Le27

Thursday, 25 May 2023 08:35

Es1c

$x(t) = A\,\delta(t)$

$\delta(t) = \frac{\partial 1(t)}{\partial t} = y'(t)$

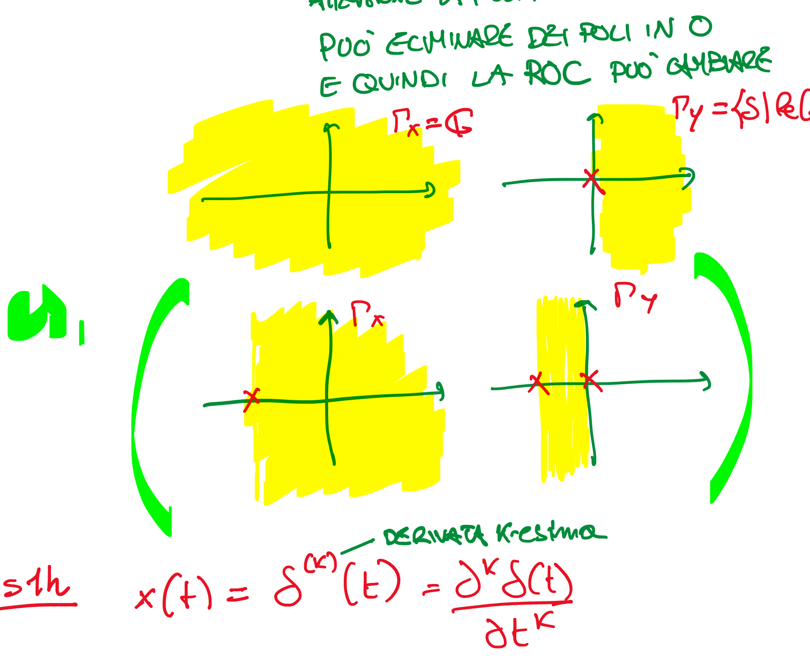
$y(t) = 1(t)$

$Y(s) = \frac{1}{s}$

$x(t) = A\,y'(t)$

$X(s) = A \cdot s \cdot Y(s) = A \cdot s \cdot \frac{1}{s} = A$

ATTENZIONE LA MOLTIPLICAZIONE PER S PUÒ ELIMINARE DEI POLI IN 0 E QUINDI LA ROC PUÒ CAMBIARE



Es1h

$x(t) = \delta^{(k)}(t) = \frac{\partial^k \delta(t)}{\partial t^k}$ — DERIVATA K-ESIMA

$\delta(t) \xrightarrow{\mathcal{L}} 1 \qquad \Gamma = \mathbb{C}$

$\delta'(t) \xrightarrow{\mathcal{L}} s$

$\delta''(t) \xrightarrow{\mathcal{L}} s^2$

$\vdots$

$\delta^{(k)}(t) \xrightarrow{\mathcal{L}} s^k \qquad \Gamma = \mathbb{C}$

X CASA TRAMITE REGOLA DI INTEGRAZIONE $1(t) \xrightarrow{\mathcal{L}} \frac{1}{s}$

$t\,1(t) = 1 * 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s^2}$

$\frac{t^2}{2} 1(t) = 1 * 1 * 1(t) \xrightarrow{\mathcal{L}} \frac{1}{s^3}$

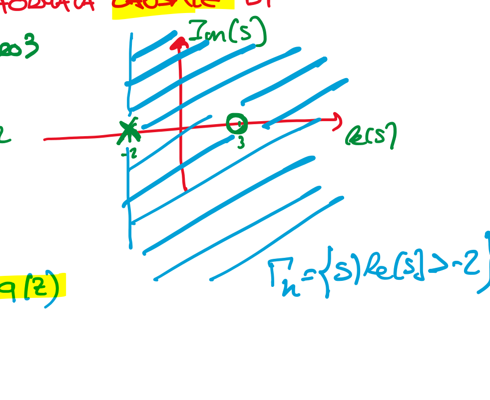
$\vdots$

$\frac{t^k}{k!} 1(t) = \underbrace{1 * 1 \cdots 1}_{k+1}(t) \xrightarrow{\mathcal{L}} \frac{1}{s^{k+1}}$

$\frac{t^k e^{s_0 t}}{k!} 1(t) \xrightarrow{\mathcal{L}} \frac{1}{(s-s_0)^{k+1}}$

Es1a TROVARE L'ANTITRASFORMATA CAUSALE DI

$H(s) = \frac{s-3}{s+2}$ — ZERO 3; POLO -2; IMPROPRIA



$s-3 : s+2 = 1 = q(s)$

$-s-2$

$/ \quad -5 = r(s)$

$H(s) = 1 - \frac{5}{s+2} = \frac{s+2-5}{s+2}$ (−3)

$h(t) = \delta(t) - 5\,e^{-2 \cdot t}\,1(t)$ — POLO $p_0 = -2$

Es1b TROVARE L'ANTITRASFORMATA CAUSALE DI

$H(s) = \frac{1}{s^3 + s^2 - 6s} = \frac{1}{s(s^2+s-6)}$

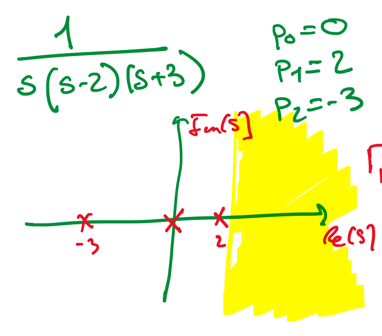
$p_0 = 0 \qquad p_{1,2} = \frac{-1 \pm \sqrt{1+6\cdot 4}}{2} = \frac{-1 \pm 5}{2} \to 2, \; -3$

$H(s) = \frac{1}{s(s-2)(s+3)}$

$p_0 = 0$

$p_1 = 2$

$p_2 = -3$



CERCHIAMO DI ESPRIMERE $H(s) = \frac{R_0}{s} + \frac{R_1}{s-2} + \frac{R_2}{s+3}$

NOTA $X(s) = \frac{X_1}{s-p_1} + \frac{X_2}{s-p_2} + \frac{X_3}{s-p_3} + \cdots$

IPOTIZZIAMO I p_i, TUTTI DIVERSI.

$X(s)(s-p_1)\Big|_{s=p_1} = \frac{X_1}{\cancel{s-p_1}}\cancel{(s-p_1)} + \frac{X_2}{p_1-p_2}\underbrace{(p_1-p_1)}_{0} + \frac{X_3}{p_1-p_3}\underbrace{(p_1-p_1)}_{0} + \cdots$ (with $p_1 - p_2 \neq 0$, $p_1 - p_3 \neq 0$)

$= X_1$

$H(s) = \frac{1}{s(s-2)(s+3)} = \frac{R_0}{s} + \frac{R_1}{s-2} + \frac{R_2}{s+3}$

$R_0 = H(s)\,s\Big|_{s=0} = \frac{1}{(s-2)(s+3)}\Big|_{s=0} = -\frac{1}{6}$

$R_1 = H(s)(s-2)\Big|_{s=2} = \frac{1}{s(s+3)}\Big|_{s=2} = \frac{1}{10}$

$R_2 = H(s)(s+3)\Big|_{s=-3} = \frac{1}{s(s-2)}\Big|_{s=-3} = \frac{1}{15}$

$H(s) = -\frac{1}{6}\frac{1}{s} + \frac{1}{10}\frac{1}{s-2} + \frac{1}{15}\frac{1}{s+3}$

$h(t) = -\frac{1}{6}1(t) + \frac{1}{10}e^{2t}1(t) + \frac{1}{15}e^{-3t}1(t)$ — (p_1, p_2)

NOTA RESIDUI NEL CASO DI MOLTEPLICITÀ DI UN POLO

$H(s) = \frac{R_0}{s-p_0} + \frac{R_1}{(s-p_0)^2} + \frac{R_2}{(s-p_0)^3} + \frac{K_0}{(s-p_i)^n} + \cdots$ — IPOTESI CHE $\mu_0 = 3$

$H(s)(s-p_0)^3\Big|_{s=p_0} = R_0\cancel{(s-p_0)^2} + R_1\cancel{(s-p_0)} + R_2 + K_0\cancel{\frac{(s-p_0)^3}{(s-p_i)^n}} + \cdots$ (molteplicità 3)

$= R_2$

$\frac{\partial H(s)(s-p_0)^3}{\partial s}\Big|_{s=p_0} = 2R_0\cancel{(s-p_0)} + R_1 + \cancel{0} + \cancel{3K_0\frac{(s-p_0)^2}{(s-p_i)^n}} \cancel{- nK_0\frac{(s-p_0)^3}{(s-p_i)^{n+1}}} + \cdots$

$= R_1$

$\frac{\partial^2 H(s)(s-p_0)^3}{\partial s}\Big|_{s=p_0} = 2R_0 + \cancel{0} + \cancel{0} + \cancel{\frac{(s-p_0)^*}{(s-p_i)^*}}$

$= 2R_0$

Es1c TROVARE L'ANTITRASFORMATA CAUSALE DI

$H(s) = \frac{4s-1}{2s^2(s-1)} = \frac{H_0}{s} + \frac{H_1}{s^2} + \frac{H_2}{s-1}$

$H_2 = H(s)(s-1)\Big|_{s=1} = \frac{4s-1}{2s^2}\Big|_{s=1} = \frac{3}{2}$

$H_1 = H(s)\,s^2\Big|_{s=0} = \frac{4s-1}{2(s-1)} = \frac{1}{2}$

$H_0 = \frac{\partial}{\partial s}\left[H(s)\,s^2\right]\Big|_{s=0} = \frac{4}{2(s-1)} - \frac{(4s-1)}{2(s-1)^2}\Big|_{s=0}$

$= -2 + \frac{1}{2} = -\frac{3}{2}$

$H(s) = -\frac{3}{2}\frac{1}{s} + \frac{1}{2}\frac{1}{s^2} + \frac{3}{2}\frac{1}{s-1}$

$h(t) = -\frac{3}{2}1(t) + \frac{1}{2}t\,1(t) + \frac{3}{2}e^{t}1(t)$

NOTA SE CONOSCETE H_1

$H(s) = \frac{H_0}{s} + \frac{H_1}{s^2} + \frac{H_2}{(s-1)}$

POTETE TOGLIERLO

$H_1(s) = H(s) - \frac{H_1}{s^2} = \frac{H_0}{s} + \frac{H_2}{s-1}$

IN CUI H_0 È FACILE DA ~~DERIVARE~~

$H_0 = H_1(s)\,s\Big|_{s=0}$

$H_1(s) = \frac{4s-1}{2s^2(s-1)} - \frac{1}{2s^2} = \frac{4s\cancel{-1} - (s\cancel{-1})}{2s^2(s-1)} = \frac{3\cancel{s}}{2s^{\cancel{2}}(s-1)}$

$= \frac{3}{2}\frac{1}{s(s-1)}$

$H_0 = H_1(s)\,s\Big|_{s=0} = \frac{3}{2}\frac{1}{(s-1)}\Big|_{s=0} = -\frac{3}{2}$