frac{1}{R_o C_o}
\]

\( R_o \): Low-frequency impedance at the port formed by node 0 and ground
\( C_o \): total capacitance between node 0 and ground

Low-impedance nodes
\( \approx \) ac ground

\( R_o = R_{0b} \parallel R_{0a} \)

\( C_o = C_{ds} + C_{gb} + C_{db} + C_{gk} + C_l \)

Low-frequency gain - differential mode

\[
\begin{align*}
\dot{i}_o &= -\frac{1}{g_m} \left( g_{m1} + g_{m2} \right) \frac{V_{id}}{2} = -\frac{V_{id}}{2} \\
&= -g_m \cdot V_{id} \quad \text{with} \quad g_m = \frac{1}{R_{source}} \\
V_o &= R_o \cdot i_o = -g_m R_o V_{id}
\end{align*}
\]
Determine the bias current $I_{D1}$ so that the unity-gain frequency of the OTA is $f_T = 20$ MHz (consider the parasitic capacitances of the OTA negligible with respect to $C_L$).

So:

$$A_{DR}(s) = \frac{A_D}{1 + s/\omega_p} = \frac{M \gamma_m R_0}{1 + s R_0 C_0}$$

with

$$R_0 = R_{DS} \parallel R_{JO}$$

and $C = C_{GS} + C_{GD} + C_{JO} + C_{Gd} + C_L$

3) Determine the bias current $I_{D1} = I_{D2}$ of $M_{1,2}$ so that the unity-gain frequency of the OTA is $f_T = 20$ MHz (consider the parasitic capacitances of the OTA negligible with respect to $C_L$).

Since $A_{DR}(s) = \frac{A_D}{1 + s/\omega_p}$, the unity-gain frequency $f_T$ of the OTA, $\omega_T = 2\pi f_T$ is given by:

$$|A_{DR}(\omega_T)| = 1 \Rightarrow \frac{|A_D|}{\sqrt{1 + \left(\frac{\omega_T}{\omega_p}\right)^2}} = 1$$

$$\omega_T = \omega_p \sqrt{\frac{A_D^2}{1 + 1}} \leq \omega_p |A_D| = \frac{1}{R_{DS} C_0} M \gamma_m R_0$$

$$\Rightarrow \frac{M \gamma_m}{C_0} \leq \frac{M \gamma_m}{C_L}$$

since parasitic capacitances are assumed to be negligible w.r.t. $C_L$

Then

$$\gamma_m = \frac{I_{D1} C_L}{\pi} = \frac{2\pi (20 \mu H_z)(5 \mu F)}{4} = 15 \mu A$$

Since the transconductance efficiency $\gamma_m = \frac{\gamma_m}{I_{D1}} = 10 \, V^{-1}$ then
Determine the size W/L of all transistors so that the low-frequency gain of the OTA is $|A_{\text{dB}}| = 40 \text{ dB}$.

4) Determine the size W/L of all transistors so that the low-frequency gain of the OTA is $|A_{\text{dB}}| = 40 \text{ dB}$.

$$|A_{\text{dB}}| = |A_{\text{B}}| = M \alpha_m R_o = 100 \text{ (40 dB)}$$

$$R_o = \frac{|A_{\text{B}}|}{\pi \alpha_m} = \frac{100}{4 \cdot 15 \mu \text{s}} = 153 \text{ k}\Omega$$

$$R_o = \frac{R_o Q || R_{o10}}{M \alpha_m L_{B} \mu_{p} L_{10}} = \frac{(M \alpha_m L_{B} \mu_{p} L_{10}) I_{D10}}{R_o}$$

$$I_{D10} = M \cdot I_{D4} = M \cdot I_{D2} = 62.8 \mu\text{A}$$

Assuming $L_{10} = L_q$ we have:

$$L_q = R_o I_{D10} \frac{\mu_n + \mu_p}{\mu_n - \mu_p} = 1.58 \text{ mm} = L_{10}$$

Since $\gamma_{np} = \frac{\alpha_m}{I_{D10}} = \frac{2}{V_{oV10}} = \frac{2}{V_{oV4}} = \frac{2}{V_{oV3}} = \gamma_{n3} = 5 \text{ V}^{-1}$

and $\gamma_{p10} = \gamma_{n3} = 5 \text{ V}^{-1}$ then

$$V_{oV8} = \frac{2}{\gamma_{n3}} = 0.4 \text{ V} \quad V_{oV10} = \frac{2}{\gamma_{p10}} = 0.4 \text{ V}$$

$$W_9 = L_q \cdot \frac{2}{\gamma_{n3}} I_{D3}^2 = 2.63 \text{ mm}$$

$$W_{10} = L_{10} \cdot \frac{2}{\gamma_{p10}} I_{D10}^2 = 12.4 \text{ mm}$$
Then

\[ W_4 = \frac{1}{M} \quad W_8 = 0.653 \text{ mm} \quad L_4 = L_8 \quad \text{(current mirror rule)} \]

\[ W_5 = \frac{1}{M} \quad W_{10} = 3.03 \text{ mm} \quad L_5 = L_{10} \]

\[ W_3 = W_4 \quad L_3 = L_4 \quad \text{for symmetry} \]

\[ W_2 = W_5 \quad (dual) \quad L_2 = L_5 \quad \text{(CP rule)} \]

\[ V_{ov6} = \frac{2}{\gamma_{n6}} = 0.4 \text{ V} \quad W_6 = L_6 \quad \frac{2 I_{s1}}{k_{m1} V_{ov6}^2} = 0.25 \text{ mm} \]

\[ W_5 = \frac{I_{s2}}{I_{s6}} \quad W_6 = \frac{I_{s1} + I_{s2}}{I_{s6}} \quad W_6 = 1.41 \text{ mm} \]

\[ L_5 = L_6 = 360 \text{ nm} \quad \text{(CP rule)} \]

\[ V_{ov1} = \frac{2}{\gamma_{n1}} = 0.2 \text{ V} \quad W_1 = L_1 \quad \frac{2 I_{s1}}{k_{p1} V_{ov1}^2} = 1.41 \text{ mm} \]

\[ W_2 = W_1 \]

In summary,
5) Compute the parasitic capacitance of the output node.

\[ C_{o, \text{par}} = C_{dbs} + C_{gds} + C_{db10} + C_{gd10} = \]
\[ = \left( C_{sdn} + C_{gmn} \right) W_g + \left( C_{sdp} + C_{gmp} \right) W_{10} = \]
\[ = 14.7 \text{ fF} \]