Stability of feedback amplifiers

Fully differential

\[
\frac{v_o}{v_i} = -\text{Add}(s) = -\left. \frac{v_{od}}{v_{id}} \right|_{v_{ic}=0}
\]

Note: \(\text{Add}(s) > 0\)

\[
\begin{cases}
  v_o = -\text{Add}(s) \cdot v_i \\
  v_c = \frac{C_1}{C_1 + C_1 + C_2} \cdot v_s + \frac{C_2}{C_1 + C_1 + C_2} v_o
\end{cases}
\]

\(\alpha < 1\)

\(\beta < 1\)
\[ v_o = \frac{-A(s)}{\beta \text{Add}(s)} \left( \alpha v_s + \beta v_o \right) \]

\[ A(s) = \frac{v_o}{v_s} = -\frac{C_1}{C_2} \frac{T(s)}{1 + T(r)} \]

\[ T(r) = \beta \text{Add}(s) \text{ loop transmission or loop gain} \]

The feedback network loads the OTA output with an equivalent capacitance \( C_{fb} = C_2(1 - \beta) \).

Example: consider a single-stage OTA

\[ \Rightarrow \text{Add}(s) = \frac{G_m \cdot R_o}{1 + \frac{s}{\omega_{p1}}} \]

\( G_m \): transconductance of the diff. pair

\( R_o \): output resistance

\( \omega_{p1} = \frac{1}{\sqrt{R_o \cdot C_L}} \)

\( C_l = C_o + C_L + C_{fb} \)

\( \text{par. cap. of the output node} \)

\( \text{ext. cap.} \)

**Stability of Feedback Amplifiers**

Amplifier \( \rightarrow \) Linear Time-Invariant system (LTI)
Bounded-Input Bounded-Output (BIBO) stability

The system is BIBO-stable if:

\[ x(t) \xrightarrow{LTI} y(t) \quad x_{\text{min}} \leq x(t) \leq x_{\text{max}} \]

then

\[ y_{\text{min}} \leq y(t) \leq y_{\text{max}} \]

In order to verify if the system is BIBO-stable we can:

- determine the position of the system poles and verify that \( \text{Re}\{p_k\} < 0 \)

  😊 general validity

  😐 computing the poles position can be complicated

- use the Nyquist criterion:

  😊 general validity

  😐 need to draw the polar plot of \( T(j\omega) \)
  and this can be complicated

- use the Bode criterion:

  😐 not always valid but in most cases involving feedback amplifiers it is valid
simple and it also gives the designer two alternate parameters that give a measure of how much the feedback amplifier is stable.

Consider a general feedback system:

\[ H(s) = \frac{A(s)}{1 + T(s)} \]

\[ T(s) = \beta(s) A(s) \]

Assume \( \beta(0) A(0) > 0 \) real positive, so we have negative feedback at low frequency.

\[ x_f = T(s) x_e \quad x_e = x - x_f \]

\[ x_e = \frac{x}{1 + T(s)} \quad \text{so at low freq: } x \text{ and } x_f \text{ are in phase} \]

\[ w_{180} : \angle T(s_{180}) = -180^\circ \]

when \( w = w_{180} \) the feedback becomes positive.

Bode criterion: if \( |T(s_{180})| < 1 \) then the system is stable.
What if $|T(j\omega_{10})| > 1$:

\[
x_f = T(j\omega_{10}) x_e = |T(j\omega_{10})| \cdot x_e \geq 1
\]

\[
= -|T(j\omega_{10})| \cdot x_c
\]

Gain Margin (GM):

\[
GM = \frac{1}{|T(j\omega_{10})|} = -20 \log_{10} |T(j\omega_{10})| \text{ dB}
\]

GM > 0 \Rightarrow \text{system stable}

GM = 0 \Rightarrow |T(j\omega_{10})| = 1 \text{ system unstable}

GM < 0 \Rightarrow |T(j\omega_{10})| > 1 \text{ system unstable}

Phase Margin (PM):

\[
\omega_c : |T(j\omega_c)| = 1 \quad \phi_c = \angle T(j\omega_c)
\]
\[ \Phi_N = \angle T(j\omega_c) \quad (-180^\circ) = \]
\[ = 180^\circ + \angle T(j\omega_c) \]

Typically, designers aim at a \( \Phi_N = 60^\circ - 20^\circ \)

\[ \Phi_N = 180^\circ + \angle T(j\omega_c) = 180^\circ + \phi_c \]

\[ \phi_c = \Phi_N - 180^\circ \]

\[ T(j\omega_c) = \frac{1}{T(j\omega_c)} e^{-j\phi_c} = e^{-j\Phi_N} e^{-j\phi_c} = -e^{-j\Phi_N} \]

\[ = -\cos(\Phi_N) - j\sin(\Phi_N) \]

\[ |H(j\omega_c)| = \left| \frac{A(j\omega_c)}{1 + \beta A(j\omega_c)} \right| = \left| \frac{1}{\beta} \right| \left| \frac{T(j\omega_c)}{1 + T(j\omega_c)} \right| \]
\[
\begin{align*}
T(\omega_c) &= \frac{1}{\beta} \cdot \frac{1}{\sqrt{\left(1 - \cos(\beta \omega_c)\right)^2 + \sin^2(\beta \omega_c)}} = \frac{1}{\beta \sqrt{2}} \cdot \frac{1}{\sqrt{1 - \cos(\beta \omega_c)}} \\
\rho \omega_c &= 90^\circ \\
\frac{1}{H(\rho \omega_c)} &= \frac{\left|H(0)\right|}{\sqrt{2}} = \frac{\left|H(0)\right|_{\text{dB}}}{-3\text{dB}} \\
\omega_c &= \text{the } -3\text{dB bandwidth of the feedback system} \\
60^\circ &= \frac{1}{\beta} = \left|H(0)\right| \\
45^\circ &= \frac{1}{\beta \sqrt{2 - \sqrt{2}}} = 1.3 \left|H(0)\right| \\
30^\circ &= \frac{1}{\beta \sqrt{2 - \sqrt{3}}} = 1.7 \left|H(0)\right| \\
\end{align*}
\]