Le30

Thursday, 1 June 2023 08:35

$-p_0^{n+1} 1_0(n) \xrightarrow{Z} \dfrac{1}{z^{-1} - 1/p_0}$ $\qquad p_0$ POLO

$(n+1) p_0^{n+2} 1_0(n) \xrightarrow{Z} \dfrac{1}{(z^{-1} - 1/p_0)^2}$

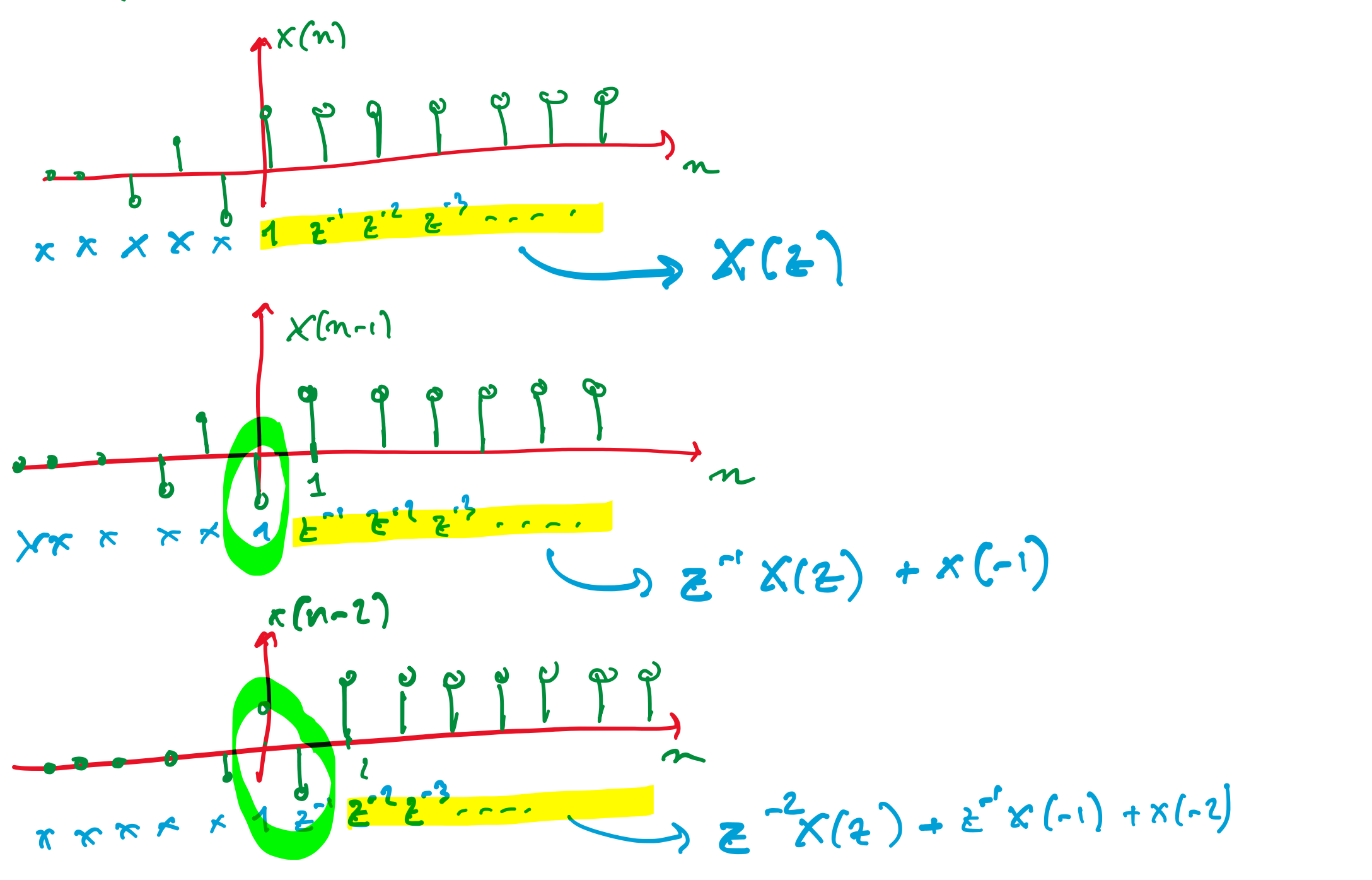
$-\dfrac{(n+1)(n+2)}{2} p_0^{n+3} 1_0(n) \longrightarrow \dfrac{1}{(z^{-1} - 1/p_0)^3}$

$\vdots$

Es $\quad s(n) = \delta(n - n_0)$

$$S(z) = \sum_{n=-\infty} \delta(n-n_0) z^{-n} = z^{-n_0}$$

REGOLA DI TRASLAZIONE



$$x(n-k) \xrightarrow{Z} z^{-k} X(z) + z^{-(k-1)} x(-1) + z^{-(k-2)} x(-2) + \dots x(-k)$$

Es 1 $\quad x(n) = y(n-2) + y(n-1) - 6y(n)$

$x(n) = A$

$y(-1) = K_1$

$y(-2) = K_2$

1) $h(n) = ?$

2) $y(n) = ?$

1) $H(z) = \dfrac{1}{z^{-2} + z^{-1} - 6}$

$$z^{-1}_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2} = \begin{cases} 2 \\ -3 \end{cases}$$

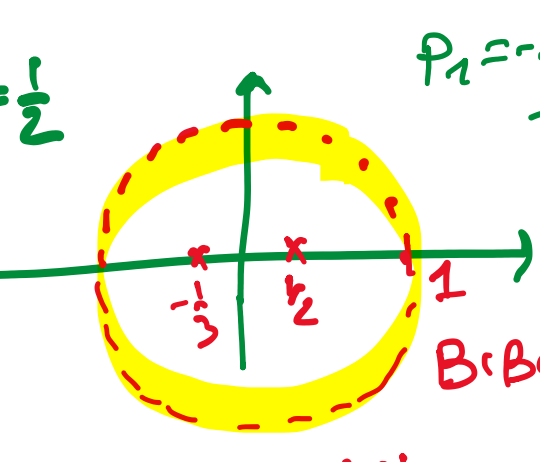
$$H(z) = \frac{1}{(z^{-1}-2)(z^{-1}+3)} = \frac{R_0}{z^{-1}-2} + \frac{R_1}{z^{-1}+3}$$

$$R_0 = H(z)(z^{-1}-2)\Big|_{z^{-1}=2} = \frac{1}{z^{-1}+3}\Big|_{z^{-1}=2} = \frac{1}{5}$$

$$R_1 = H(z)(z^{-1}+3)\Big|_{z^{-1}=-3} = \frac{1}{z^{-1}-2}\Big|_{z^{-1}=-3} = -\frac{1}{5}$$

$$H(z) = \frac{1}{5}\frac{1}{z^{-1}-2} - \frac{1}{5}\frac{1}{z^{-1}+3}$$

$p_0 = \frac{1}{2}$ $\qquad p_1 = -\frac{1}{3}$



BIBO STABILE!

$$h(n) = \frac{1}{5} \cdot -\left(\frac{1}{2}\right)^{n+1} 1_0(n) + \frac{1}{5}\left(-\frac{1}{3}\right)^{n+1} 1_0(n)$$

$$= -\frac{1}{5}\left(\frac{1}{2}\right)^{n+1} 1_0(n) + \frac{1}{5}\left(-\frac{1}{3}\right)^{n+1} 1_0(n)$$

2) $x(n) = y(n-2) + y(n-1) - 6y(n)$

$Z \downarrow$

$X(z) = z^{-2} Y(z) + z^{-1} y(-1) + y(-2)$ $\quad (y(-1) = K_1,\ y(-2) = K_2)$

$+ z^{-1} Y(z) + y(-1) - 6Y(z)$ $\quad (y(-1) = K_1)$

$$Y(z) = \frac{X(z)}{z^{-2}+z^{-1}-6} - \frac{K_1 + K_2 + z^{-1}K_1}{z^{-2}+z^{-1}-6}$$

$$Y(z) = \frac{X(z)}{(z^{-1}-2)(z^{-1}+3)} - \frac{K_1 + K_2 + z^{-1}K_1}{(z^{-1}-2)(z^{-1}+3)}$$

$x(n) = A$

$\hookrightarrow x'(n) = A\, 1_0(n) \xrightarrow{Z} X(z) = \dfrac{-A}{z^{-1}-1}$

$-p_0^{n+1} 1_0(n) \xrightarrow{Z} \dfrac{1}{z^{-1} - 1/p_0}$

$p_0 = 1$

$\hookrightarrow -1_0(n) \xrightarrow{Z} \dfrac{1}{z^{-1}-1}$

$$Y_f(z) = \frac{-A}{(z^{-1}-2)(z^{-1}+3)(z^{-1}-1)} = \frac{R_0}{z^{-1}-2} + \frac{R_1}{z^{-1}+3} + \frac{R_2}{z^{-1}-1}$$

$$R_0 = \frac{-A}{(z^{-1}+3)(z^{-1}-1)}\Big|_{z^{-1}=2} = -\frac{A}{5}$$

$$R_1 = \frac{-A}{(z^{-1}-2)(z^{-1}-1)}\Big|_{z^{-1}=-3} = -\frac{A}{20}$$

$$R_2 = \frac{-A}{(z^{-1}-2)(z^{-1}+3)}\Big|_{z^{-1}=1} = \frac{A}{4}$$

$$y_f(n) = +\frac{A}{5} \cdot +\left(\frac{1}{2}\right)^{n+1} 1_0(n) + \frac{A}{20} \cdot +\left(-\frac{1}{3}\right)^{n+1} 1_0(n)$$

$$-\frac{A}{4} \cdot -1_0(n)$$

$$= \left(\frac{A}{5}\left(\frac{1}{2}\right)^{n+1} + \frac{A}{20}\left(-\frac{1}{3}\right)^{n+1} - \frac{A}{4}\right) 1_0(n)$$

$y_e(n) = ?$ x CASA

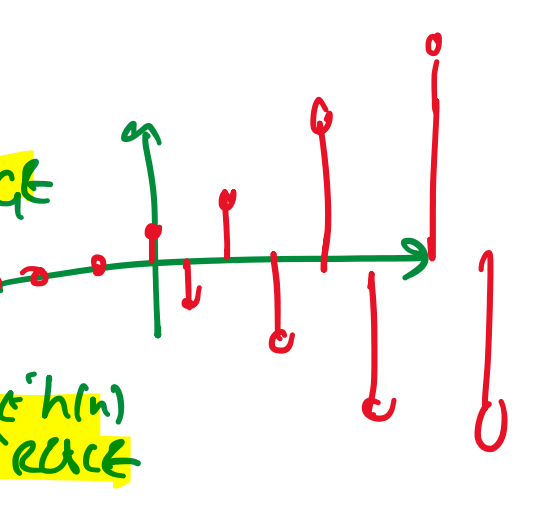
Es 2 $\quad h(n) = (1+2n)(-1)^n 1_0(n) + \frac{1}{2}\left(-\frac{1}{2}\right)^n 1_0(n)$ $\quad$ (DIVERGE)

$x(n) = \frac{1}{3}\left(-\frac{1}{3}\right)^n 1_0(n)$

COND. INIZIALI NULLE!

1) BIBO STABILE? NO XCHÉ h(n) DIVERGE ✓

REALE? SÌ XCHÉ h(n) È REALE



2) EQ. ALLE DIFFERENZE

3) EV. LIBERA $y_e(n) = 0$ + RISPOSTA FORZATA

$x(n)\ 1_0(n) \rightarrow \boxed{h(\cdot)} \rightarrow y_f(n) = y(n)$

2) $H(z) = ?$

$h(n) = (1+2n)(-1)^n 1_0(n) + \frac{1}{2}\left(-\frac{1}{2}\right)^n 1_0(n)$

$-p_0^{n+1} 1_0(n) \xrightarrow{Z} \dfrac{1}{z^{-1} - 1/p_0}$

$(n+1) p_0^{n+2} 1_0(n) \xrightarrow{Z} \dfrac{1}{(z^{-1} - 1/p_0)^2}$

$2n+1 : n+1 = 2$

$\underline{-2n-2}$

-1

$2n+1 = 2(n+1) - 1$

$$h(n) = 2(n+1)(-1)^{n+2} 1_0(n) + (-1)^{n+1} 1_0(n) + \left(-\frac{1}{2}\right)^{n+1} 1_0(n)$$

$$H(z) = 2 \cdot \frac{1}{(z^{-1}+1)^2} - 1 \cdot \frac{1}{z^{-1}+1} + 1 \cdot \frac{1}{z^{-1}+2}$$

$$= \frac{2(z^{-1}+2) - (z^{-1}+1)(z^{-1}+2) + (z^{-1}+1)^2}{(z^{-1}+1)^2 (z^{-1}+2)}$$

$-(z^{-1}+1)[z^{-1}+1 - z^{-1} - 2]$

$$= \frac{2z^{-1} + 4 - z^{-1} - 1}{(z^{-2}+2z^{-1}+1)(z^{-1}+2)}$$

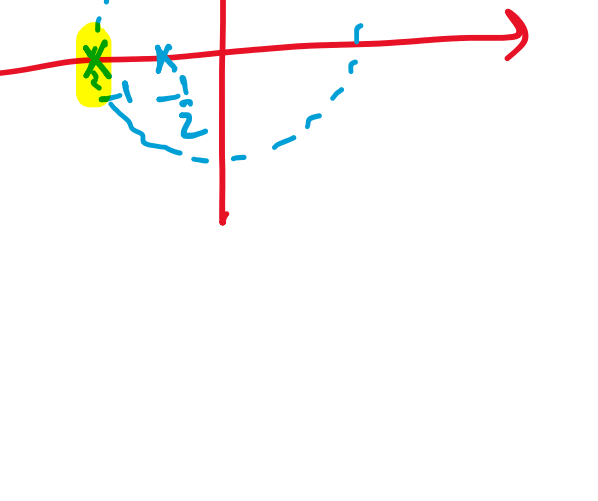
$$= \frac{z^{-1}+3}{z^{-3} + 2z^{-2} + 2z^{-2} + 4z^{-1} + z^{-1} + 2}$$

$$= \frac{z^{-1}+3}{z^{-3}+4z^{-2}+5z^{-1}+2}$$

$$x(k-1) + 3x(k) = y(k-3) + 4y(k-2) + 5y(k-1) + 2y(k)$$

3) RIS. FORZATA $\quad Y_f(z) = X(z) H(z)$

$H(z) = \dfrac{z^{-1}+3}{(z^{-1}+1)^2 (z^{-1}+2)}$



$X(z) = ? = \dfrac{1}{z^{-1}+3}$

$x(n) = \frac{1}{3}\left(-\frac{1}{3}\right)^{n+1} 1_0(n)$

$$Y_f(z) = \frac{1}{(z^{-1}+1)^2 (z^{-1}+2)} = \frac{R_0}{(z^{-1}+1)^2} + \frac{R_1}{(z^{-1}+1)} + \frac{R_2}{z^{-1}+2}$$