Le26

Wednesday, 24 May 2023 10:34

Es 1a $x(t) = e^{s_0 t} 1(t)$ $s_0 \in \mathbb{C}$

$X(s) = ?$ $s_0 = \sigma_0 + j\omega_0$

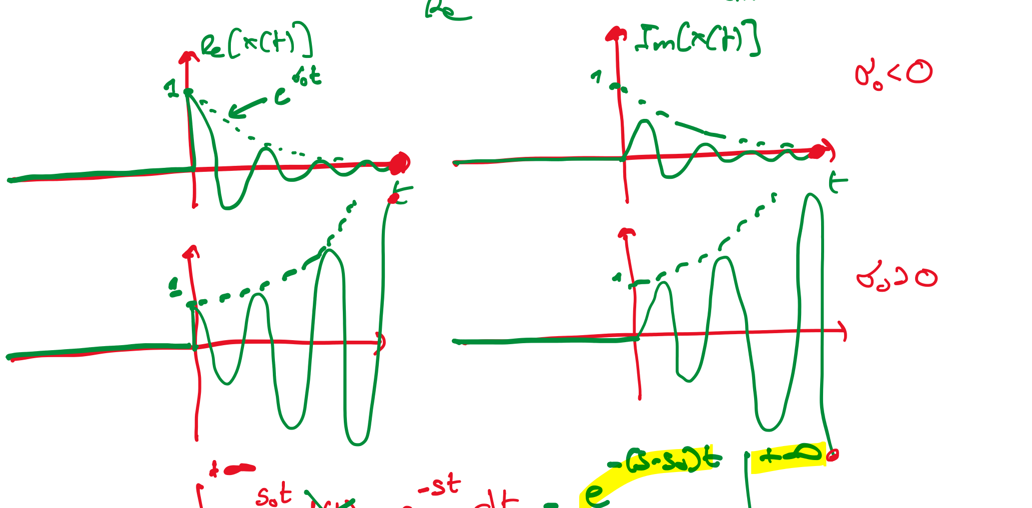
$x(t) = e^{\sigma_0 t + j\omega_0 t} 1(t)$

$= e^{\sigma_0 t} \cdot e^{j\omega_0 t} 1(t)$

$= e^{\sigma_0 t} (\cos(\omega_0 t) + j \sin(\omega_0 t)) 1(t)$

$= e^{\sigma_0 t} \cos(\omega_0 t) 1(t) + j e^{\sigma_0 t} \sin(\omega_0 t) 1(t)$

Re Im



$X(s) = \int_0^{+\infty} e^{s_0 t} 1(t) \cdot e^{-st} dt = \left. \frac{e^{-(s-s_0)t}}{-(s-s_0)} \right|_0^{+\infty}$

$e^{-(s-s_0)t}$

$\lim_{t \to \infty} e^{-(s-s_0)t} = \lim_{t \to \infty} e^{-Re[s-s_0]t} e^{-j Im[s-s_0]t}$

CONDIZIONE DI CONVERGENZA DELL'INTEGRALE

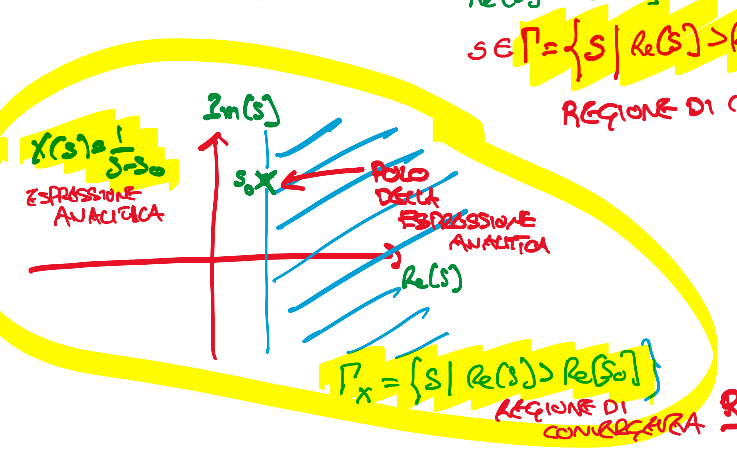
$= \begin{cases} 0 & \text{per } Re[s-s_0] > 0 \\ \infty/\text{indeterminato} & \text{per } Re[s-s_0] < 0 \end{cases}$

$X(s) = \frac{1}{s-s_0}$ per $Re[s-s_0] > 0$

$Re(s) > Re(s_0)$

$s \in \Gamma = \{s \mid Re(s) > Re(s_0)\}$

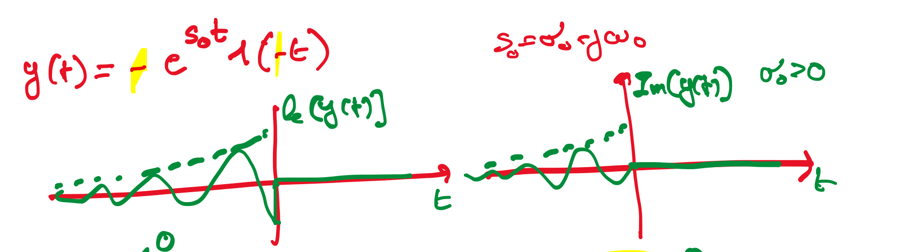
REGIONE DI CONVERGENZA



$\Gamma_x = \{s \mid Re(s) > Re(s_0)\}$ REGIONE DI CONVERGENZA **ROC**

PROPRIETÀ: ALL'INTERNO DELLA REGIONE DI CONVERGENZA $X(s)$ È UNA FUNZIONE ANALITICA

Es 1b $y(t) = -e^{s_0 t} 1(-t)$ $s_0 = \sigma_0 + j\omega_0$



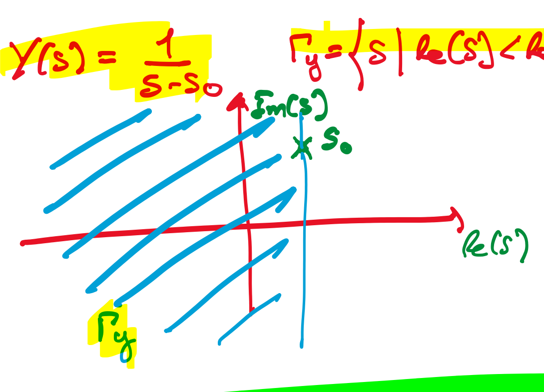
$Y(s) = \int_{-\infty}^{+\infty} -e^{s_0 t} 1(-t) e^{-st} dt = \left. \frac{e^{-(s-s_0)t}}{s-s_0} \right|_{-\infty}^{0}$

$-e^{-(s-s_0)t}$

converge per $Re[s-s_0] < 0$

$Re[s] < Re[s_0]$

$Y(s) = \frac{1}{s-s_0}$ $\Gamma_y = \{s \mid Re(s) < Re(s_0)\}$



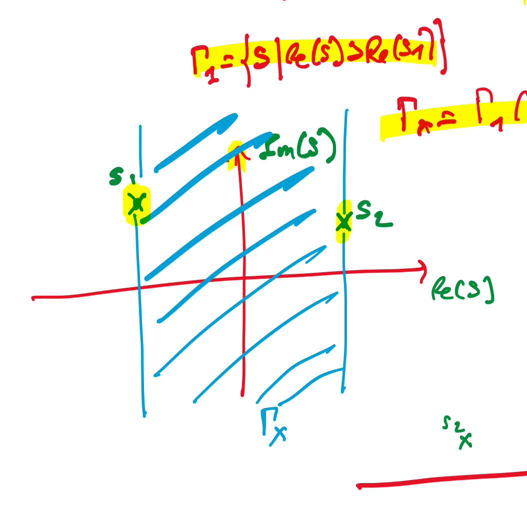
TRASF. DI LAPLACE DI $x(t)$ = ESPRESSIONE ANALITICA $X(s)$ + ROC Γ_x

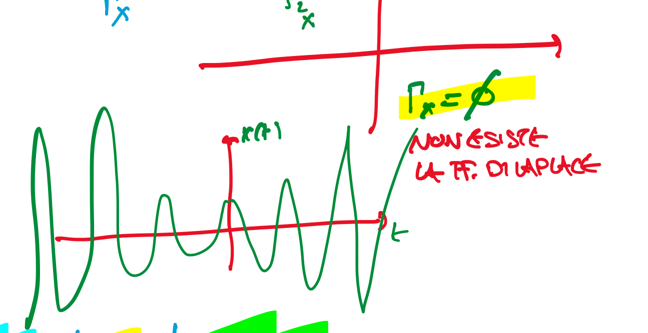
Es 1c $x(t) = e^{s_1 t} 1(t) + e^{s_2 (t)} 1(-t)$

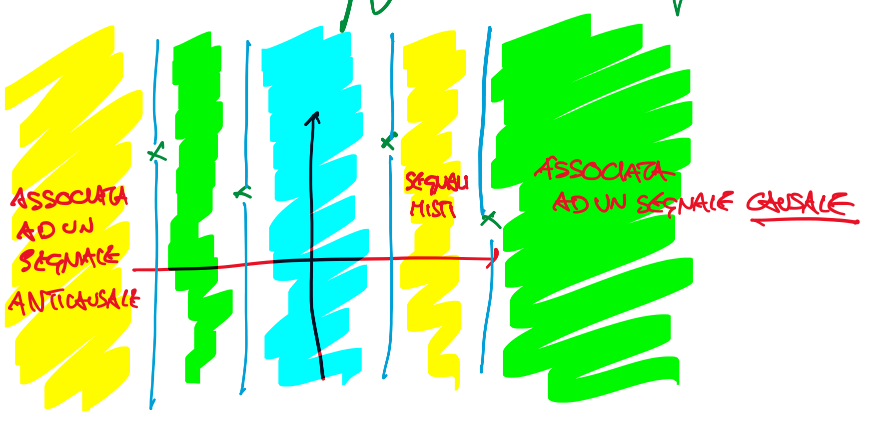
$X(s) = \frac{1}{s-s_1} - \frac{1}{s-s_2}$ $\Gamma_2 = \{s \mid Re(s) < Re(s_2)\}$

$\Gamma_1 = \{s \mid Re(s) > Re(s_1)\}$

$\Gamma_x = \Gamma_1 \cap \Gamma_2$

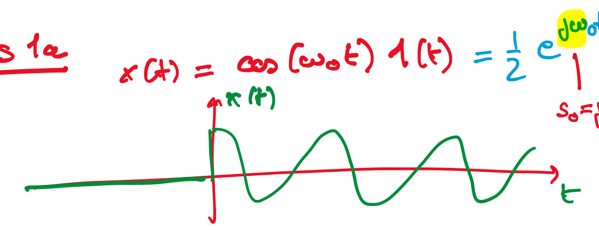






Es 1a $x(t) = \cos(\omega_0 t) 1(t) = \frac{1}{2} e^{j\omega_0 t} 1(t) + \frac{1}{2} e^{-j\omega_0 t} 1(t)$

$s_0 = j\omega_0$ $s_0 = -j\omega_0$

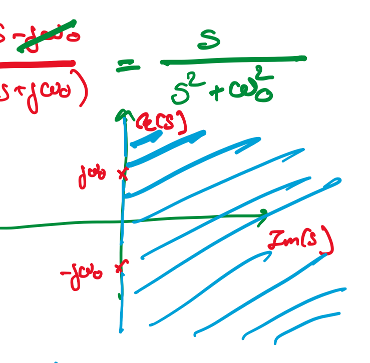


$X(s) = \frac{1/2}{s-j\omega_0} + \frac{1/2}{s+j\omega_0}$

$Re(s) > Re(j\omega_0) = 0$ $Re(s) > Re[-j\omega_0] = 0$

$\Gamma_x = \{s \mid Re(s) > 0\}$

$X(s) = \frac{1}{2} \frac{s + j\omega_0 + s - j\omega_0}{(s-j\omega_0)(s+j\omega_0)} = \frac{s}{s^2 + \omega_0^2}$

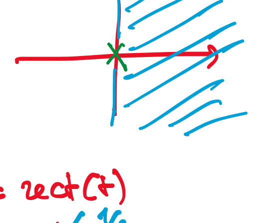


x CASA 1b $x(t) = \sin(\omega_0 t) 1(t)$

$X(s) = \frac{\omega_0}{s^2 + \omega_0^2}$

Es 1c $x(t) = 1(t) = e^{s_0 t} 1(t)$ $s_0 = 0$

$X(s) = \frac{1}{s}$ $\Gamma_x = \{s \mid Re(s) > 0\}$



Es 1d $x(t) = rect(t)$

$X(s) = \int_{-1/2}^{+1/2} rect(t) e^{-st} dt$

$= \left. \frac{e^{-st}}{-s} \right|_{-1/2}^{1/2} = \frac{e^{-s/2} - e^{s/2}}{-s}$

$X(s) = \begin{cases} \frac{e^{s/2} - e^{-s/2}}{s} & s \neq 0 \\ \int_{-\infty}^{+\infty} rect(t) dt = 1 & s = 0 \end{cases}$

$\Gamma_x = \mathbb{C}$

Es 1e $x(t) = \delta(t)$

$X(s) = \int_{-\infty}^{+\infty} \delta(t) e^{-st} dt = 1$ $\Gamma_x = \mathbb{C}$

Es 1f $x(t) = \delta(t - t_0)$

$X(s) = 1 \cdot e^{-st_0}$ ← REGOLA DI TRASLAZ. $\Gamma_x = \mathbb{C}$

$= \int_{-\infty}^{+\infty} \delta(t-t_0) e^{-st} dt = e^{-st_0}$ $\Gamma_x = \mathbb{C}$

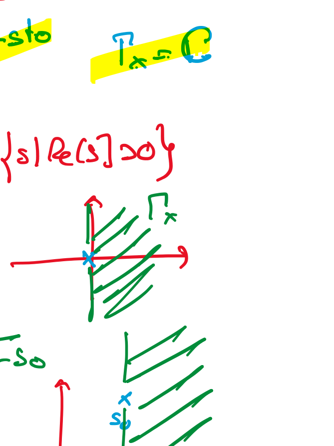
Es $x(t) = 1(t) \xrightarrow{\mathcal{L}} X(s) = \frac{1}{s}$ $\Gamma_x = \{s \mid Re(s) > 0\}$

$y(t) = e^{s_0 t} 1(t) = x(t) e^{s_0 t}$

↓ REGOLA DI MOD.

$Y(s) = X(s - s_0) = \frac{1}{s-s_0}$

$\Gamma_y = \Gamma_x + s_0$



$\Gamma_y = \{s \mid Re(s) > Re(s_0)\}$

Es 1g $x(t) = t^K e^{s_0 t} 1(t)$

$X(s) = ?$

$K=0$ $x_0(t) = e^{s_0 t} 1(t)$ $X_0(s) = \frac{1}{s-s_0}$ $\Gamma_x = \{Re(s) > Re(s_0)\}$

$K=1$ $x_1(t) = t\, x_0(t)$

$X_1(s) = -X_0'(s) = -1 \cdot \frac{-1}{(s-s_0)^2} = \frac{1}{(s-s_0)^2}$

$K=2$ $x_2(t) = t^2 x_0(t) = t\, x_1(t)$

$X_2(s) = -X_1'(s) = -1 \cdot \frac{-2}{(s-s_0)^3} = \frac{2}{(s-s_0)^3}$

$K=3$ $x_3(t) = t\, x_2(t)$

$X_3(s) = -X_2'(s) = -1 \cdot 2 \cdot \frac{-3}{(s-s_0)^4} = \frac{3!}{(s-s_0)^4}$

$K=4$ $x_4(t) = t\, x_3(t)$

$X_4(s) = -X_3'(s) = -1 \cdot 3! \cdot \frac{-4}{(s-s_0)^5} = \frac{4!}{(s-s_0)^5}$

⋮

K $X(s) = \frac{K!}{(s-s_0)^{K+1}}$

$\frac{t^K e^{s_0 t} 1(t)}{K!} \xrightarrow{\mathcal{L}} \frac{1}{(s-s_0)^{K+1}}$ $\Gamma = \{Re(s) > Re(s_0)\}$