Le21

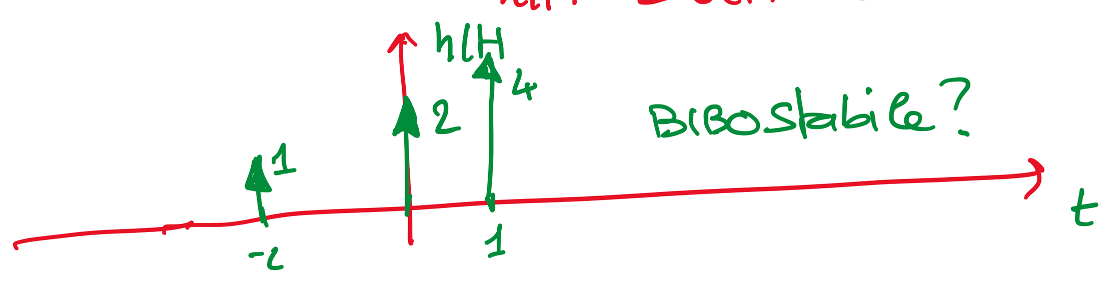
Monday, 8 May 2023 10:37

Es 3 CONTINUA

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$\downarrow \mathcal{F}^{-1}$$

$$h(t) = 2\delta(t) + \delta(t+2) + 4\delta(t-1)$$



BIBO stabile?

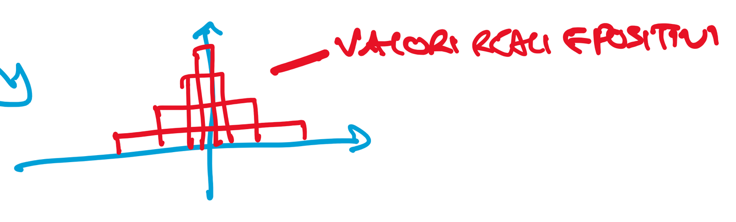
$$\int |h(t)|\, dt \overset{?}{<} \infty$$

$$y(t) = x * h(t) = 2x(t) + x(t+2) + 4x(t-1)$$

$$|y(t)| \le 2|x(t)| + |x(t+2)| + 4|x(t-1)| \le 7L_x < \infty$$

$\&\ |x(t)| \le L_x < \infty$

BIBO STABILE!



VALORI REALI E POSITIVI

$$|h(t)| = h(t)$$

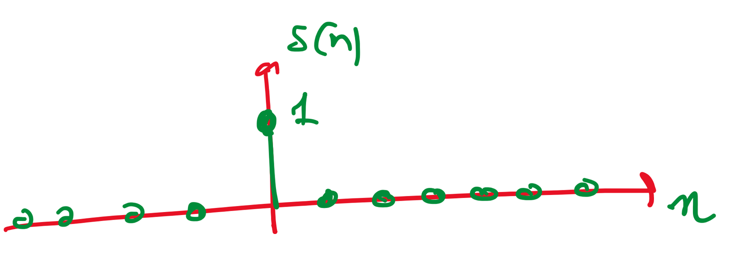
$$\int |h(t)|\,dt = \int h(t)\,dt = \int 2\delta(t) + \delta(t+2) + 4\delta(t-1)\,dt$$

$$= 2 + 1 + 4 = 7 < +\infty$$

BIBO STABILE

Es 1a $s(n) = \delta(n)$

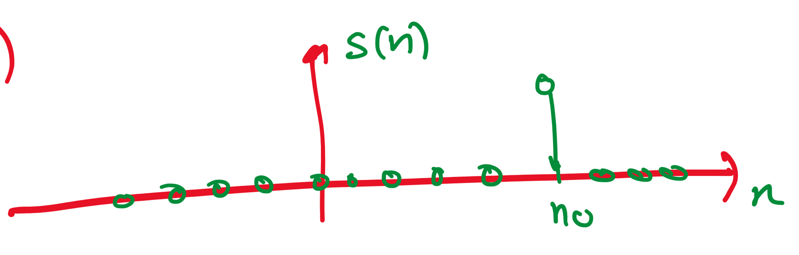
$S(e^{j\theta}) = ?$



$$S(e^{j\theta}) = \sum_{n=-\infty}^{+\infty} \delta(n)\, e^{-j\theta n} = 1$$

Es 1b $s(n) = \delta(n-n_0)$

$S(e^{j\theta}) = ?$



$$S(e^{j\theta}) = \sum_{n=-\infty}^{+\infty} \delta(n-n_0)\, e^{-j\theta n} = e^{-j\theta n_0}$$

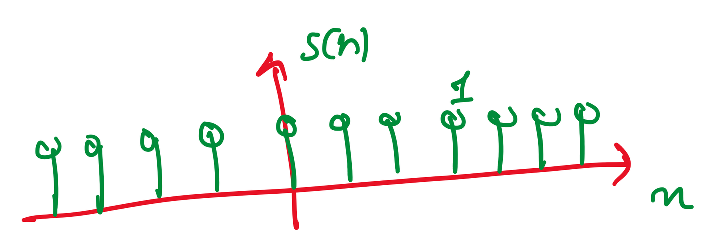
SI PUÒ ANCHE RICAVARE DALLA REGOLA DI TRASLAZIONE NEL TEMPO

$$1 \cdot e^{-j\theta n_0}$$

TRASF. $\delta(n)$ MODULAZIONE

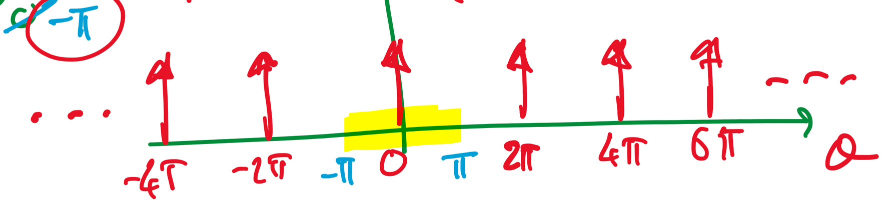
Es 1c $s(n) = 1$

$S(e^{j\theta}) = ?$



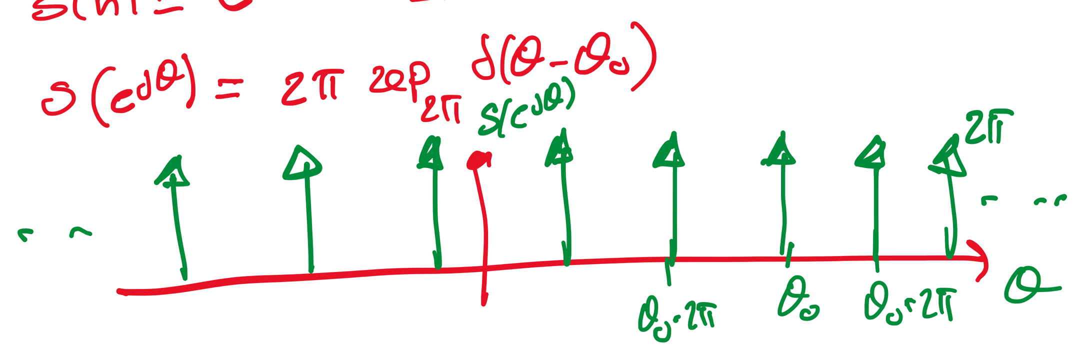
$$S(e^{j\theta}) = 2\pi\, \mathrm{rep}_{2\pi}\, \delta(\theta)$$

$$s(n) = \frac{1}{\cancel{2\pi}} \int_{-\pi}^{+\pi} \cancel{2\pi}\, \cancel{\mathrm{rep}_{2\pi}}\, \delta(\theta)\, e^{j\theta n}\, d\theta = e^{j\theta n}\Big|_{\theta=0} = 1$$



Es 1c bis $s(n) = e^{j\theta_0 n} \cdot 1$

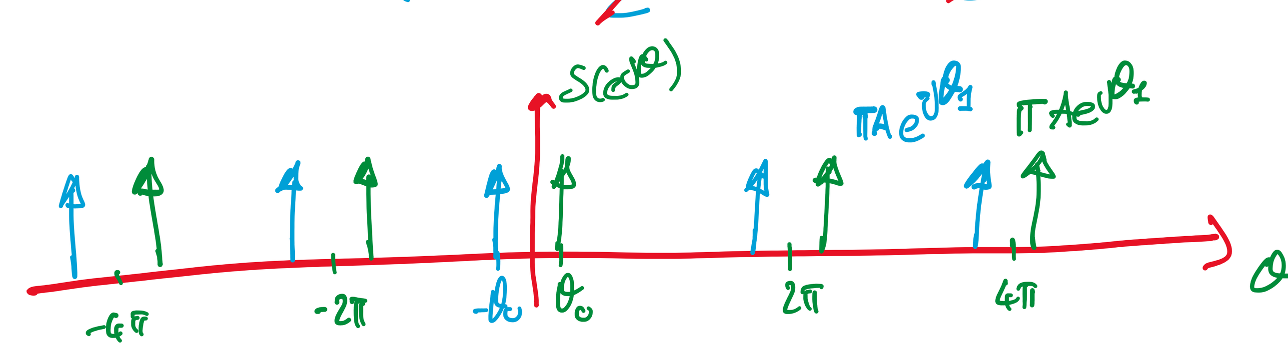
$$S(e^{j\theta}) = 2\pi\, \mathrm{rep}_{2\pi}\, \delta(\theta-\theta_0)$$

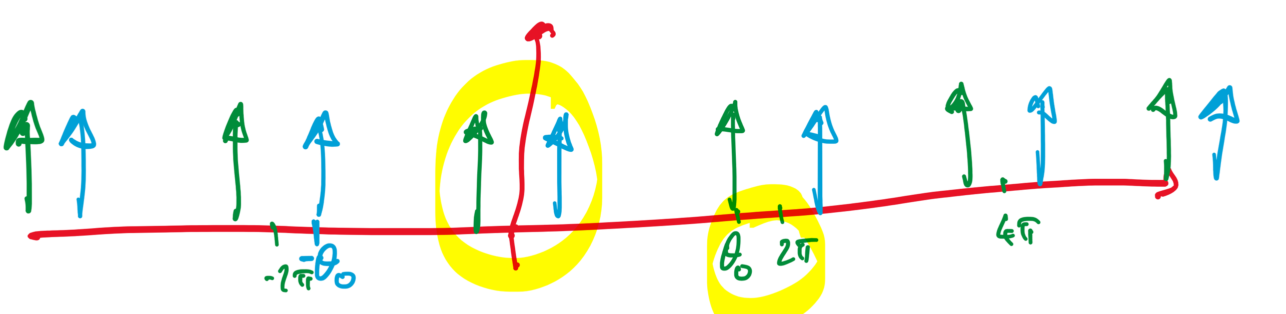


Es 1d $s(n) = A\cos(\theta_0 n + \theta_1) = \frac{Ae^{j\theta_1}}{2} \cdot e^{j\theta_0 n} + \frac{Ae^{-j\theta_1}}{2} \cdot e^{-j\theta_0 n}$

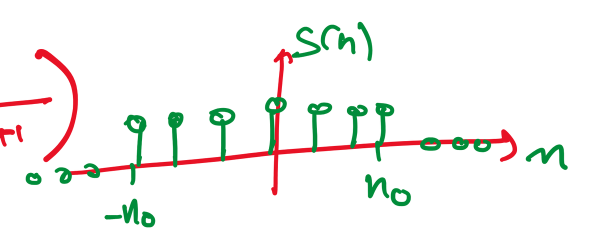
$S(e^{j\theta}) = ?$

$$S(e^{j\theta}) = \frac{Ae^{j\theta_1}}{\cancel{2}}\, \cancel{2}\pi\, \mathrm{comb}_{2\pi}(\theta-\theta_0) + \frac{Ae^{-j\theta_1}}{\cancel{2}}\, \cancel{2}\pi\, \mathrm{comb}_{2\pi}(\theta+\theta_0)$$





x casa Es 1e $s(n) = \mathrm{rect}\left(\frac{n}{2n_0+1}\right)$

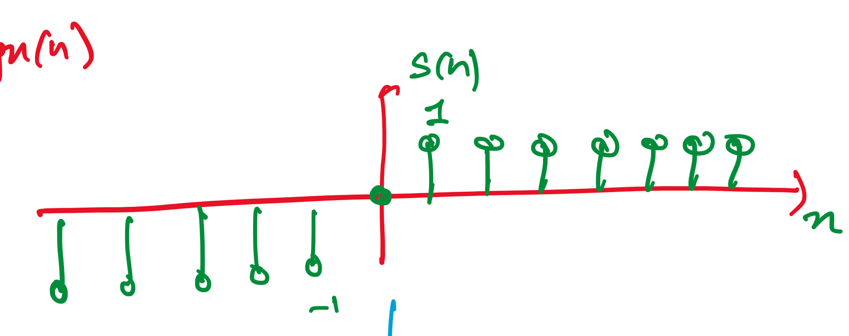


PROVARE CHE

$$S(e^{j\theta}) = \frac{\sin(\theta(n_0+1/2))}{\sin(\theta/2)}$$

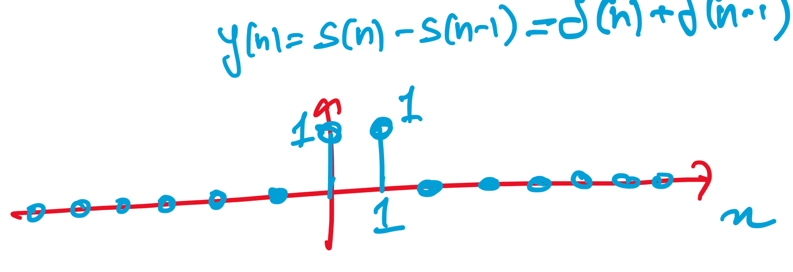
Es 1g $s(n) = \mathrm{sign}(n)$

$S(e^{j\theta}) = ?$



INCREMENTO

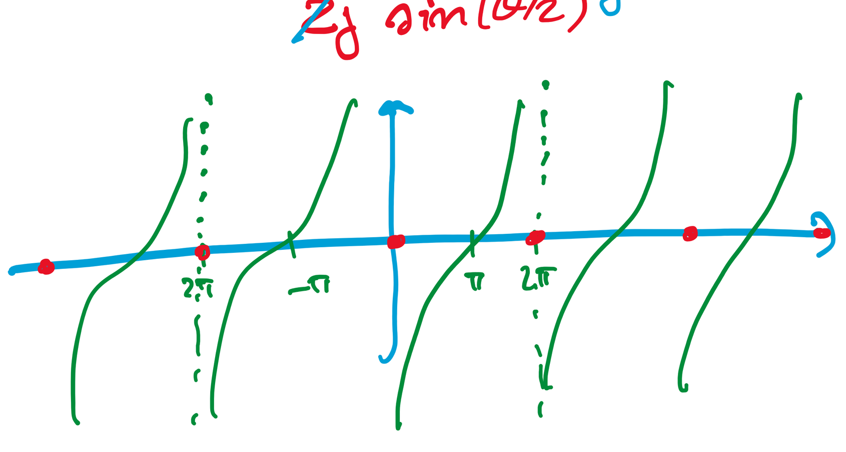
$$y(n) = s(n) - s(n-1) = \delta(n) + \delta(n-1)$$



$$Y(e^{j\theta}) = 1 + e^{-j\theta} = S(e^{j\theta})\left(1 - e^{-j\theta}\right)$$

$$S(e^{j\theta}) = \frac{1+e^{-j\theta}}{1-e^{-j\theta}}\, \frac{e^{j\theta/2}}{e^{j\theta/2}} = \frac{e^{j\theta/2}+e^{-j\theta/2}}{e^{j\theta/2}-e^{-j\theta/2}}$$

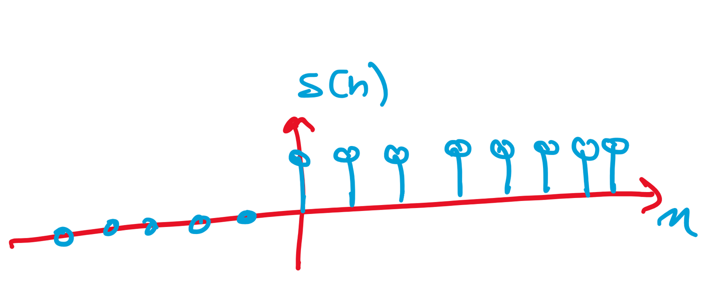
$$= \frac{\cancel{2}\cos(\theta/2)\,(-j)}{\cancel{2}j\,\sin(\theta/2)\,(-j)} = -j\cot(\theta/2)$$



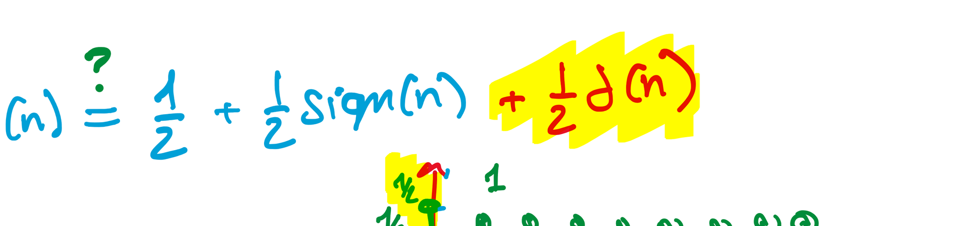
$$S(e^{j\theta}) = \begin{cases} -j\cot(\theta/2) & \theta \ne 2\pi \cdot K \\ A_{s(n)} = 0 & \theta = 2\pi \cdot K \end{cases}$$

Es 1h $s(n) = 1_0(n)$

$S(e^{j\theta}) = ?$



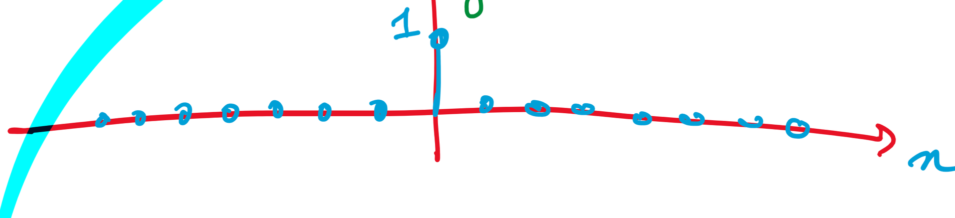
$$1_0(n) \overset{?}{=} \frac{1}{2} + \frac{1}{2}\mathrm{sign}(n) + \frac{1}{2}\delta(n)$$



$$S(e^{j\theta}) = \frac{1}{\cancel{2}} \cdot \cancel{2}\pi\, \mathrm{comb}_{2\pi}(\theta) - \frac{j}{2}\cot\left(\frac{\theta}{2}\right) + \frac{1}{2}$$

E USANDO LA REGOLA DELL'INCREMENTO?

$$y(n) = s(n) - s(n-1) = \delta(n)$$



COINCIDONO

$$Y(e^{j\theta}) = 1 = S(e^{j\theta})\left(1 - e^{-j\theta}\right)$$

$$S(e^{j\theta}) = \begin{cases} \dfrac{1}{1-e^{-j\theta}} & \theta \ne 2\pi \cdot K \\ A_{s(n)} & \theta = 2\pi \cdot K \end{cases}$$

DEVONO COINCIDERE

$$m_s \xrightarrow{\mathcal{F}} \cancel{m_s} \cdot \cancel{2}\pi\, \mathrm{comb}_{2\pi}(\theta)$$

$$= \frac{1}{2}$$