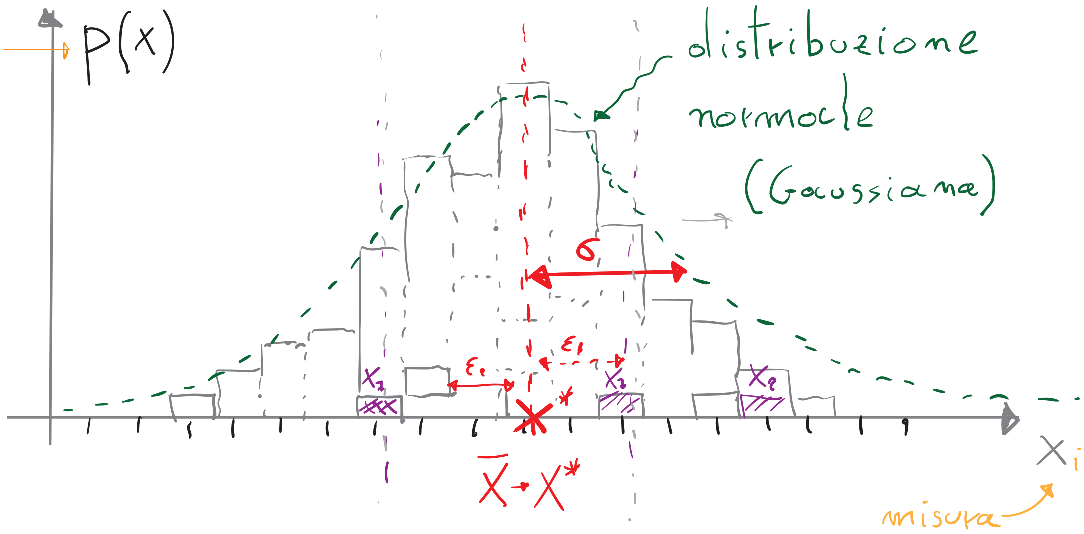
$\frac{\nu_j}{M} = f_j$

$M \to \infty$

numero risultati possibili



$N \to \infty$

$\underbrace{[x_{j+1} - x_j] \to \emptyset}_{\text{sensibilità infinita}}$

varianza

$$\sigma^2 = \frac{1}{n} \sum_{i=0}^{n-1} \varepsilon_i^2 = \frac{1}{n} \sum_{i=\emptyset}^{n-1} \overbrace{(x_i - \bar{x})}^{\text{scarto}}{}^2$$

$\Sigma > \emptyset$

probabilità

$$\lim_{n \to \infty} p(x_i) = \frac{1}{\boxed{\sigma}\sqrt{2\pi}} e^{\frac{-(x_i - \bar{x})}{2\sigma^2}}$$

σ → deviazione standard / → errore } stessa unità di misura della grandezza fisica

↳ errore SINGOLA misura

$\bar{x}$ $\quad \lim_{n \to \infty} \bar{x} \to x^*$

↳ σ ?

$$\boxed{\bar{x} = \frac{1}{N} \sum_{i=\emptyset}^{N-1} x_i}$$

↳ def.

Th. di Bernoulli

$\underbrace{\hookrightarrow N \to \infty \Rightarrow \bar{x} \to x^*}$

[ ]

$$\sigma_x = \sqrt{\sigma_x^2}$$

errore

∇ $\quad$ Vero $\Leftrightarrow$ fenomeno casuale

errori casuali

Errore di funzioni: somma

$\sigma_g \qquad g(x,y) = x + y$ — σ_x, σ_y

$$\sigma_g^2 = \frac{1}{N}\sum_{i=\emptyset}^{N-1}(g_i-\bar g)^2 = \frac{1}{N}\sum_{i=\emptyset}^{N-1}\left(x_i+y_i-\bar x-\bar y\right)^2$$

$$= \frac{1}{N}\sum_{i=\emptyset}^{N-1}\left(x_i-\bar x+y_i-\bar y\right)^2 = \frac{1}{N}\sum\left(\varepsilon_{ix}+\varepsilon_{iy}\right)^2$$

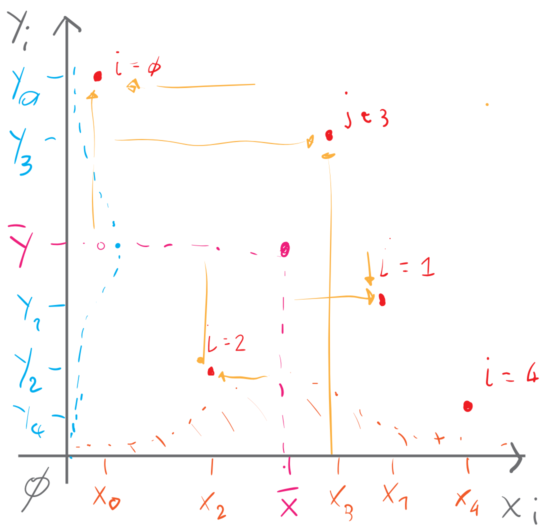
$\varepsilon_{ix} = x_i-\bar x$, $\varepsilon_{iy} = y_i-\bar y$

$$= \frac{1}{N}\sum_{i=\emptyset}^{N-1}\left(\varepsilon_{ix}^2+\varepsilon_{iy}^2+2\varepsilon_{ix}\varepsilon_{iy}\right)$$

$$= \underbrace{\frac{1}{N}\sum_{i=\emptyset}^{N-1}\varepsilon_{ix}^2}_{\sigma_x^2} + \underbrace{\frac{1}{N}\sum_{i=\emptyset}^{N-1}\varepsilon_{iy}^2}_{\sigma_y^2} + \frac{1}{N}\sum_{i=\emptyset}^{N-1}2\varepsilon_{ix}\varepsilon_{iy}$$

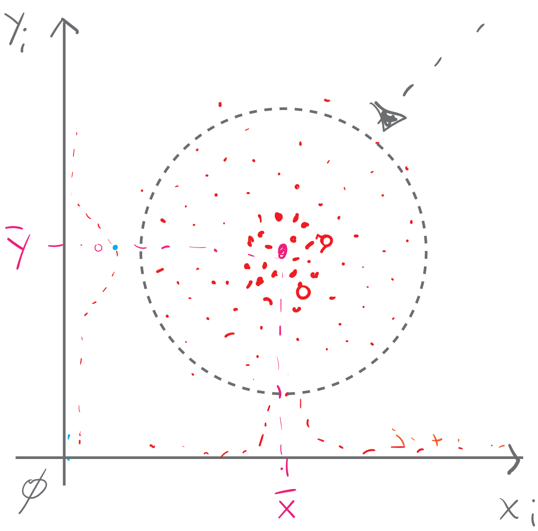
$$\underbrace{\frac{1}{N}\sum_{i=\emptyset}^{N-1}2\left(x_i-\bar x\right)\left(y_i-\bar y\right)}_{\emptyset}$$

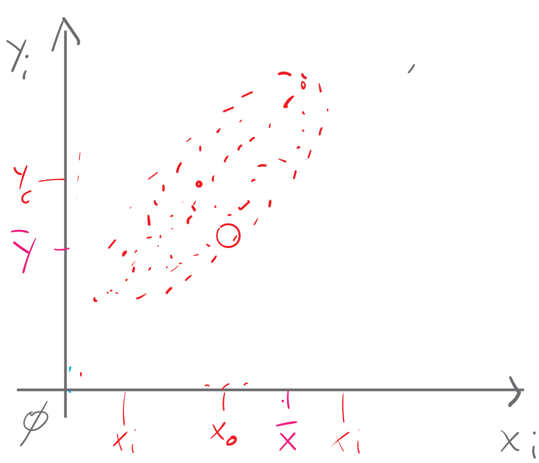
i-esima



Covarianza

x, y — Indipendenti

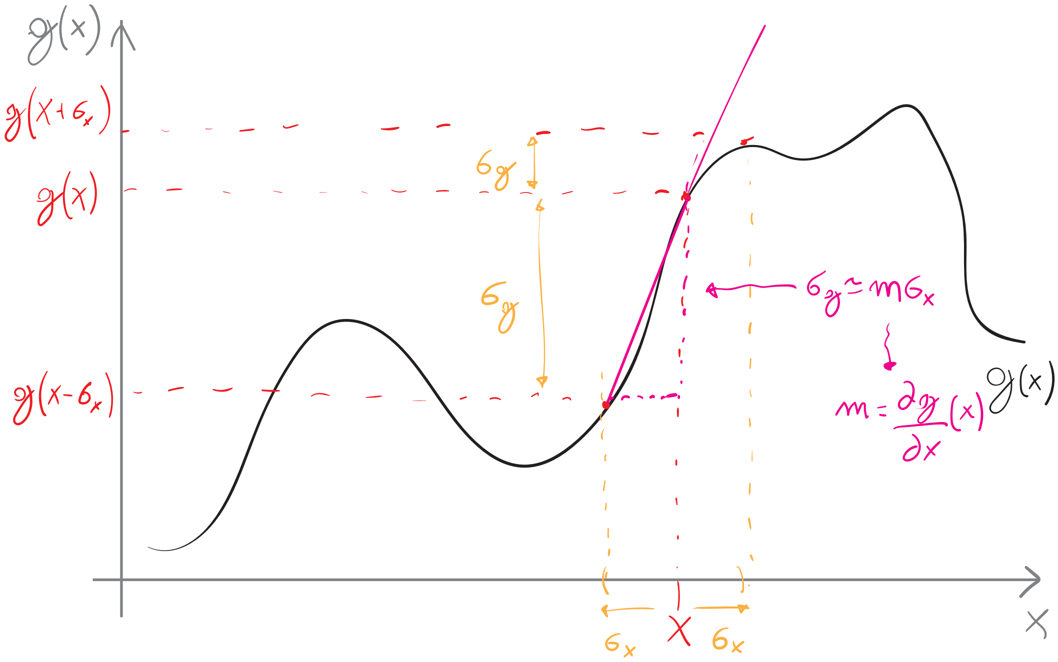




Variabili (misure) indipendenti

$$g(x_0, x_1, \dots x_{N-1}) = x_0 + x_1 + \dots \dots x_{N-1}$$

$$\sigma_g^2 = \sigma_{x_0}^2 + \sigma_{x_1}^2 + \dots \dots + \sigma_{x_{N-1}}^2 \qquad \sigma_g = \sqrt{\sigma_{x_0}^2 + \sigma_{x_1}^2 + \dots + \sigma_{x_{N-1}}^2}$$

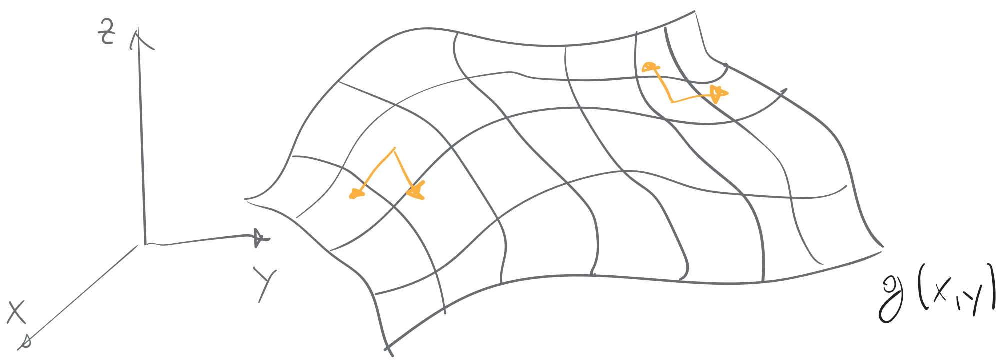


$$\sigma_g = \frac{\partial}{\partial x} g(x)\, \sigma_x$$

$$\sigma_g^2 = \left(\frac{\partial}{\partial x_0} g(x)\right)^2 \sigma_{x_0}^2 + \left(\frac{\partial}{\partial x_1} g(x)\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial}{\partial x_2} g(x)\right)^2 \sigma_{x_2}^2 + \dots\dots$$

Deriv. parziale

$$g(x,y) = x + y \qquad \frac{\partial g}{\partial x} = \frac{\partial}{\partial x}(x+y) = \frac{\partial}{\partial x} x + \frac{\partial}{\partial x} y = 1 + 0 = 1$$



Università degli Studi di Padova

Fisica 1

Lezione 2 : Misura

Prof. Giubilato

Errore sulla media

$$\bar{x} = \frac{1}{N}\sum_{i=\phi}^{N-1} x_i = \overbrace{\frac{x_\phi}{N} + \frac{x_1}{N} + \frac{x_2}{N} + \cdots + \frac{x_{N-1}}{N}}^{\text{Insieme di misure}}$$

$$\sigma_{\bar{x}}^2 = \left[\frac{\partial}{\partial x_0}\left(\frac{x_0}{N}\right)\right]^2 \sigma_{x_0}^2 + \left[\frac{\partial}{\partial x_1}\left(\frac{x_1}{N}\right)\right]^2 \sigma_{x_1}^2 + \cdots + \left[\frac{\partial}{\partial x_{N-1}}\left(\frac{x_1}{N}\right)\right]^2 \sigma_{x_{N+1}}^2$$

$$= \frac{\sigma_{x_0}^2}{N^2} + \frac{\sigma_{x_1}^2}{N^2} + \cdots + \frac{\sigma_{x_{N-1}}^2}{N^2}$$

$$= N\left[\frac{\sigma_x^2}{N^2}\right] = \frac{\sigma_x^2}{N}$$

$$\sigma_{\bar{x}} = \sqrt{\frac{\sigma_x^2}{N}} = \frac{\sigma_x}{\sqrt{N}}$$

errore singola misura

errore media misure

$N \to \infty$

$$\left.\begin{array}{l} \\ \sigma_{\bar{x}} \to \phi \end{array}\right\} \bar{x} \to x^*$$