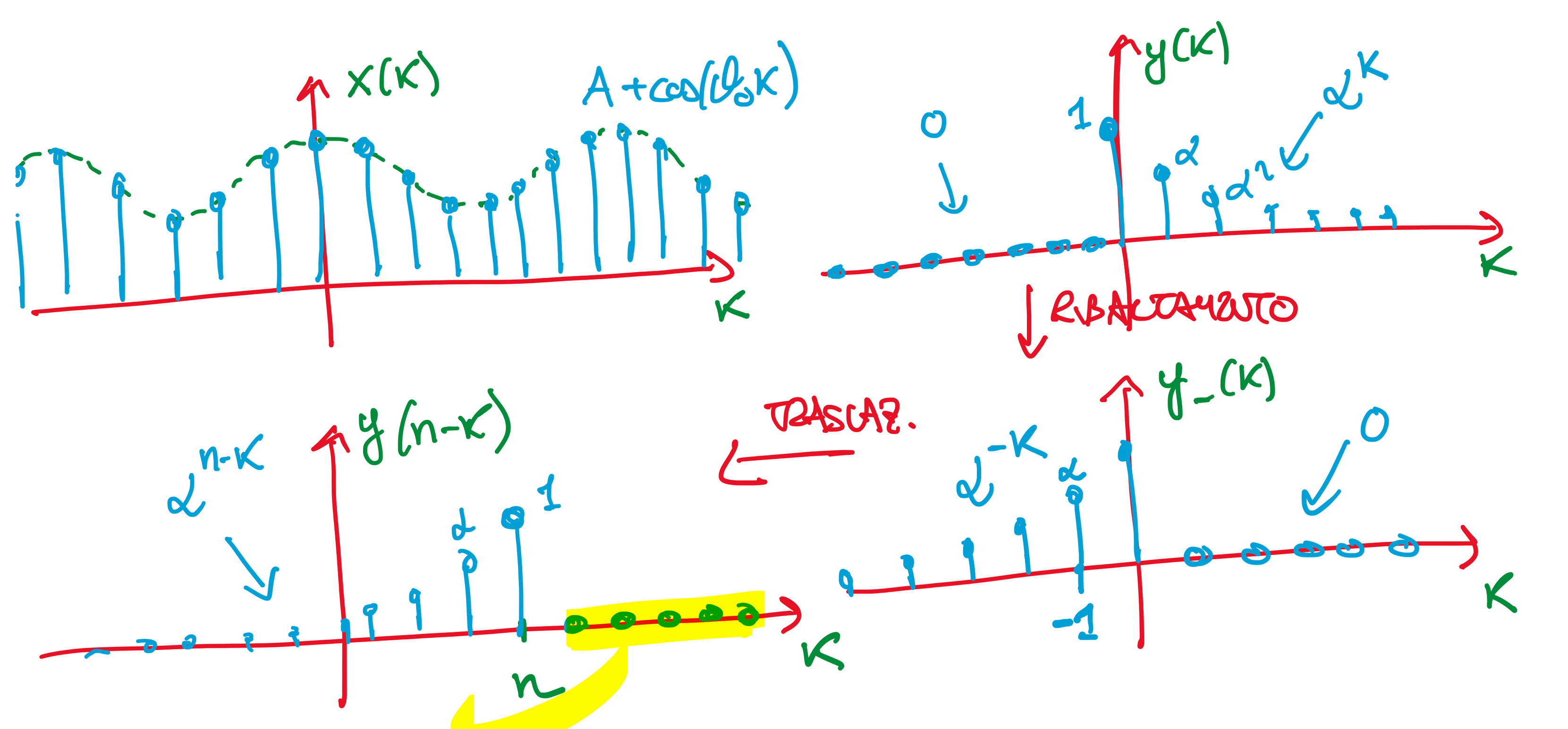
# Le9

Wednesday, 15 March 2023 22:46

**ES1** CALCOLARE $z(n) = x * y(n)$

CON $x(n) = A + \cos(\theta_o n)$ $\theta_o = 2\pi f_o T$

E $y(n) = 1_0(n)\,\alpha^n$ $\alpha$ REALE $|\alpha| < 1$




$$z(n) = \sum_{K=-\infty}^{\cancel{+\infty}\; n} \cancel{x(K)}\, \cancel{y(n-K)} \;\; (A+\cos(\theta_o K))\,\alpha^{n-K}$$

$$= \sum_{K=-\infty}^{n} \alpha^{n-K} \left(A + \cos(\theta_o K)\right) \qquad m = n-K \qquad \cos(\theta_o K) = \frac{e^{j\theta_o K} + e^{-j\theta_o K}}{2}$$

$$= \sum_{m=0}^{+\infty} \alpha^m \left(A + \frac{1}{2} e^{j\theta_o (n-m)} + \frac{1}{2} e^{-j\theta_o (n-m)}\right)$$

$$= \sum_{m=0}^{+\infty} \underbrace{A}_{\text{COST}} \alpha^m + \underbrace{\frac{e^{j\theta_o n}}{2}}_{\text{COST.}} \cdot \underbrace{e^{-j\theta_o m} \alpha^m}_{(\alpha e^{-j\theta_o})^m} + \underbrace{\frac{e^{-j\theta_o n}}{2}}_{\text{COST}} \cdot \underbrace{e^{j\theta_o m} \alpha^m}_{(\alpha e^{j\theta_o})^m}$$

$$= \frac{A}{1-\alpha} + \frac{1}{2} \frac{e^{j\theta_o n}}{1-\alpha e^{-j\theta_o}} + \frac{1}{2} \frac{e^{-j\theta_o n}}{1-\alpha e^{j\theta_o}}$$

$$= \frac{A}{1-\alpha} + \mathrm{Re}\left[\frac{e^{j\theta_o n}}{1-\alpha e^{-j\theta_o}}\right] \qquad \beta = 1-\alpha e^{-j\theta_o} \qquad \beta = |\beta| e^{j\theta_\beta}$$

NOTA $\displaystyle\sum_{m=0}^{\infty} \alpha^m = \lim_{N\to\infty} \sum_{m=0}^{N-1} \alpha^m = \lim_{N\to\infty} \frac{1-\alpha^N}{1-\alpha} = \frac{1}{1-\alpha}$

se $|\alpha| < 1$

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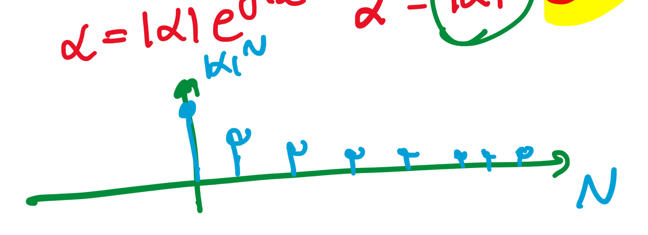
$\alpha = |\alpha| e^{j\theta_\alpha}$ $\alpha^N = |\alpha|^N e^{j\theta_\alpha N}$

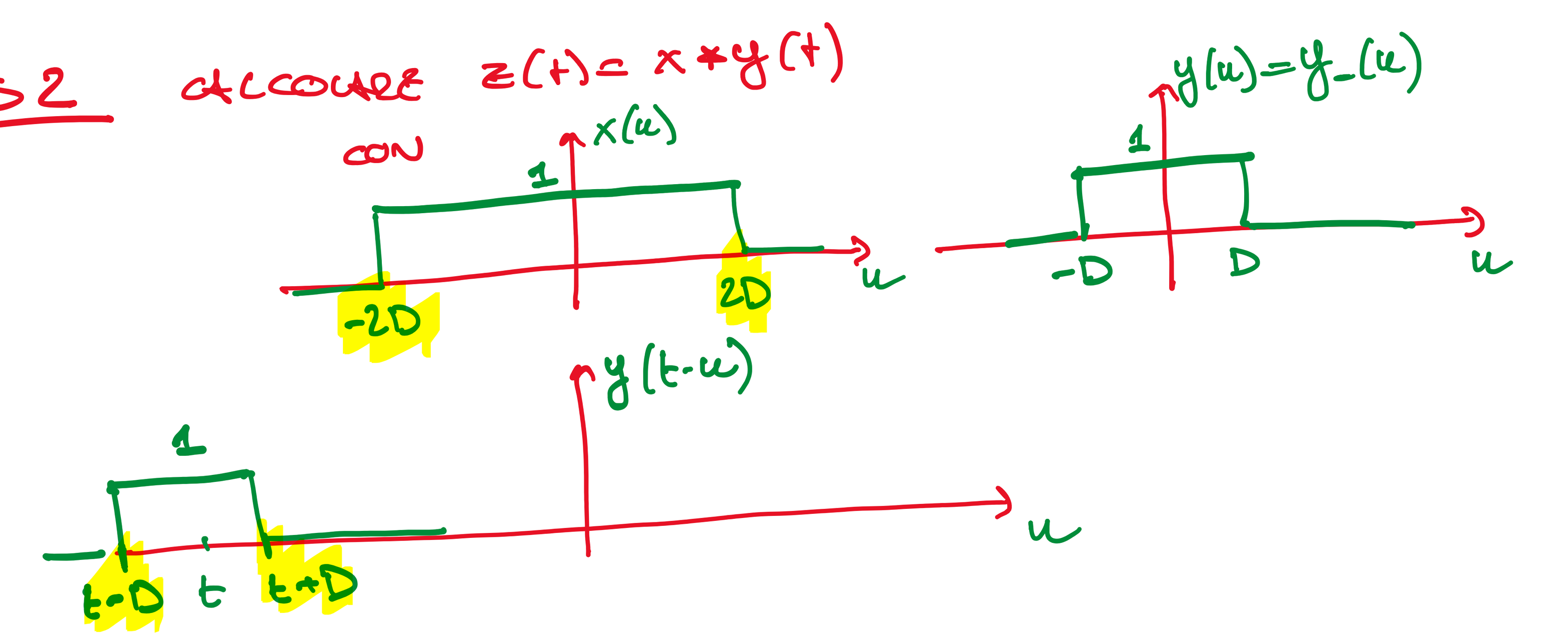



$$z(n) = \frac{A}{1-\alpha} + \mathrm{Re}\left[\frac{e^{j(\theta_o n - \theta_\beta)}}{|\beta|}\right]$$

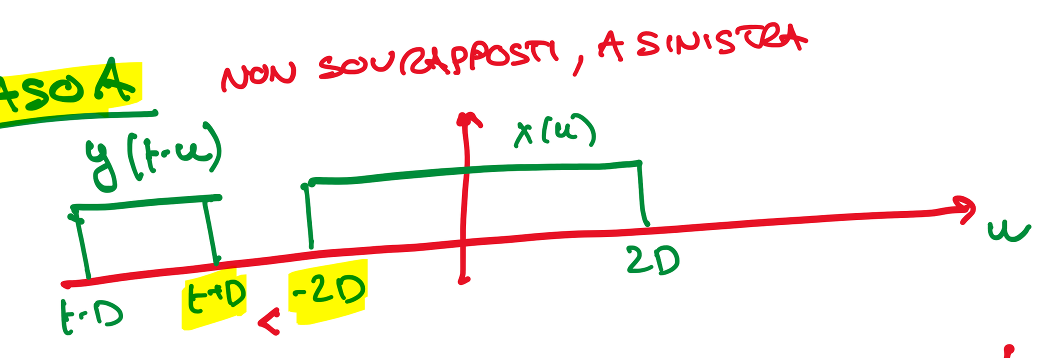
$$= \frac{A}{1-\alpha} + \frac{1}{|\beta|} \cos(\theta_o n - \theta_\beta)$$

**ES2** CALCOLARE $z(t) = x * y(t)$

CON



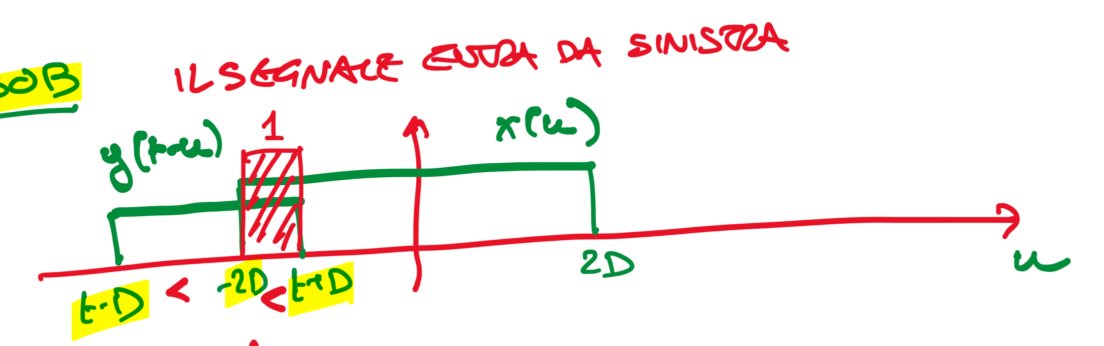

**CASO A** NON SOVRAPPOSTI, A SINISTRA




$z(t) = 0$ $t+D < -2D$

$t < -3D$

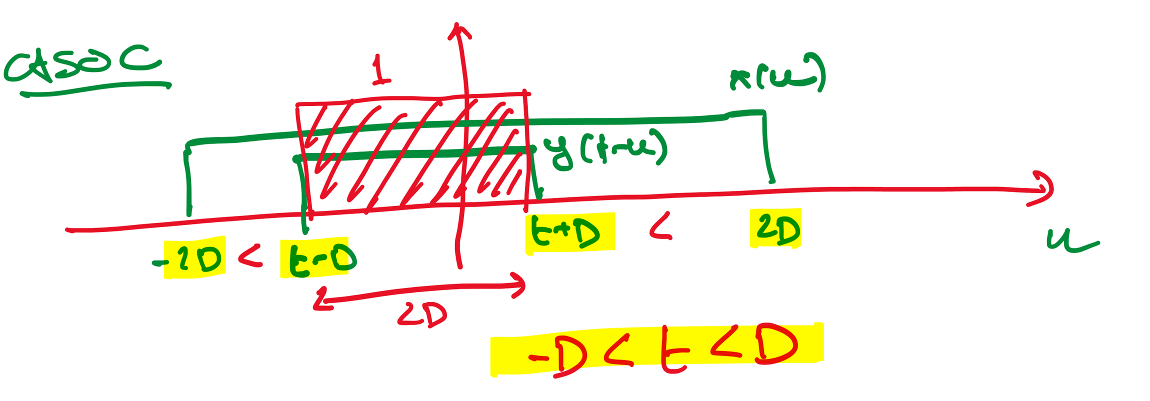
**CASO B** IL SEGNALE ENTRA DA SINISTRA




$t-D < -2D$
$-2D < t+D$ $\rightarrow$ $-3D < t < -D$

$$z(t) = \int_{-2D}^{t+D} 1\, du = t + 3D$$

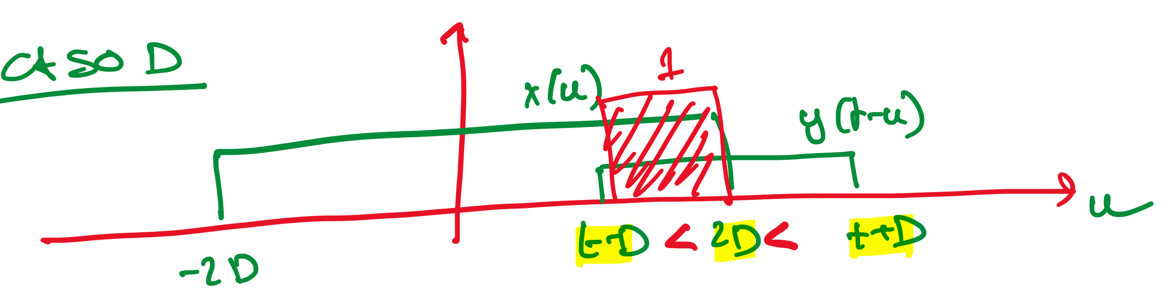
**CASO C**




$-D < t < D$

$$z(t) = \int_{t-D}^{t+D} 1\, du = 2D$$

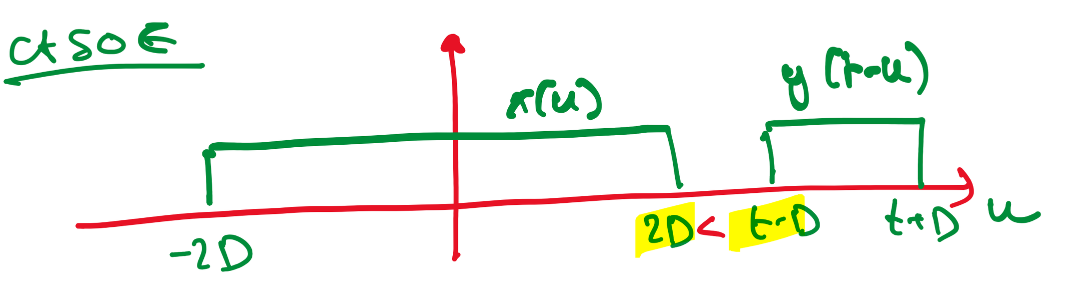
**CASO D**




$D < t < 3D$

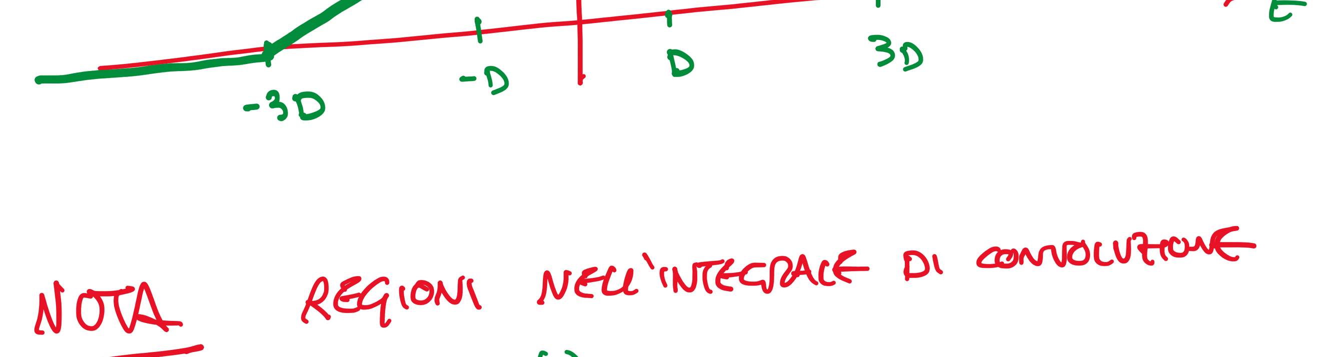
$$z(t) = \int_{t-D}^{2D} 1 \cdot du = 3D - t$$

**CASO E**

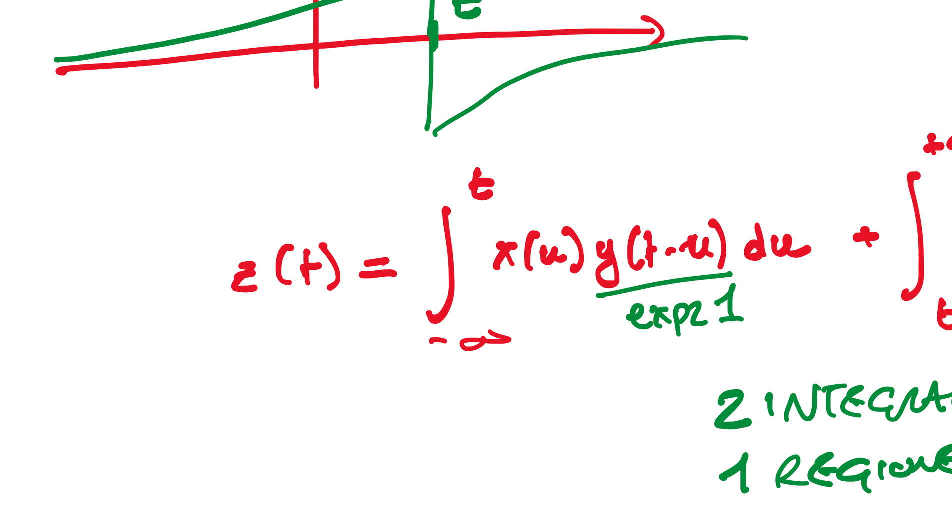



$z(t) = 0$ $t > 3D$

$$z(t) = \begin{cases} 0 & t < -3D \\ 3D + t & -3D < t < -D \\ 2D & -D < t < D \\ 3D - t & D < t < 3D \\ 0 & t > 3D \end{cases}$$

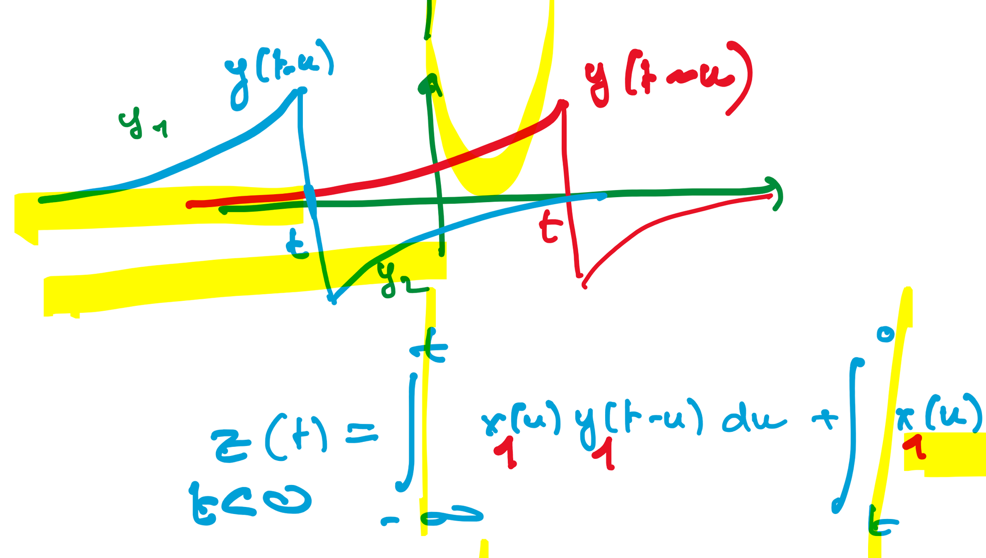



**NOTA** REGIONI NELL'INTEGRALE DI CONVOLUZIONE




$$z(t) = \int_{-\infty}^{t} x(u) \underbrace{y(t-u)}_{\text{expr } 1} du + \int_{t}^{+\infty} x(u) \underbrace{y(t-u)}_{\text{expr } 2} du$$

2 INTEGRALI
1 REGIONE x $z(t)$




$$\underset{t<0}{z(t)} = \int_{-\infty}^{t} x_1(u)\, y_1(t-u)\, du + \int_{t}^{0} x_1(u)\, y_2(t-u)\, du + \int_{0}^{+\infty} x_2(u)\, y_2(t-u)\, du$$

$$\underset{t>0}{z(t)} = \int_{-\infty}^{0} x_1(u)\, y_1(t-u)\, du + \int_{0}^{t} x_2(u)\, y_1(t-u)\, du + \int_{t}^{+\infty} x_2(u)\, y_2(t-u)\, du$$

3 INTEGRALI
2 REGIONI PER $z(t)$