Le28

Monday, 29 May 2023 11:19

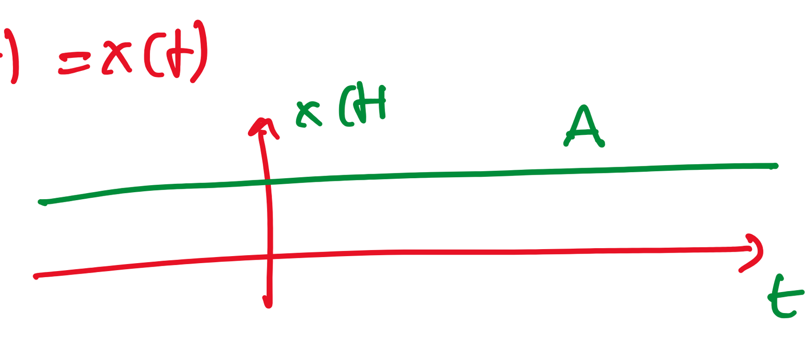
Es1 FILTRO RC

$y(t) + RC\,y'(t) = x(t)$

$x(t) = A$

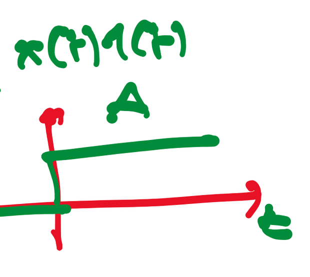
$y(0^-) = V_0$

$y(t) = ?$



UNILATERA

$$\underbrace{Y(s) + RC\left(sY(s) - \cancel{y(0^-)}_{V_0}\right)} = \underbrace{X(s)}_{A/s}$$



$$Y(s)\,\cancel{(1+sRC)} - \cancel{RC V_0} = \frac{\frac{A}{s} + RC V_0}{1+sRC}$$

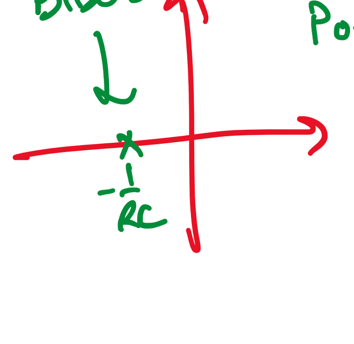
$$Y(s) = \frac{A/RC}{s(1+s\cancel{RC})_{RC}} + \frac{\cancel{RC}V_0}{1+s\cancel{RC}_{RC}}$$

$$Y(s) = \underbrace{\frac{A}{RC}\cdot\frac{1}{s\left(s+\frac{1}{RC}\right)}}_{\text{RISPOSTA FORZATA}} + \underbrace{\frac{V_0}{s+1/RC}}_{\text{EVOLUZIONE LIBERA}}$$

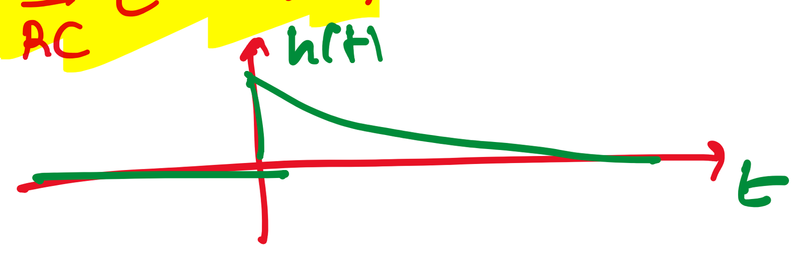
$$Y(s) = \frac{A}{s}\cdot\underbrace{\frac{1/RC}{s+1/RC}}_{H(s)} + \frac{V_0}{s+1/RC}$$

BIBO STABILE!

$P_0 = -\frac{1}{RC}$



$h(t) = \frac{1}{RC}\, e^{-t/RC}\, 1(t)$



$y_e(t) = V_0\, e^{-t/RC}\, 1(t)$

$$Y_f(s) = \frac{A}{RC}\cdot\frac{1}{s\left(s+\frac{1}{RC}\right)} = \frac{K_0}{s} + \frac{K_1}{s+\frac{1}{RC}}$$

$$K_0 = Y_f(s)\,s\big|_{s=0} = \frac{A}{RC}\cdot\frac{1}{s+\frac{1}{RC}}\bigg|_{s=0} = A$$

$$K_1 = Y_f(s)\left(s+\frac{1}{RC}\right)\bigg|_{s=-\frac{1}{RC}} = \frac{A}{RC}\cdot\frac{1}{s}\bigg|_{s=-\frac{1}{RC}} = -A$$

$$Y_f(s) = \frac{A}{s} - \frac{A}{s+1/RC}$$

$y_f(t) = A\,1(t) - A\,e^{-t/RC}\,1(t)$

$y(t) = A\,1(t) + (V_0 - A)\,e^{-t/RC}\,1(t)$

per $t \gg 0$

$y(t) = x * h(t)$

$x(t) = A$

$h(t) = \frac{1}{RC} e^{-t/RC} 1(t)$

$x * h(t) = A \cdot \text{area}_h = A$

Es3

$y''(t) - y'(t) - 6\,y(t) = x'(t) - 3x(t)$

1. $H(s) = ?$
2. BIBO STABILE ?
3. RISPOSTA FORZATA CON $x(t) = 1(t)$
4. $x(t) = A\cos(\omega_0 t + \varphi_0)$
   E LE CONDIZIONI INIZIALI SU $y(t)$ SIANO NULLE
   TROVARE ω_0 CHE GARANTISCA UN COMPORTAMENTO A REGIME DEL TIPO
   $y(t) = \frac{1}{5} x(t - t_0)$

$$H(s) = \frac{s-3}{s^2 - s - 6}$$

$$p_{1,2} = \frac{1 \pm \sqrt{\cancel{1+24}}^{\,5}}{2} = \begin{cases} 3 \\ -2 \end{cases}$$

$$H(s) = \frac{\cancel{s-3}}{\cancel{(s-3)}(s+2)} = \frac{1}{s+2}$$

BIBO STABILE

eq. differenziale semplificata

$\boxed{x(t) = y'(t) + 2y(t)}$

(3) $Y_f(s) = H(s)\underset{\text{UNIL.}}{X(s)} = \frac{1}{s+2}\,\frac{1}{s} = \frac{R_0}{s} + \frac{R_1}{s+2}$

$$R_0 = Y_f(s)\,s\big|_{s=0} = \frac{1}{s+2}\bigg|_{s=0} = \frac{1}{2}$$

$$R_1 = Y_f(s)(s+2)\big|_{s=-2} = \frac{1}{s}\bigg|_{s=-2} = -\frac{1}{2}$$

$$Y_f(s) = \frac{1}{2}\,\frac{1}{s} - \frac{1}{2}\,\frac{1}{s+2}$$

$y_f(t) = \frac{1}{2} 1(t) - \frac{1}{2} e^{-2t}\, 1(t)$

(4) $x(t) = A\cos(\omega_0 t + \varphi_0)$ COMPORTAMENTO USCITA A REGIME ($t \gg 0$)

$x(t) \to \boxed{h(\cdot)} \to y(t)$

$H(s) = \frac{1}{s+2}$

$h(t) = e^{-2t}\, 1(t)$

$A\cos(\omega_0 t + \varphi_0) \to \boxed{h(\cdot)} \to A\,|H(j\omega_0)|\cos(\omega_0 t + \varphi_0 + \angle H(j\omega_0))$

AMPLIFICAZIONE

TRASLAZIONE

$$|H(j\omega_0)|^2 = \left|\frac{1}{j\omega_0 + 2}\right|^2 = \frac{1}{4+\omega_0^2} = \frac{1}{25}$$

$\omega_0^2 = 21$

$\omega_0 = \sqrt{21}$