Le29

Wednesday, 31 May 2023 10:34

Es2

$x(t) = K y(t) + m y''(t)$

$x(t) = F_0 \cos(\omega_0 t)$

$y(0^-) = y_0$

$y'(0^-) = v_0$

$y(t) = ?$

$\mathcal{L}$

$\frac{1}{m} X(s) = \frac{K}{m} Y(s) + \cancel{m}\left(s^2 Y(s) - s\cancel{y(0^-)} - \cancel{y'(0^-)}\right)$ (with y_0, v_0)

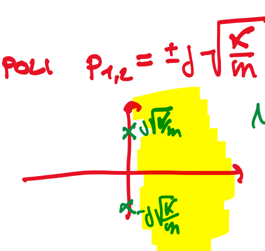
$Y(s)\left(s^2 + \frac{K}{m}\right) = \frac{1}{m} X(s) + s y_0 + v_0$

$$Y(s) = \underbrace{\frac{1/m}{s^2 + K/m}}_{H(s)} X(s) + \underbrace{\frac{s y_0 + v_0}{s^2 + \frac{K}{m}}}_{Y_e(s)}$$

$x(t) = F_0 \cos(\omega_0 t) \xrightarrow{\mathcal{L}} X(s) = \frac{F_0 s}{s^2 + \omega_0^2}$

NOTA
$\cos(\omega_0 t) 1(t) \xrightarrow{\mathcal{L}} \frac{s}{s^2 + \omega_0^2}$
$\sin(\omega_0 t) 1(t) \xrightarrow{\mathcal{L}} \frac{\omega_0}{s^2 + \omega_0^2}$

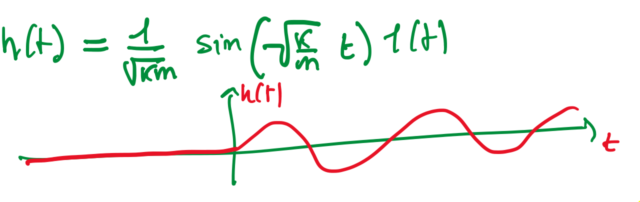
$H(s) = \frac{1/m}{s^2 + K/m} \longrightarrow$ POLI $p_{1,2} = \pm j\sqrt{\frac{K}{m}}$



NON BIBO STABILE
$Re[p_i] = 0$

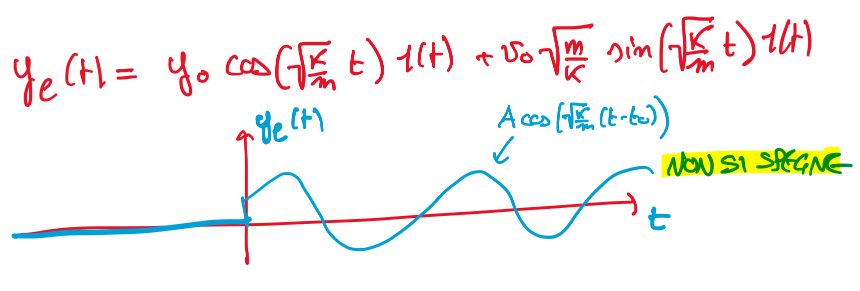
$H(s) = \frac{\sqrt{K/m}}{s^2 + K/m} \cdot \frac{1}{\sqrt{Km}}$

$h(t) = \frac{1}{\sqrt{Km}} \sin\left(\sqrt{\frac{K}{m}}\, t\right) 1(t)$



$Y_e(s) = \frac{s y_0 + v_0}{s^2 + \frac{K}{m}} = y_0 \frac{s}{s^2 + \frac{K}{m}} + v_0 \sqrt{\frac{m}{K}} \cdot \frac{\sqrt{K/m}}{s^2 + \frac{K}{m}}$

$y_e(t) = y_0 \cos\left(\sqrt{\frac{K}{m}}\, t\right) 1(t) + v_0 \sqrt{\frac{m}{K}} \sin\left(\sqrt{\frac{K}{m}}\, t\right) 1(t)$



$Y_f(s) = \frac{1/m}{s^2 + K/m} \cdot \frac{F_0 s}{s^2 + \omega_0^2} = \frac{F_0}{m} \cdot \frac{s}{\left(s^2 + \frac{K}{m}\right)\left(s^2 + \omega_0^2\right)}$

$\frac{1}{\left(s^2 + \frac{K}{m}\right)\left(s^2 + \omega_0^2\right)} = \frac{R_0}{s^2 + \frac{K}{m}} + \frac{R_1}{s^2 + \omega_0^2}$

$R_0 = \frac{1}{s^2 + \omega_0^2}\Big|_{s^2 = -\frac{K}{m}} = \frac{1}{\omega_0^2 - \frac{K}{m}}$

$R_1 = -R_0 = \frac{1}{s^2 + \frac{K}{m}}\Big|_{s^2 = -\omega_0^2} = -\frac{1}{\omega_0^2 - \frac{K}{m}}$

$Y_f(s) = \frac{F_0/m}{\omega_0^2 - \frac{K}{m}} \left(\frac{s}{s^2 + \frac{K}{m}} - \frac{s}{s^2 + \omega_0^2} \right)$

$y_f(t) = \frac{F_0/m}{\omega_0^2 - K/m} \left(\cos\left(\sqrt{\frac{K}{m}}\, t\right) - \cos(\omega_0 t) \right) 1(t)$

DOMANDA: e se $\omega_0 = \sqrt{\frac{K}{m}}$?

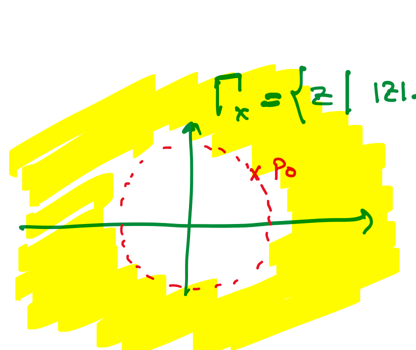
Es 1

$x(n) = p_0^n 1_0(n)$ $\qquad p_0 \in \mathbb{C}$

$X(z) = \sum_{n=\cancel{-\infty}\,0}^{+\infty} p_0^n \cancel{1_0(n)} z^{-n} = \sum_{n=0}^{\infty} (p_0 z^{-1})^n$

$= \frac{1}{1 - p_0 z^{-1}}$ $\qquad |p_0 z^{-1}| < 1 \Rightarrow \left|\frac{p_0}{z}\right| < 1 \rightarrow |p_0| < |z|$

$\Gamma_x = \{z \mid |z| > |p_0|\}$



p_0 è un polo!

$X(z) = \frac{z}{z - p_0}$

Es 1b

$x(n) = p_0^n 1_0(-n)$

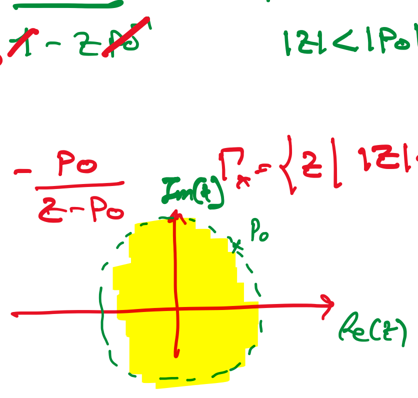
$X(z) = \sum_{n=-\infty}^{\cancel{+\infty}\,0} p_0^n \cancel{1_0(-n)} z^{-n}$

$= \sum_{n=-\infty}^{0} (p_0 z^{-1})^n = \sum_{m=0}^{+\infty} (p_0 z^{-1})^{-m}$ $\quad m = -n$

$= \sum_{m=0}^{\infty} (z p_0^{-1})^m$

$= \frac{1 \cdot p_0}{p_0 \cancel{1} - z \cancel{p_0}}$ $\qquad |z p_0^{-1}| < 1$, $\quad |z| < |p_0|$

$X(z) = -\frac{p_0}{z - p_0}$ $\qquad \Gamma_x = \{z \mid |z| < |p_0|\}$



Es 1c

$x(n) = \cos(\theta_0 n) 1_0(n)$

$= \frac{1}{2} \underbrace{e^{j\theta_0 n}}_{p_0^n} 1_0(n) + \frac{1}{2} \underbrace{e^{-j\theta_0 n}}_{p_1^n} 1_0(n)$

$p_0 = e^{j\theta_0}$ $\qquad p_1 = e^{-j\theta_0}$

$p_0^n 1_0(n) \xrightarrow{\mathcal{Z}} \frac{1}{1 - p_0 z^{-1}}$

$X(z) = \frac{1}{2} \frac{1}{1 - e^{j\theta_0} z^{-1}} + \frac{1}{2} \frac{1}{1 - e^{-j\theta_0} z^{-1}}$

$= \frac{1}{2} \frac{1 - e^{-j\theta_0} z^{-1} + 1 - e^{j\theta_0} z^{-1}}{(1 - e^{j\theta_0} z^{-1})(1 - e^{-j\theta_0} z^{-1})}$

$$X(z) = \frac{1 - z^{-1} \cos\theta_0}{1 - z^{-1} \cdot 2\cos\theta_0 + z^{-2}}$$

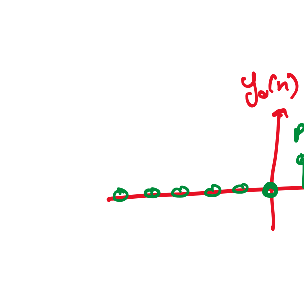
x CASA $\quad x(n) = \sin(\theta_0 n) 1_0(n)$

$X(z) = ?$

Es 2a

$x_0(n) = -p_0^{n+1} 1_0(n) = -p_0 \cdot p_0^n 1_0(n)$

$X_0(z) = ? = -p_0 \cdot \frac{1}{1 - p_0 z^{-1}}$



$X_0(z) = \frac{1}{z^{-1} - p_0^{-1}}$ $\qquad$ polo $z = p_0$

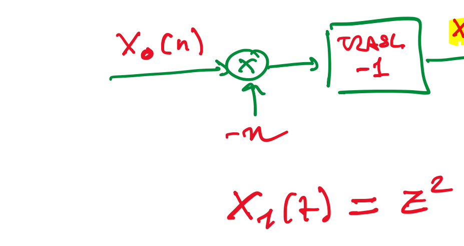
$= \frac{1}{\frac{1}{z} - \frac{1}{p_0}} = \frac{p_0 z}{p_0 - z}$

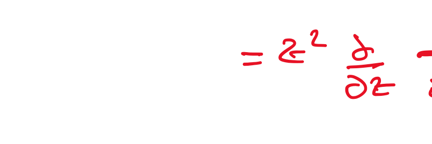
Es 2b

$x_1(n) = (n+1) p_0^{n+2} 1_0(n)$

$y_0(n) = -n\, x_0(n) = n\, p_0^{n+1} 1_0(n\cancel{-1})$

$x_1(n) = y_0(n+1) = (n+1) p_0^{n+2} 1_0(n \cancel{+1})$





$X_1(z) = z^2 X_0'(z)$

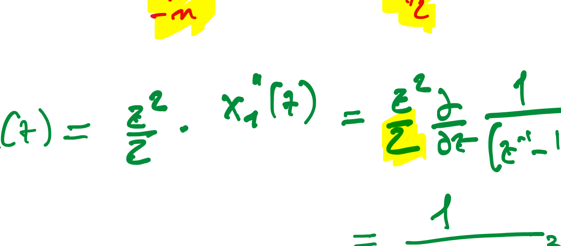
$= z^2 \frac{\partial}{\partial z} \frac{1}{z^{-1} - 1/p_0}$

$= \cancel{z^2} \cdot \frac{\cancel{-1}}{(z^{-1} - 1/p_0)^2} \cdot \cancel{-1 \cdot z^{-2}}$

$$X_1(z) = \frac{1}{(z^{-1} - 1/p_0)^2}$$

Es 2c

$x_2(n) = -\frac{1}{2}(n+1)(n+2)\, p_0^{n+3} 1_0(n)$



$X_2(z) = \frac{z^2}{2} \cdot X_1'(z) = \frac{z^2}{2} \frac{\partial}{\partial z} \frac{1}{(z^{-1} - 1/p_0)^2}$

$= \frac{1}{(z^{-1} - 1/p_0)^3}$