Le22

Tuesday, 9 May 2023 23:37

INVERSIONE DELL'INCREMENTO (NEL DOMINIO DI FOURIER)

$$\underset{\substack{\uparrow\\ \text{INCREMENTO}}}{Y(e^{j\theta})} = \underset{\substack{\uparrow\\ \text{SEGNALE ORIGINARIO}}}{X(e^{j\theta})}\left(1-e^{-j\theta}\right)$$

$$X(e^{j\theta}) = \begin{cases} \dfrac{Y(e^{j\theta})}{1-e^{-j\theta}} & \theta \neq 2k\pi \\ m_x\, 2\pi\, \mathrm{comb}_{2\pi}(\theta) & \theta = 2\pi k \end{cases}$$

$\mathrm{comb}_{2\pi}(\theta) \leftarrow \mathrm{rep}_{2\pi}\delta(\theta)$

Es 1i

$s(n) = \mathrm{sinc}(nd)$

$S(e^{j\theta}) = ?$

$x(t) = \mathrm{sinc}(td) \rightarrow \boxed{\downarrow}_{T=1} \rightarrow s(n) = \mathrm{sinc}(nd)$

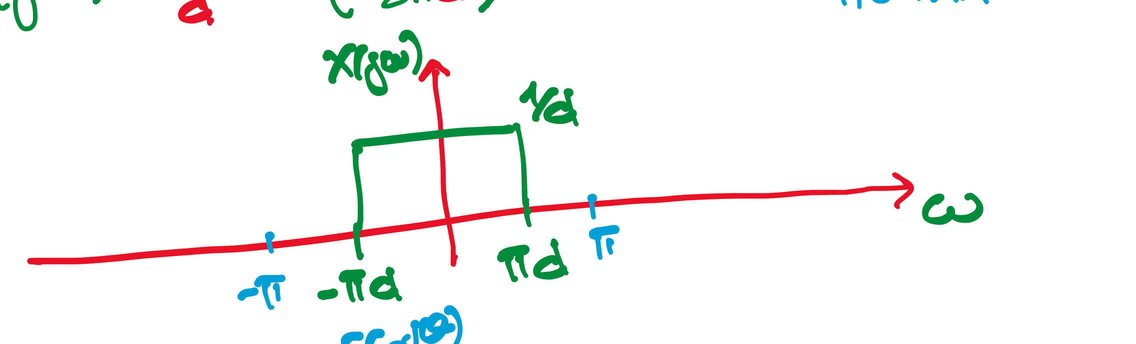
$\downarrow \mathcal{F} \qquad \downarrow \mathcal{F}$

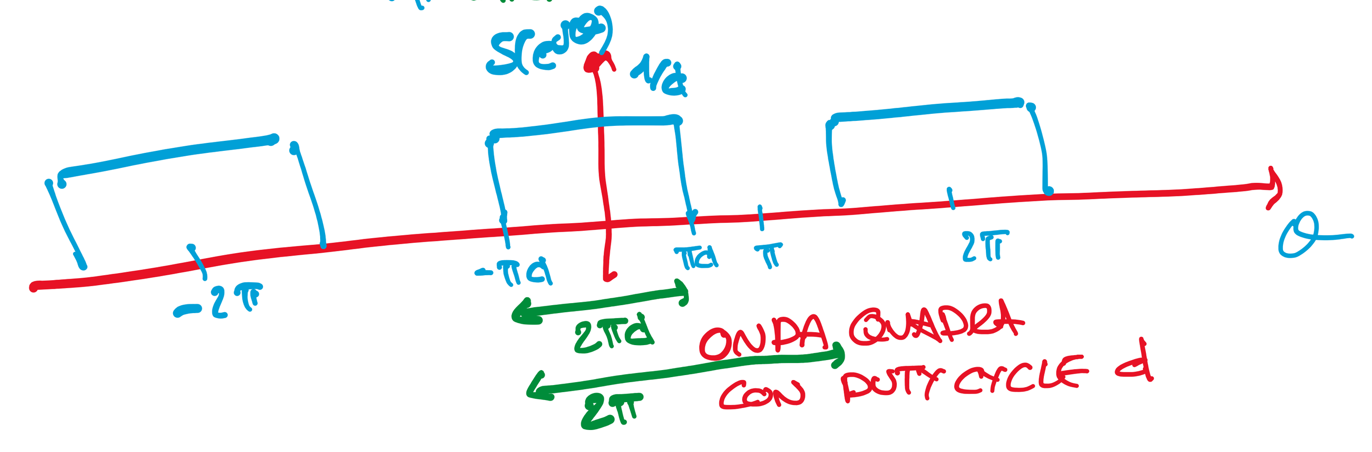
$X(j\omega) \rightarrow \boxed{\mathrm{rep}_{2\pi}} \rightarrow S(e^{j\theta})$

$x(t) = \mathrm{sinc}\left(\frac{t}{1/d}\right)$

$X(j\omega) = \frac{1}{d}\,\mathrm{rect}\left(\frac{\omega}{2\pi d}\right)$

IPOTIZZIAMO $d<1$





ONDA QUADRA CON DUTY CYCLE d

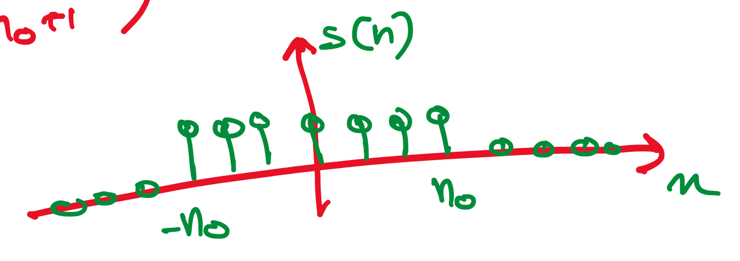
XCASA $s(n) = \mathrm{sinc}^2(nd)$

$S(e^{j\theta}) = ?$

ES 1c

$s(n) = \mathrm{rect}\left(\frac{n}{2n_0+1}\right)$ n_0 intero positivo

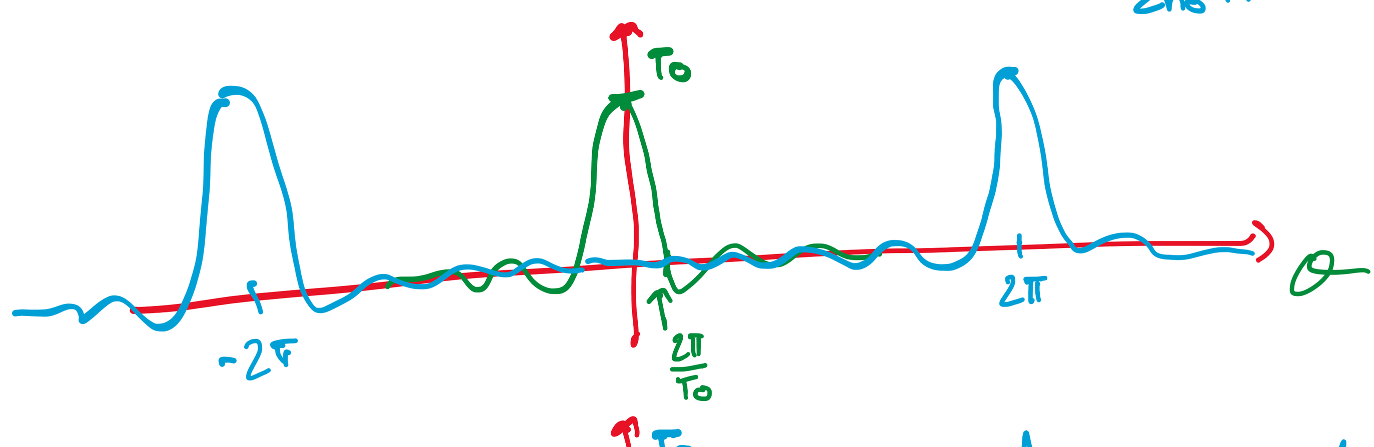
$S(e^{j\theta}) = ?$

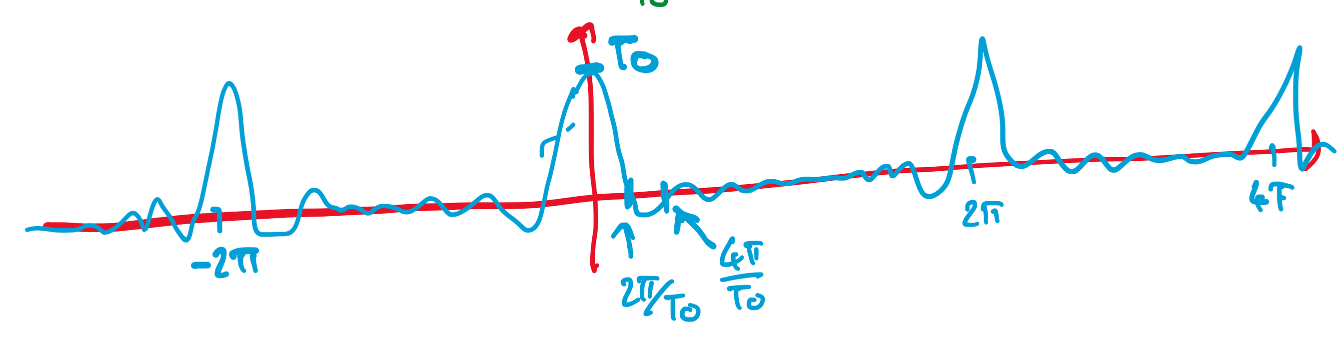


$x(t) = \mathrm{rect}\left(\frac{t}{T_0}\right)$ $T_0 = 2n_0+1$

$X(j\omega) = T_0\,\mathrm{sinc}\left(\frac{\omega T_0}{2\pi}\right)$

$S(e^{j\theta}) = \mathrm{rep}_{2\pi}\, T_0\,\mathrm{sinc}\left(\frac{\theta}{2\pi/T_0}\right)$ $\quad \frac{2\pi}{2n_0+1}$





Es 1d REVISED

$s(n) = A\cos(\theta_0 n + \theta_1)$

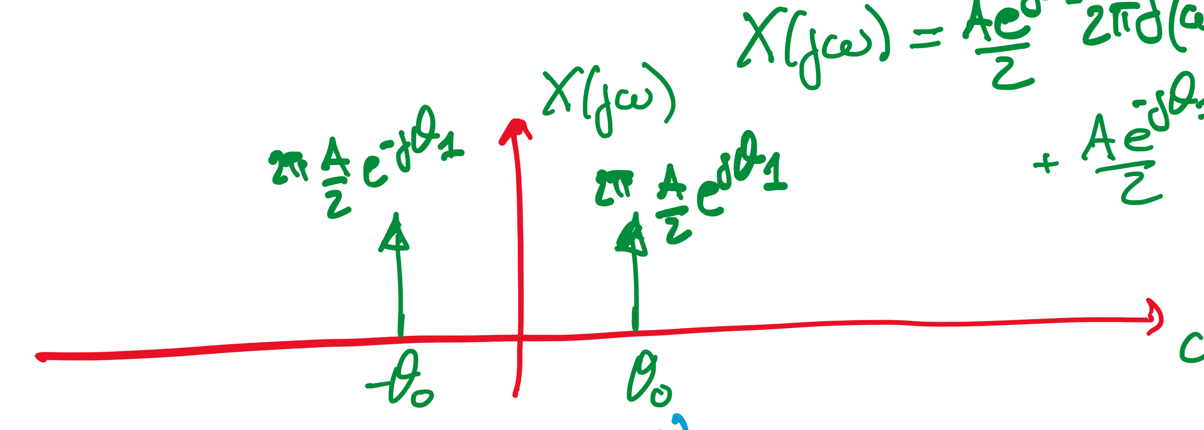
$S(e^{j\theta}) = ?$

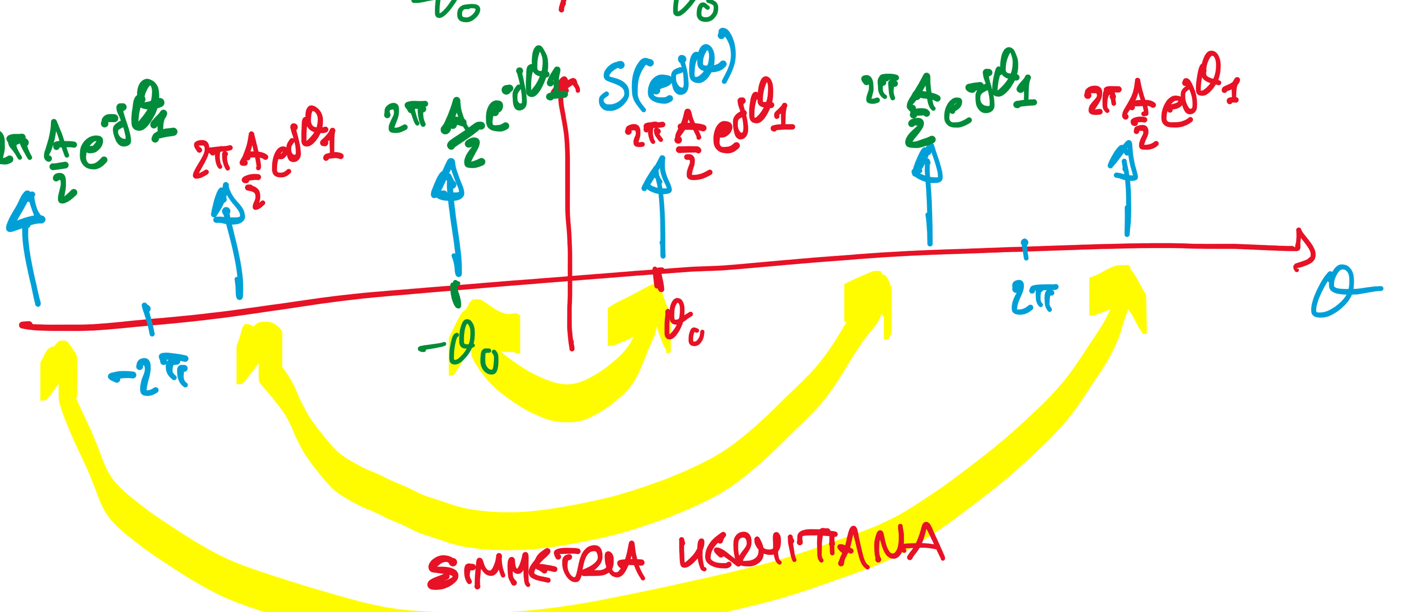
$x(t) = A\cos(\theta_0 t + \theta_1) = \frac{Ae^{j\theta_1}}{2}e^{j\theta_0 t} + \frac{Ae^{-j\theta_1}}{2}e^{-j\theta_0 t}$

$S(e^{j\theta}) = \mathrm{rep}_{2\pi}\, X(j\theta)$

$\mathcal{F}$

$X(j\omega) = \frac{Ae^{j\theta_1}}{2}2\pi\delta(\omega-\theta_0) + \frac{Ae^{-j\theta_1}}{2}2\pi\delta(\omega+\theta_0)$





$S(e^{j\theta}) = \pi A e^{j\theta_1}\,\mathrm{comb}_{2\pi}(\theta-\theta_0) + \pi A e^{-j\theta_1}\,\mathrm{comb}_{2\pi}(\theta+\theta_0)$

ES

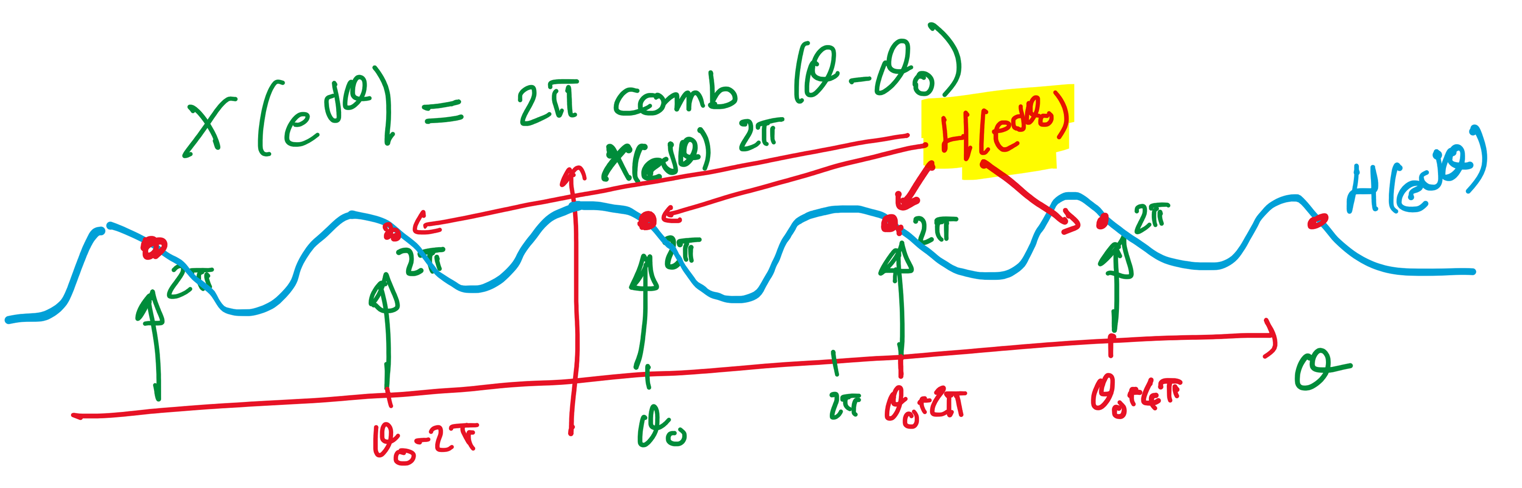
DIMOSTRARE CHE I SEGNALI $x(n) = e^{j\theta_0 n}$ SONO AUTOFUNZIONI DEI FILTRI

$x(n) = e^{j\theta_0 n} \rightarrow \boxed{h(n)}_{\text{FILTRO}} \rightarrow y(n) \overset{?}{=} A\,x(n) = A e^{j\theta_0 n}$

$\downarrow \mathcal{F} \qquad \downarrow \mathcal{F}$

$X(e^{j\theta}) \rightarrow \otimes \rightarrow Y(e^{j\theta})$, $\quad \uparrow H(e^{j\theta})$

$X(e^{j\theta}) = 2\pi\,\mathrm{comb}_{2\pi}(\theta-\theta_0)$



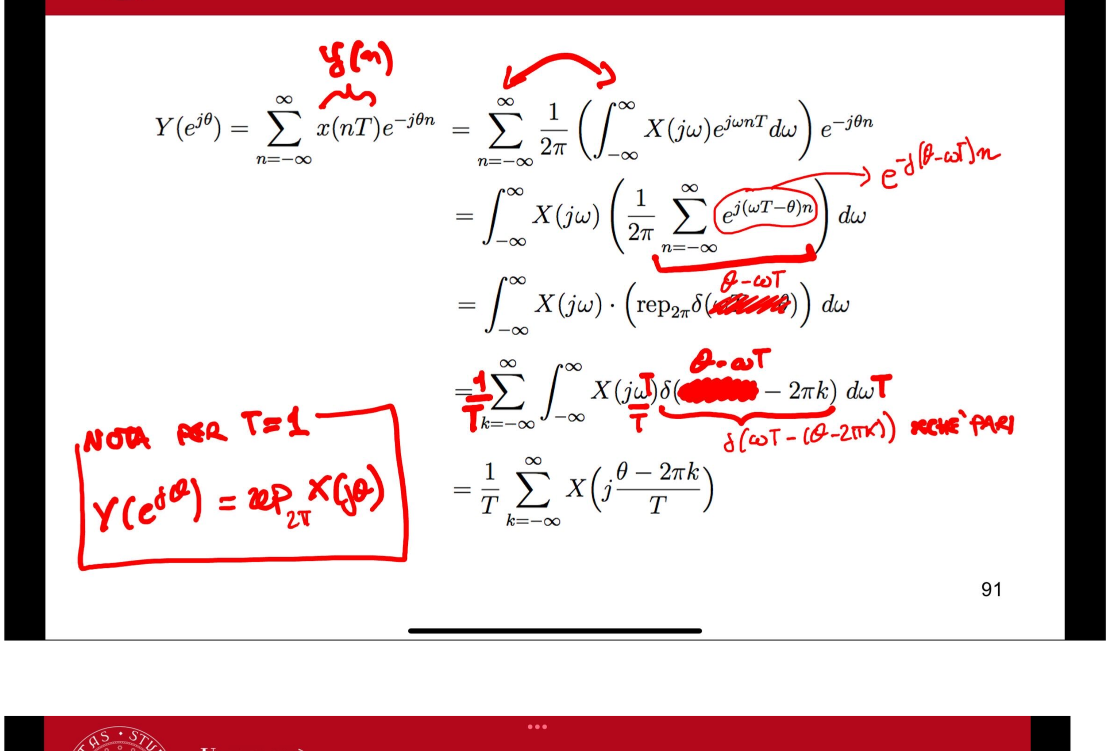
$Y(e^{j\theta}) = \underset{\substack{\uparrow\\ \text{COSTANTE}}}{H(e^{j\theta_0})} \cdot \underbrace{2\pi\,\mathrm{comb}_{2\pi}(\theta-\theta_0)}_{X(e^{j\theta})}$

$y(n) = H(e^{j\theta_0})\,x(n)$

$= H(e^{j\theta_0})\,\underbrace{e^{j\theta_0 n}}_{\text{È AUTOFUNZIONE DI UN QUALUNQUE FILTRO}}$

X CASA DIMOSTRARE CHE

$\cos(\theta_0 n + \theta_1) \rightarrow \boxed{h(n)}_{\text{FILTRO REALE}} \rightarrow |H(e^{j\theta_0})|\cos(\theta_0 n + \theta_1 + \angle H(e^{j\theta_0}))$



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Dimostrazione

$$Y(e^{j\theta}) = \sum_{n=-\infty}^{\infty} \underbrace{x(nT)}_{y(n)} e^{-j\theta n} = \sum_{n=-\infty}^{\infty}\frac{1}{2\pi}\left(\int_{-\infty}^{\infty}X(j\omega)e^{j\omega nT}d\omega\right)e^{-j\theta n}$$

$$= \int_{-\infty}^{\infty}X(j\omega)\left(\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}e^{j(\omega T-\theta)n}\right)d\omega \quad \rightarrow e^{-j(\theta-\omega T)n}$$

$$= \int_{-\infty}^{\infty}X(j\omega)\cdot\left(\mathrm{rep}_{2\pi}\delta(\theta-\omega T)\right)d\omega$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}X(j\omega)\,\underbrace{\delta(\theta-\omega T-2\pi k)}_{\delta(\omega T-(\theta-2\pi k))\text{ PERCHÉ PARI}}\,d\omega T$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty}X\left(j\frac{\theta-2\pi k}{T}\right)$$

NOTA PER $T=1$: $\boxed{Y(e^{j\theta}) = \mathrm{rep}_{2\pi}X(j\theta)}$

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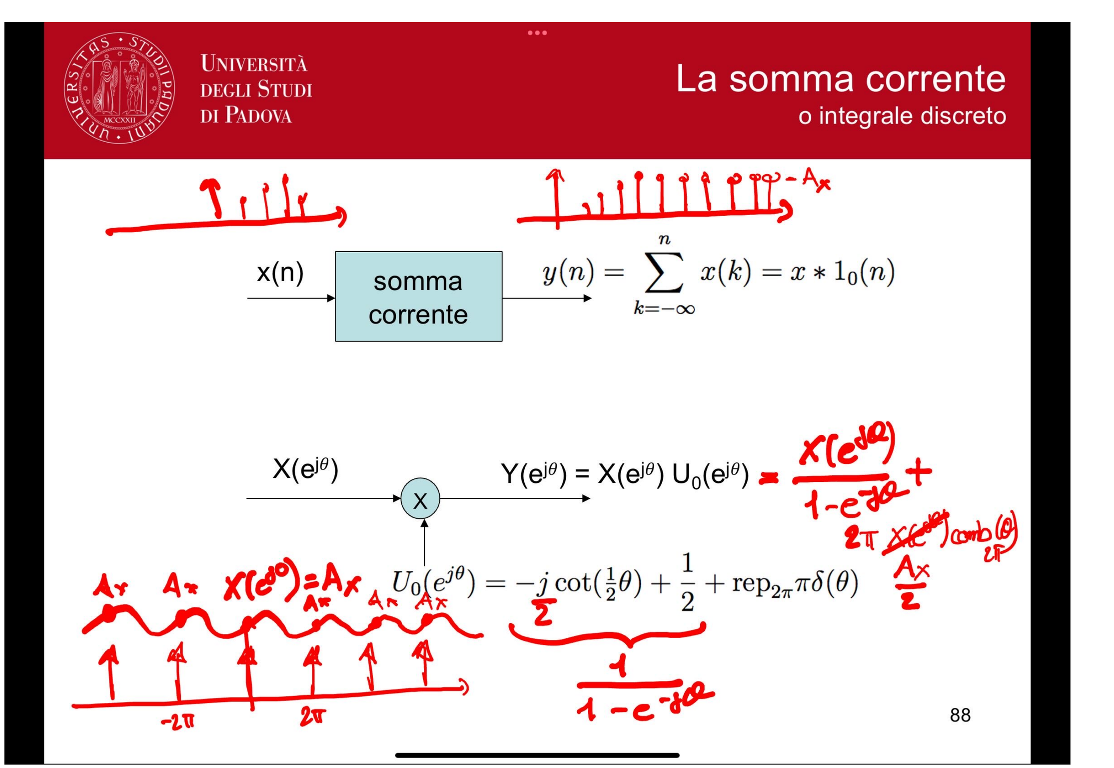

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La somma corrente
o integrale discreto

x(n) → somma corrente → $y(n) = \sum_{k=-\infty}^{n} x(k) = x * 1_0(n)$

$-A_x$

X(e^{jθ}) → ⊗ → $Y(e^{j\theta}) = X(e^{j\theta})\,U_0(e^{j\theta}) = \frac{X(e^{j\theta})}{1-e^{-j\theta}} + 2\pi\,\frac{A_x}{2}\,\mathrm{comb}_{2\pi}(\theta)$

$U_0(e^{j\theta}) = \underbrace{-\frac{j}{2}\cot(\frac{1}{2}\theta) + \frac{1}{2}}_{\frac{1}{1-e^{-j\theta}}} + \mathrm{rep}_{2\pi}\pi\delta(\theta)$

$A_x \quad A_x \quad X(e^{j0}) = A_x$

-2π 2π

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