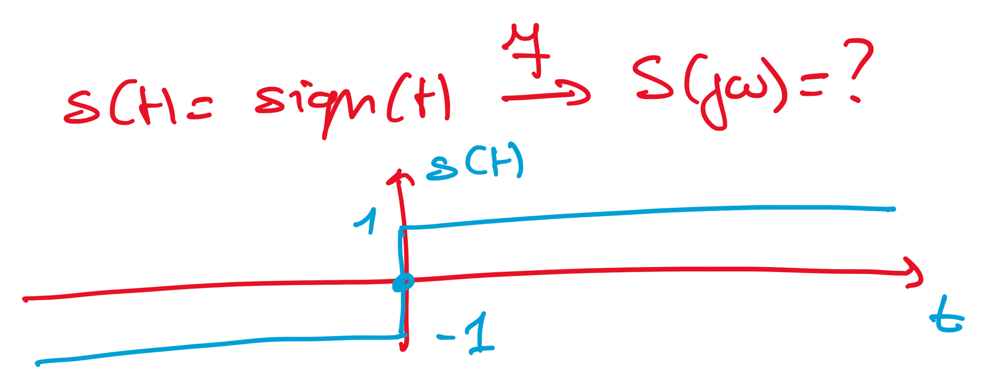
Le19

Thursday, 27 April 2023 08:45

Es1n

$s(t) = \text{sign}(t) \xrightarrow{\mathcal{F}} S(j\omega) = ?$



$$S(j\omega) = \cancel{\int_{-\infty}^{+\infty} \text{sign}(t)\, e^{-j\omega t}\, dt}$$

$$\cancel{= \int_{-\infty}^{0} -1\, e^{-j\omega t}\, dt + \int_{0}^{+\infty} 1\, e^{-j\omega t}\, dt}$$

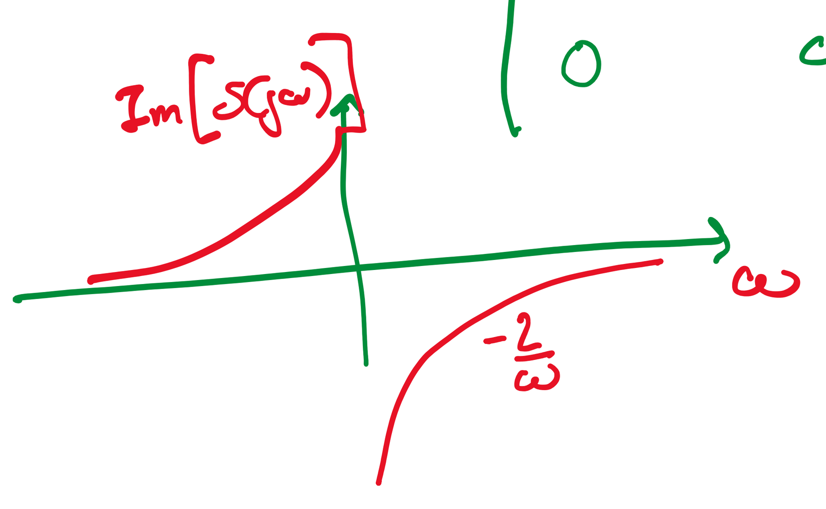
$$\cancel{= -\frac{e^{-j\omega t}}{-j\omega}\Big]_{-\infty}^{0} + \frac{e^{-j\omega t}}{-j\omega}\Big]_{0}^{+\infty}}$$

$$y(t) = s'(t) = 2\delta(t) \xrightarrow{\mathcal{F}} Y(j\omega) = 2 = j\omega S(j\omega)$$

regola di derivazione nel tempo

$$S(j\omega) = \begin{cases} \dfrac{2}{j\omega} = -j\dfrac{2}{\omega} & \omega \neq 0 \\ 0 & \omega = 0 \end{cases}$$

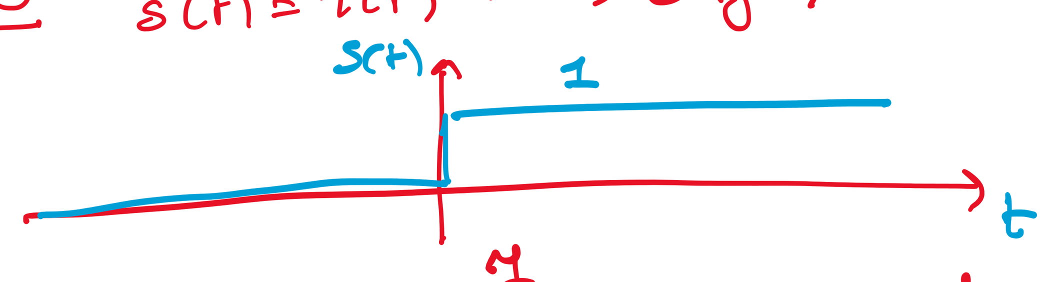
$\text{Im}[S(j\omega)]$



$-\frac{2}{\omega}$

Es10

$s(t) = 1(t) \xrightarrow{\mathcal{F}} S(j\omega) = ?$



$$s'(t) = \delta(t) \xrightarrow{\mathcal{F}} j\omega S(j\omega) = 1$$

$$S(j\omega) = \begin{cases} \dfrac{1}{j\omega} & \omega \neq 0 \\ A_s \; +\infty & \omega = 0 \end{cases}$$

$$s(t) = 1(t) = \frac{1}{2} \cdot 1 + \frac{1}{2}\text{sign}(t)$$

$$S(j\omega) = \frac{1}{\cancel{2}} \cdot \cancel{2}\pi\delta(\omega) + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{j\omega}$$

$$= \pi\delta(\omega) + \frac{1}{j\omega}$$

CONTRIBUTO DEL VALORE MEDIO

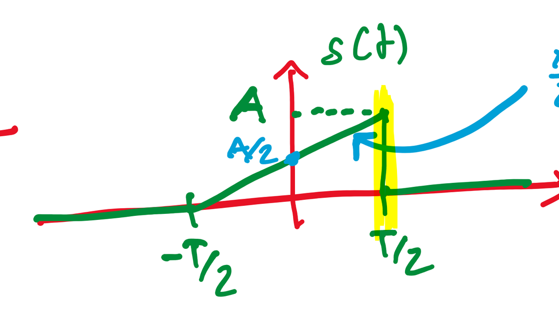
$$m_s \xrightarrow{\mathcal{F}} m_s \cdot 2\pi\delta(\omega)$$

UTILIZZO REGOLA DI DERIVAZIONE

$$y(t) = s'(t) \longrightarrow Y(j\omega) = j\omega S(j\omega)$$

$$S(j\omega) = \frac{Y(j\omega)}{j\omega} + m_s\, 2\pi\delta(\omega)$$

Es 2



$\frac{A}{2} + \frac{2t}{T}\frac{A}{2} = \frac{A}{2} + \frac{At}{T}$

$S(j\omega) = ?$

$$s(t) = \left(\frac{A}{2} + \frac{At}{T}\right) \cdot \text{rect}\left(\frac{t}{T}\right)$$

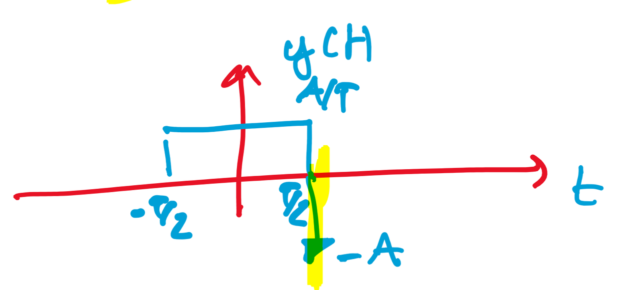
$$= \frac{A}{2}\,\text{rect}\left(\frac{t}{T}\right) + \frac{A}{T}\, t\, \text{rect}\left(\frac{t}{T}\right)$$

PROPRIETÀ DI DERIVAZIONE IN ω

$$S(j\omega) = \frac{A}{2} \cdot T\text{sinc}\left(\frac{T\omega}{2\pi}\right) + \frac{A}{\cancel{T}} \cdot j\frac{d}{d\omega}\left(\cancel{T}\text{sinc}\left(\frac{T\omega}{2\pi}\right)\right)$$

$$= \frac{AT}{2}\,\text{sinc}\left(\frac{T\omega}{2\pi}\right) + j\frac{AT}{2\pi}\,\text{sinc}'\left(\frac{T\omega}{2\pi}\right)$$

$y(t) = s'(t)$



$$y(t) = \frac{A}{T}\,\text{rect}\left(\frac{t}{T}\right) - A\,\delta(t - T/2)$$

$$Y(j\omega) = \frac{A}{T} \cdot T\text{sinc}\left(\frac{T\omega}{2\pi}\right) - A\, e^{-j\omega T/2}$$

$$= j\omega S(j\omega)$$

VALOR MEDIO = 0

$$S(j\omega) = \frac{Y(j\omega)}{j\omega} + \cancel{m_s\, 2\pi\delta(\omega)}$$

$$= \frac{A\,\text{sinc}(T\omega/2\pi)}{j\omega} - A\frac{e^{-j\omega T/2}}{j\omega}$$

$\cos(\omega T/2) - j\sin(\omega T/2)$

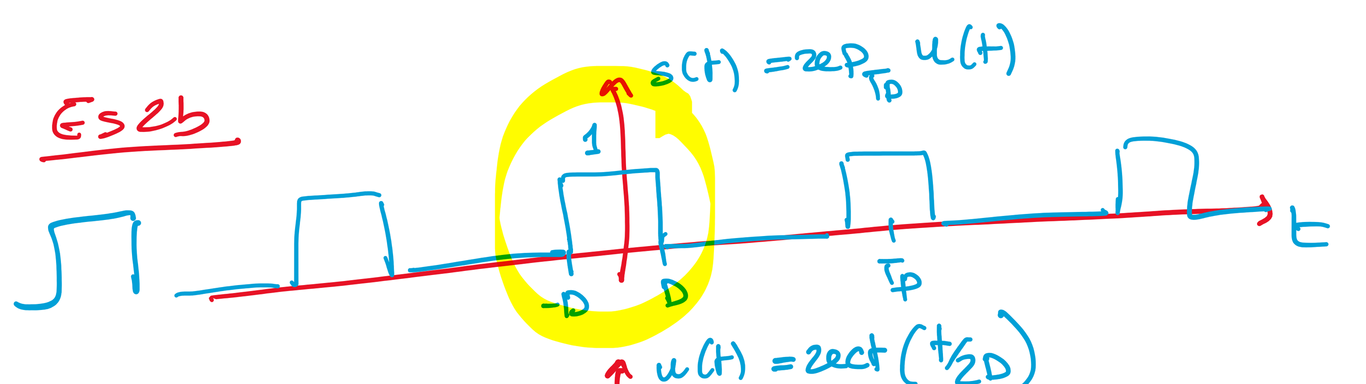
$$= -j\frac{A\,\text{sinc}(T\omega/2\pi)}{\omega} + jA\frac{\cos(\omega T/2)}{\omega}$$

$$+ \frac{AT}{2}\frac{\sin(\omega T/2)}{\omega T/2}$$

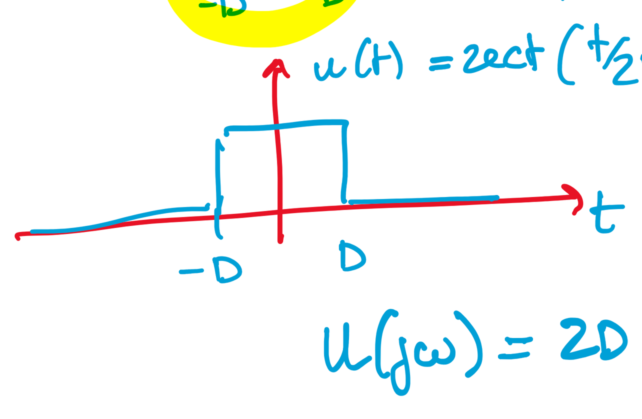
$$\frac{AT}{2} \cdot \text{sinc}\left(\frac{\omega T}{2\pi}\right)$$

Es2b

$s(t) = \text{rep}_{T_p}\, u(t)$



$u(t) = \text{rect}(t/2D)$



$$U(j\omega) = 2D\,\text{sinc}\left(\frac{2D\omega}{2\pi}\right)$$

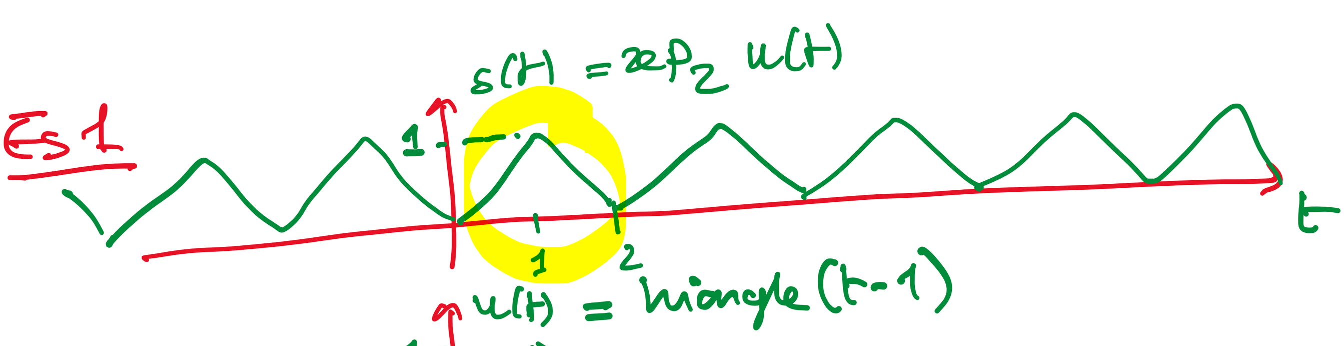
$$S_k = \frac{1}{T_p} U(j\omega)\Big|_{\omega = k\omega_0} \qquad \omega_0 = \frac{2\pi}{T_p}$$

$$= \frac{1}{T_p} \cdot 2D\,\text{sinc}\left(\frac{2D}{\cancel{2\pi}} \cdot k\frac{\cancel{2\pi}}{T_p}\right) \qquad d = \frac{2D}{T_p}$$

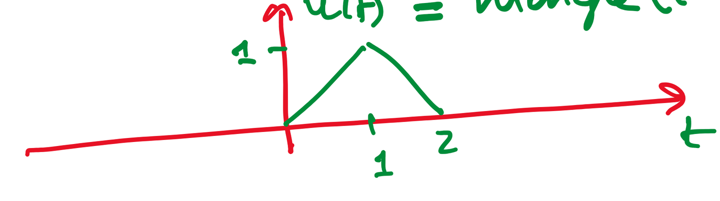
$$= d\,\text{sinc}(kd)$$

Es1

$s(t) = \text{rep}_2\, u(t)$



$u(t) = \text{triangle}(t-1)$



$$U(j\omega) = \text{sinc}^2\left(\frac{\omega}{2\pi}\right) \cdot e^{-j\omega \cdot 1}$$

$$S_k = \frac{1}{\cancel{T_p}\,2} U(j\omega)\Big|_{\omega = k\omega_0} \qquad \omega_0 = \frac{2\pi}{\cancel{T_p}\,2} = \pi$$

$$= \frac{1}{2}\,\text{sinc}^2\left(\frac{k\cancel{\pi}}{2\cancel{\pi}}\right) \underbrace{e^{-jk\pi}}_{(-1)^k}$$

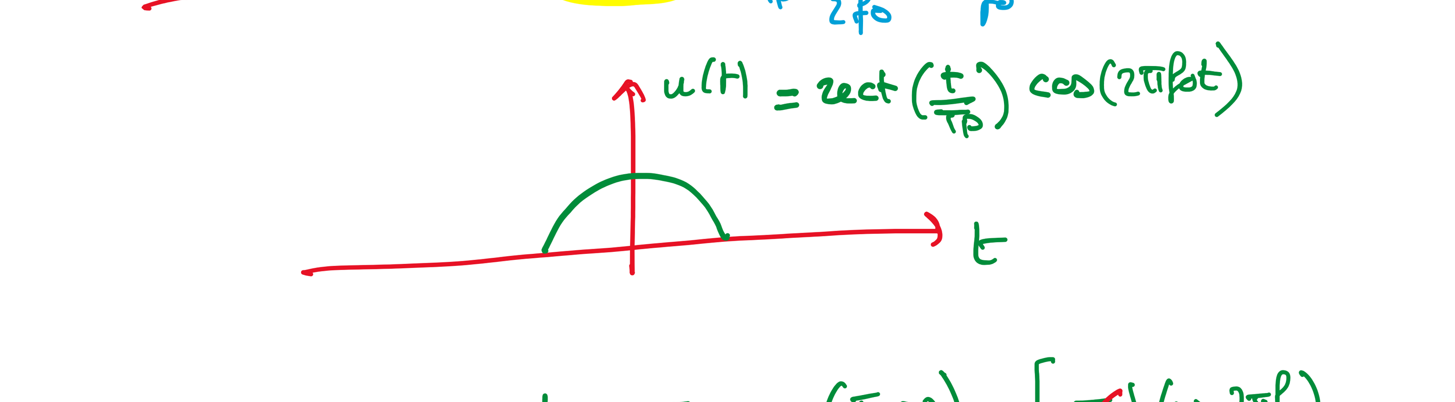
$$= \frac{1}{2}\,\text{sinc}^2\left(\frac{k}{2}\right)(-1)^k$$

Es2a

$s(t) = |\cos(2\pi f_0 t)|$

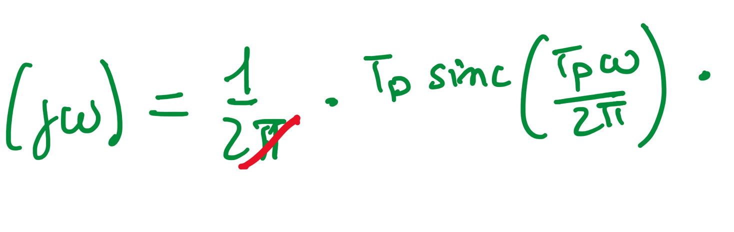
$= \text{rep}_{T_p}\, u(t)$

$f_0 > 0$



$T_p = \frac{1}{2f_0}$ $\quad \frac{1}{f_0}$

$u(t) = \text{rect}\left(\frac{t}{T_p}\right)\cos(2\pi f_0 t)$



$$U(j\omega) = \frac{1}{2\cancel{\pi}} \cdot T_p\,\text{sinc}\left(\frac{T_p\omega}{2\pi}\right) \cdot \left[\cancel{\pi}\delta(\omega - 2\pi f_0) + \cancel{\pi}\delta(\omega + 2\pi f_0)\right]$$

$$= \frac{T_p}{2}\,\text{sinc}\left(\frac{T_p}{2\pi}(\omega - 2\pi f_0)\right) + \frac{T_p}{2}\,\text{sinc}\left(\frac{T_p}{2\pi}(\omega + 2\pi f_0)\right)$$

$$S_k = \frac{1}{T_p} U(j\omega)\Big|_{\omega = k\omega_0} \qquad \omega_0 = \frac{2\pi}{T_p} \qquad T_p = \frac{1}{2f_0}$$

$$= \frac{1}{\cancel{T_p}}\frac{\cancel{T_p}}{2}\,\text{sinc}\left(\frac{T_p}{2\pi}\left(k\frac{2\pi}{T_p} - 2\pi f_0\right)\right) + \frac{1}{\cancel{T_p}}\frac{\cancel{T_p}}{2}\,\text{sinc}\left(\frac{T_p}{2\pi}\left(k\frac{2\pi}{T_p} + 2\pi f_0\right)\right)$$

$$= \frac{1}{2}\,\text{sinc}\left(k - \underbrace{\cancel{T_p f_0}}_{1/2}\right) + \frac{1}{2}\,\text{sinc}\left(k + \underbrace{\cancel{T_p f_0}}_{1/2}\right)$$