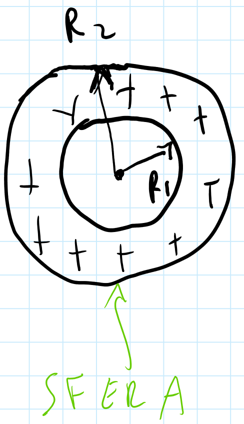
Tutorato 12

giovedì 1 giugno 2023 12:49

3.18

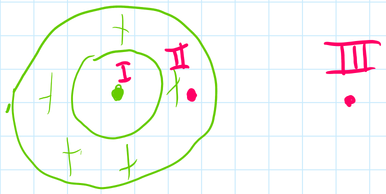


$R_1 = 0{,}1\ m$

$R_2 = 0{,}2\ m$

$\rho = 26{,}58 \cdot 10^{-8}\ C/m^3$

$E(r) = ?$



III) $r > R_2$ $\quad E(r) = \dfrac{q(r)}{4\pi \varepsilon_0 r^2}$ (come carica puntiforme)

$q(r) = \rho \cdot V_{conduttore} \rightarrow \left(\frac{4}{3}\pi R_2^3 - \frac{4}{3}\pi R_1^3\right)$

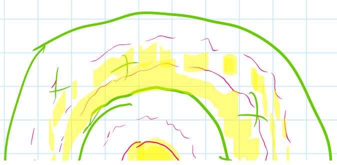
$= \rho \cdot \frac{4}{3}\pi \left(R_2^3 - R_1^3\right)$

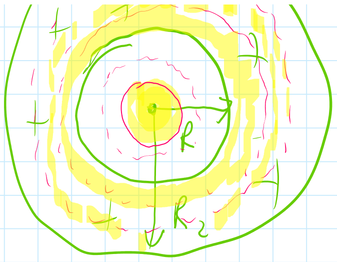
$= 26.58 \cdot 10^{-8} \cdot \frac{4}{3}\pi \left(0{,}2^3 - 0{,}1^3\right)$

$= 7{,}79 \cdot 10^{-9}\ C$

$$E(r) = \frac{q(r)}{4\pi\varepsilon_0 r^2} = \frac{7{,}79 \cdot 10^{-9}}{4\pi\varepsilon_0 \cdot r^2} = \frac{70}{r^2}\ V/m \quad (N/C)$$

$\Phi_E = \dfrac{q}{\varepsilon_0} \leftarrow$ CARICA NELLA SUPERFICIE





$$\Phi_E = \frac{q}{\varepsilon_0}$$

$$\uparrow$$

$$\oint_\Sigma \vec{E} \cdot \hat{n} \, d\Sigma$$

sup. sfera

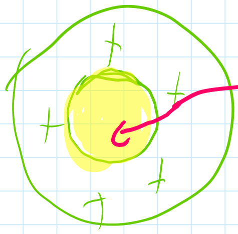
$$\rightarrow E \cdot \text{Superficie} = E \cdot 4\pi r^2$$

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

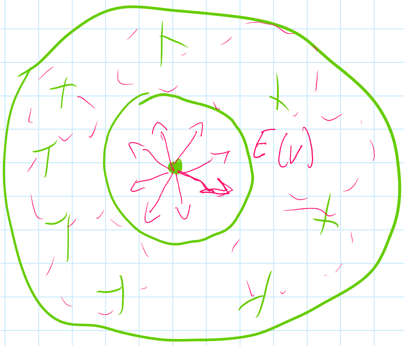
I) $r \leq R_1$ NO APPROSSIMAZIONE CARICA PUNTIFORME

Uso teorema di Gauss

$$\Phi_E = \frac{q(r)}{\varepsilon_0} \qquad E \cdot 4\pi r^2 = \frac{q(r)}{\varepsilon_0} \rightarrow E(r) = 0$$

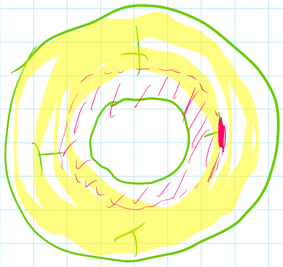


$q = 0$



AD OCCHIO SI VEDE

II) $R_1 < r < R_2 \rightsquigarrow$ Teo Gauss



$$\phi_E = \frac{q(r)}{\varepsilon_0} \rightsquigarrow \text{No è costante}$$

$$q(r) = \rho \cdot V \to \frac{4}{3}\pi r^3 - \frac{4}{3}\pi R_1^3$$

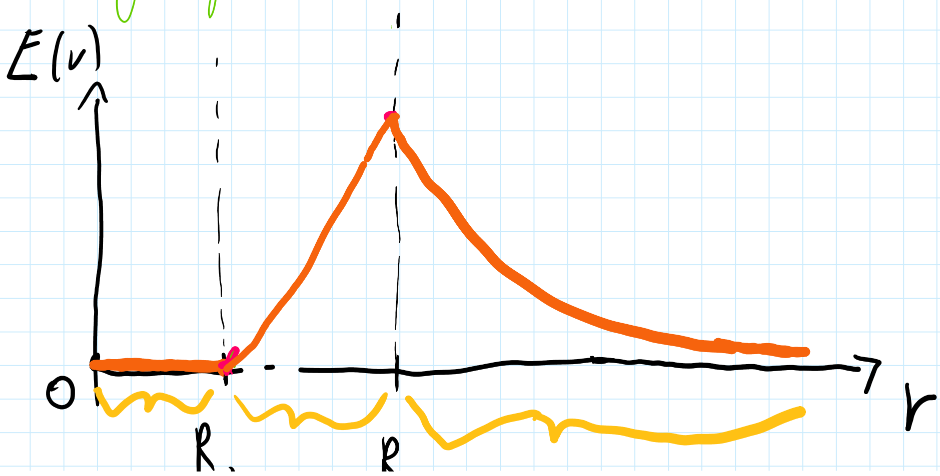
$$= \rho \cdot \frac{4}{3}\pi \left(r^3 - R_1^3\right)$$

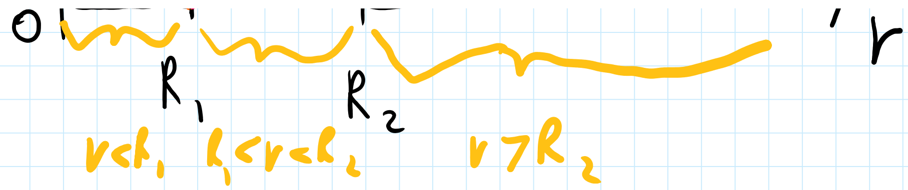
lineare quadratica

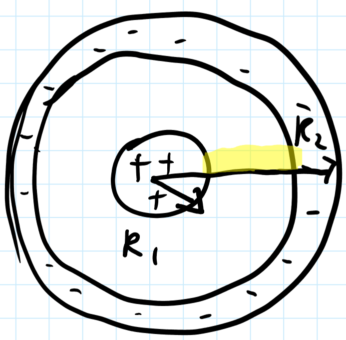
$$E \cdot 4\pi r^2 = \frac{\rho \cdot \frac{4}{3}\pi (r^3 - R_1^3)}{\varepsilon_0} = \left(\frac{\rho r}{3\varepsilon_0} - \frac{\rho R_1^3}{3\varepsilon_0 r^2}\right)$$

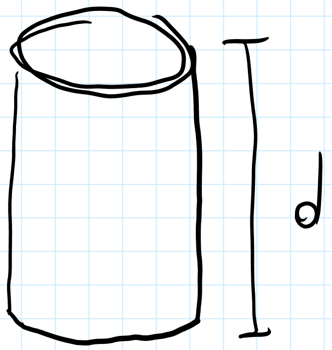
$$E(r) = \frac{\rho \cdot \frac{\cancel{4}}{3}\cancel{\pi} (r^3 - R_1^3)}{\cancel{4}\cancel{\pi} r^2 \varepsilon_0} = \frac{\rho \cdot (r^3 - R_1^3)}{3 r^2 \varepsilon_0}$$

Rappresentare graficamente andamento di $E(r)$









Capacità?

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}\,\hat{u}_r \qquad C = \frac{q}{\Delta V}$$

$$\Delta V = \frac{q}{C}$$

$$\Delta V \Rightarrow V_2 - V_1 \rightsquigarrow -(V_1 - V_2)$$

$$V_1 - V_2 = +\int_{R_1}^{R_2} \vec{E}\cdot d\vec{r} = \int_{R_1}^{R_2} \frac{\lambda}{2\pi\varepsilon_0 \cdot r}\,dr = \frac{\lambda}{2\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{1}{r}\,dr$$

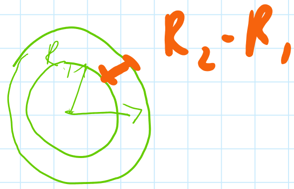
$$\rightsquigarrow = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)$$

$$\boxed{\lambda = \frac{q}{d} \rightsquigarrow q = \lambda \cdot d}$$

$$2\pi\varepsilon_0 \qquad \left| R_1 \right| \qquad d$$

$$C = \frac{q}{V_1 - V_2} = \frac{q}{\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)} = \frac{\lambda \cdot d}{\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)} = \boxed{\frac{2\pi\varepsilon_0 \cdot d}{\ln\left(\frac{R_2}{R_1}\right)}}$$

se $R_2 - R_1$ è MOLTO MINORE DI R_1 o R_2 cosa si può dire?



Capacità $\uparrow \rightsquigarrow C = ?$

$R_2 - R_1$ piccolo $\quad \boxed{h = R_2 - R_1}$

$$\ln\left(\frac{R_2}{R_1}\right) = \ln\left(1 + \frac{R_2 - R_1}{R_1}\right) \qquad \ln(1+x) \sim x \quad x \to 0$$

$$\sim \frac{R_2 - R_1}{R_1} = \frac{R_2}{R_1} - 1 = \frac{h}{R_1}$$

$$\left(C \cong \frac{2\pi\varepsilon_0 d}{\frac{R_2}{R_1} - 1} = \frac{2\pi\varepsilon_0 d R}{h} = \frac{\varepsilon_0 \Sigma}{h} \right)$$