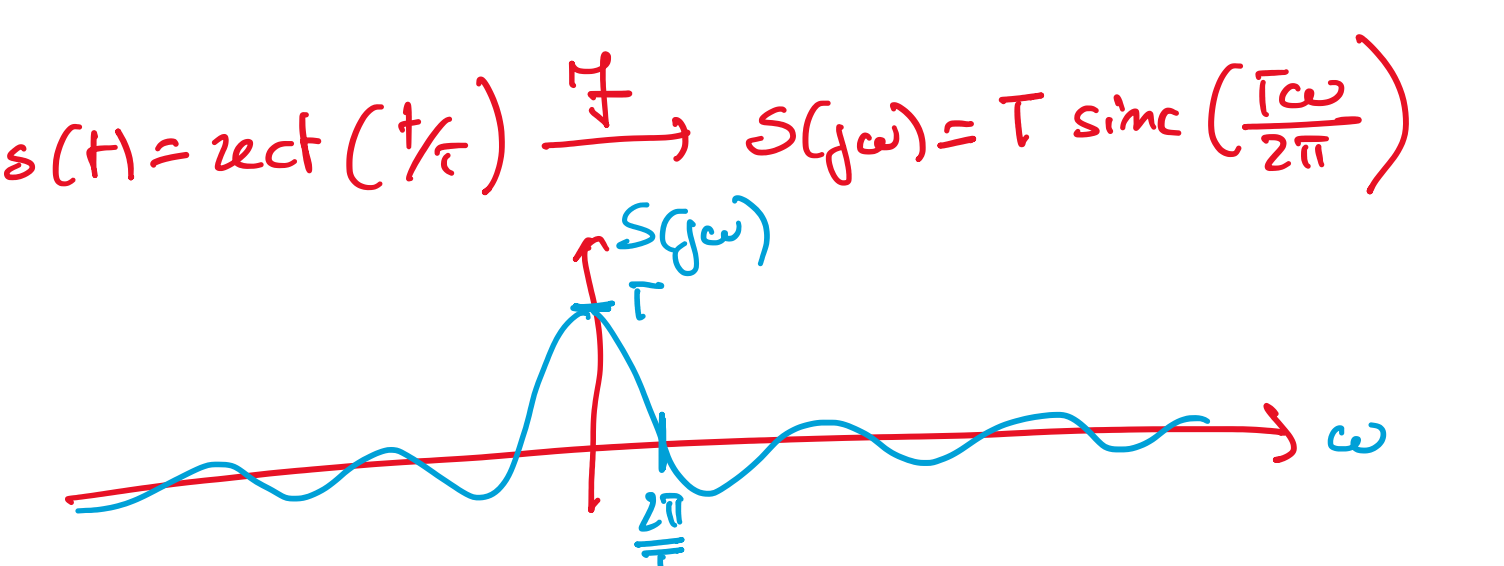
Le18

Wednesday, 26 April 2023 09:55

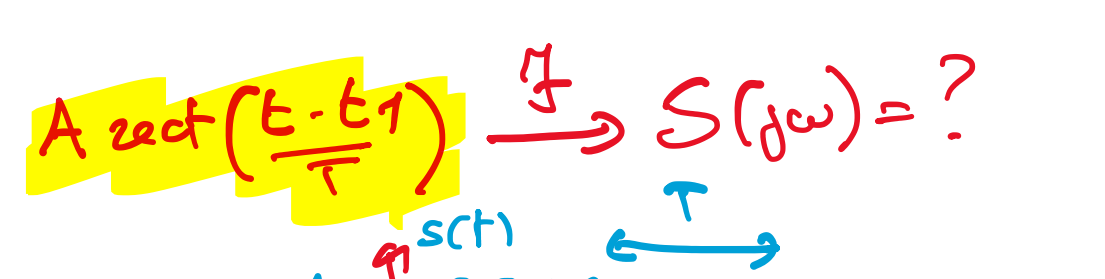
$\text{rect}(t) \xrightarrow{\mathcal{F}} \text{sinc}\left(\frac{\omega}{2\pi}\right)$

$\text{sinc}(t) \xrightarrow{\mathcal{F}} \text{rect}\left(\frac{\omega}{2\pi}\right)$

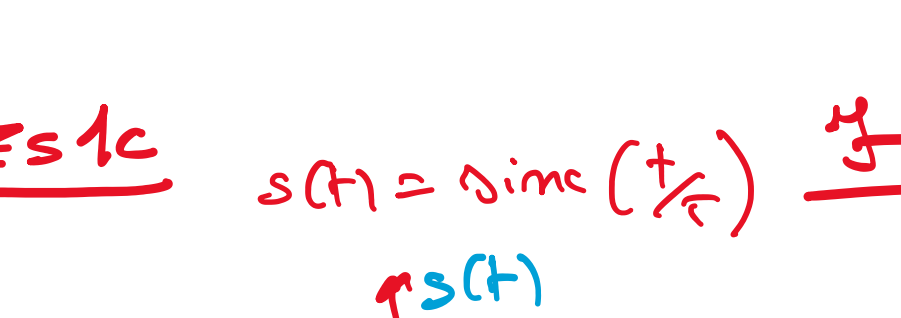
Es 1b $s(t) = \text{rect}(t/\tau) \xrightarrow{\mathcal{F}} S(j\omega) = T\,\text{sinc}\left(\frac{T\omega}{2\pi}\right)$



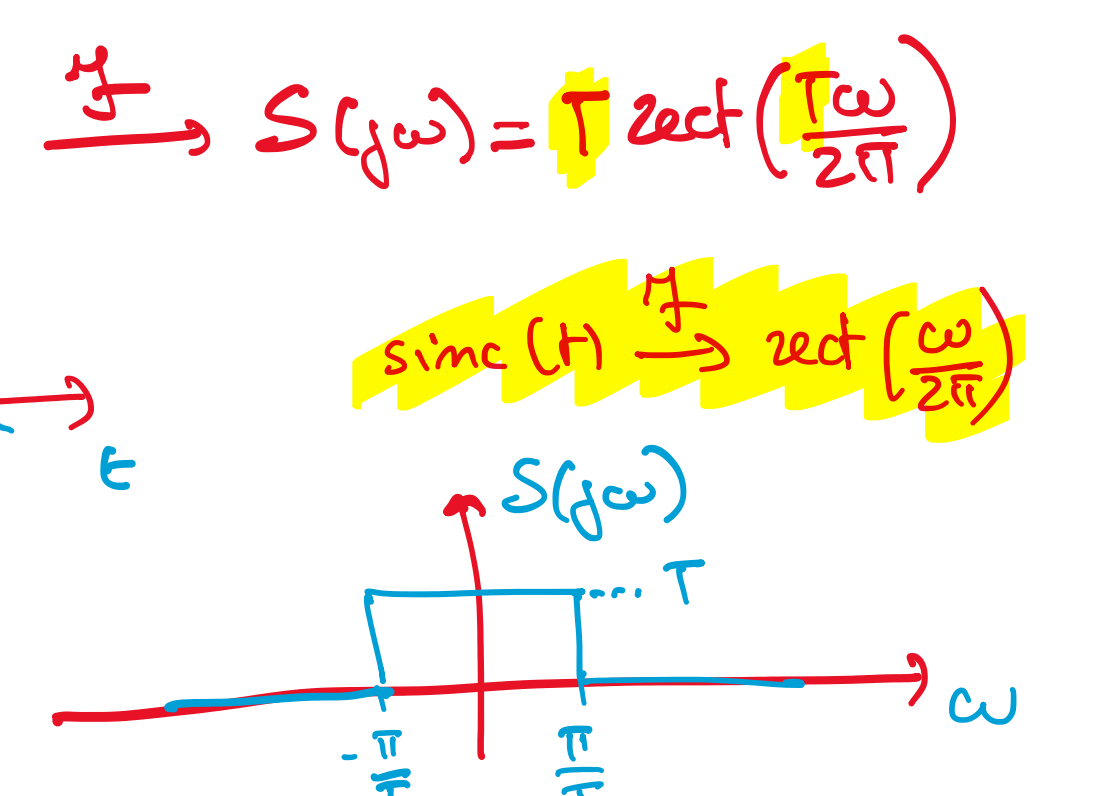
XCASA $s(t) = A\,\text{rect}\left(\frac{t-t_1}{T}\right) \xrightarrow{\mathcal{F}} S(j\omega) = ?$



Es 1c $s(t) = \text{sinc}(t/\tau) \xrightarrow{\mathcal{F}} S(j\omega) = T\,\text{rect}\left(\frac{T\omega}{2\pi}\right)$



$\text{sinc}(t) \xrightarrow{\mathcal{F}} \text{rect}\left(\frac{\omega}{2\pi}\right)$



XCASA $s(t) = A\,\text{sinc}\left(\frac{t-t_1}{T}\right) \xrightarrow{\mathcal{F}} ?$

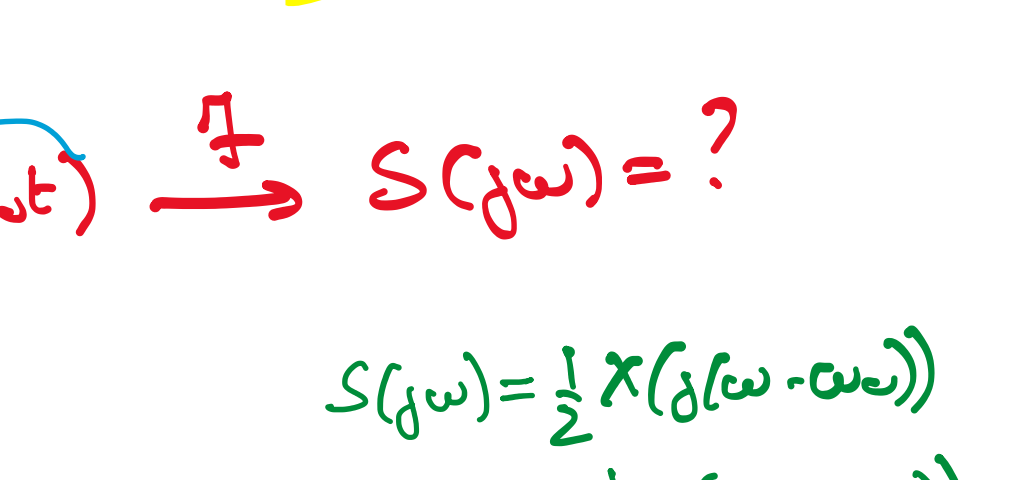
NOTA negli esercizi su elearning si usa la notazione
$S(f) = S(j2\pi f) \qquad \omega = 2\pi f$

$S(j\omega) = S(f)\big|_{f=\frac{\omega}{2\pi}}$

Es 1j $s(t) = x(t)\underbrace{\cos(\omega_0 t)}_{y(t)} \xrightarrow{\mathcal{F}} S(j\omega) = ?$

$\omega_0 \gg \omega_1$

$S(j\omega) = \frac{1}{2}X(j(\omega-\omega_0)) + \frac{1}{2}X(j(\omega+\omega_0))$



$x(t) \xrightarrow{\mathcal{F}} X(j\omega)$

$y(t) = \cos(\omega_0 t) \xrightarrow{\mathcal{F}} Y(j\omega) = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$

$s(t) = x(t)y(t) \xrightarrow{\mathcal{F}} S(j\omega) = \frac{1}{2\pi} X * Y(j\omega)$

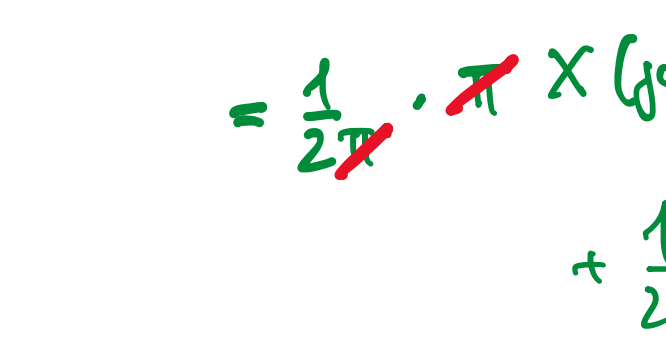
$S(j\omega) = \frac{1}{2\pi} X(j\omega) * [\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)]$

$= \frac{1}{2\pi}\cdot\pi\, X(j\omega) * \delta(\omega-\omega_0)$

$+ \frac{1}{2\pi}\cdot\pi\, X(j\omega) * \delta(\omega+\omega_0)$

$= \frac{1}{2}X(j(\omega-\omega_0)) + \frac{1}{2}X(j(\omega+\omega_0))$

Es 1i $s(t) = \text{triangle}(t/D) \rightarrow S(j\omega) = ?$



REGOLA CONV.

$\underbrace{\text{triangle}(t)}_{z(t)} = \underbrace{\text{rect}}_{x(t)} * \underbrace{\text{rect}(t)}_{y(t)} \xrightarrow{\mathcal{F}} Z(j\omega) = X(j\omega)Y(j\omega)$

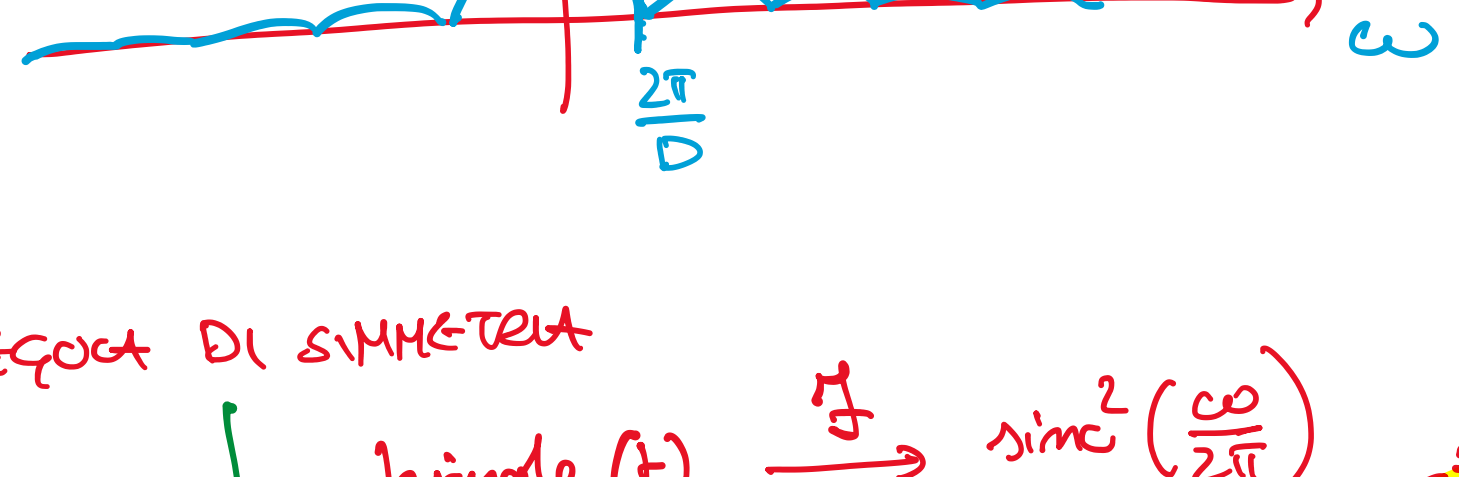
$= X^2(j\omega)$

$= \text{sinc}^2\left(\frac{\omega}{2\pi}\right)$

REGOLA SCALA

$s(t) = z(t/D) \xrightarrow{\mathcal{F}} S(j\omega) = D\,Z(j\omega D)$

$= D\,\text{sinc}^2\left(\frac{D\omega}{2\pi}\right)$



REGOLA DI SIMMETRIA

$\text{triangle}(t) \xrightarrow{\mathcal{F}} \text{sinc}^2\left(\frac{\omega}{2\pi}\right)$

$\text{sinc}^2\left(\frac{t}{2\pi}\right) \xrightarrow{\mathcal{F}} 2\pi\,\text{triangle}(-\omega)$ PARI

$= 2\pi\,\text{triangle}(\omega)$

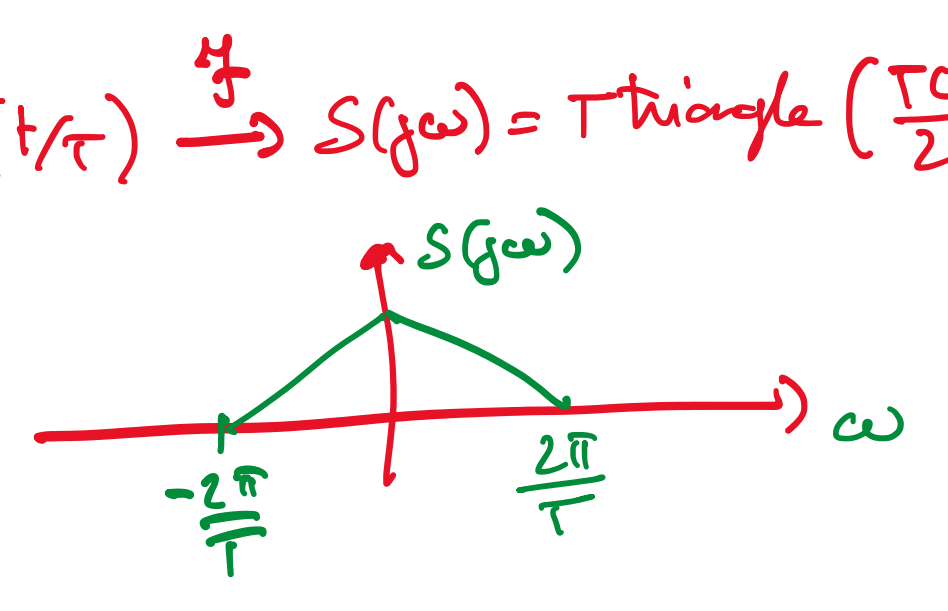
regola di scala $a = 1/2\pi$

$\text{sinc}^2(t) \xrightarrow{\mathcal{F}} \frac{1}{2\pi}\cdot 2\pi\,\text{triangle}\left(\frac{\omega}{2\pi}\right)$

$\text{triangle}(t) \xrightarrow{\mathcal{F}} \text{sinc}^2\left(\frac{\omega}{2\pi}\right)$

$\text{sinc}^2(t) \xrightarrow{\mathcal{F}} \text{triangle}\left(\frac{\omega}{2\pi}\right)$

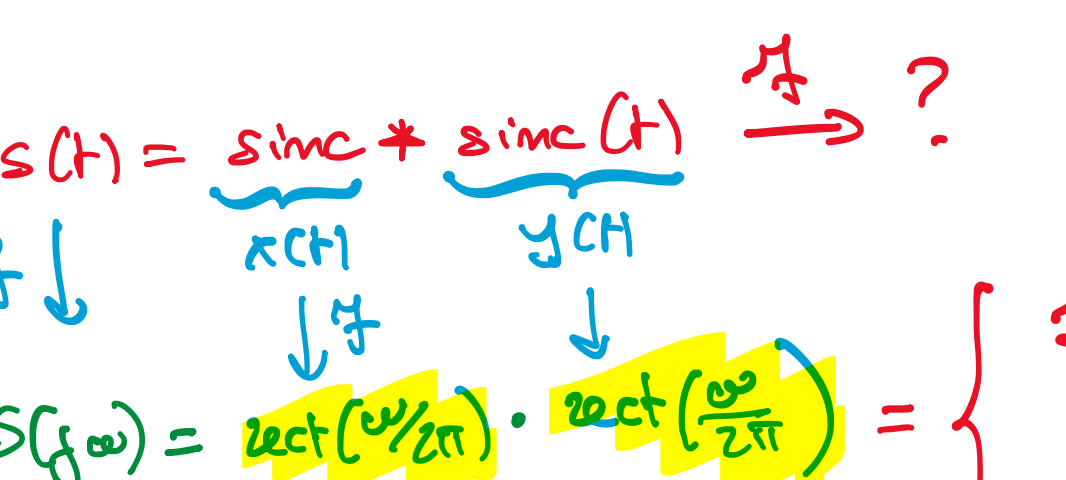
Es 1h $s(t) = \text{sinc}^2(t/\tau) \xrightarrow{\mathcal{F}} S(j\omega) = T\,\text{triangle}\left(\frac{T\omega}{2\pi}\right)$



Es 1e $s(t) = \underbrace{\text{sinc}}_{x(t)} * \underbrace{\text{sinc}(t)}_{y(t)} \xrightarrow{\mathcal{F}} ?$

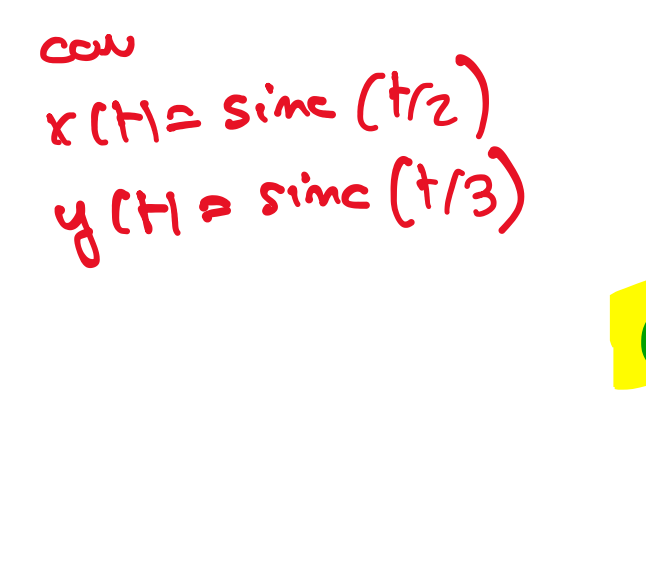
regola di conv.

$S(j\omega) = \text{rect}(\omega/2\pi)\cdot\text{rect}\left(\frac{\omega}{2\pi}\right) = \begin{cases} 1 & |\omega| < \pi \\ 0 & |\omega| \geq \pi \end{cases}$



$S(j\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)$

$s(t) = \text{sinc}(t) = \text{sinc} * \text{sinc}(t)$ RISULTATO NOTEVOLE



XCASA $z(t) = x * y(t) \xrightarrow{\mathcal{F}} ?$ ①
con
$x(t) = \text{sinc}(t/2)$
$y(t) = \text{sinc}(t/3)$

USARE FOURIER

② $z(t) = ?$

Es 3a CALCOLARE AREA DI $s(t) = \text{sinc}(t)$

$A_s = \int_{-\infty}^{+\infty} \text{sinc}(t)\,dt = \int_{-\infty}^{+\infty} \frac{\sin \pi t}{\pi t}\,dt$

$A_s = S(j\omega)\big|_{\omega=0} = \text{rect}\left(\frac{0}{2\pi}\right) = 1$

$S(j\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)$

Es 4 CALCOLARE ENERGIA DI $s(t) = \text{sinc}(t)$

$E_s = \int_{-\infty}^{+\infty} \text{sinc}^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |S(j\omega)|^2\,d\omega$

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \text{rect}\left(\frac{\omega}{2\pi}\right) d\omega$ (rect = 1)

$= \frac{2\pi}{2\pi} = 1$ TRAMITE PARSEVAL

$E_{\text{sinc}} = A_{\underbrace{\text{sinc}^2}_{y(t)}} = Y(j\omega)\big|_{\omega=0} = \text{triangle}\left(\frac{0}{2\pi}\right) = 1$

$Y(j\omega) = \text{triangle}\left(\frac{\omega}{2\pi}\right)$

Es 3b AREA DI $s(t) = \text{sinc}^3(t)$

$= \underbrace{\text{sinc}(t)}_{x(t)}\,\underbrace{\text{sinc}^2(t)}_{y(t)}$

$s(t) = x(t)\,y(t) \xrightarrow{\mathcal{F}} S(j\omega) = \frac{1}{2\pi} X * Y(j\omega)$

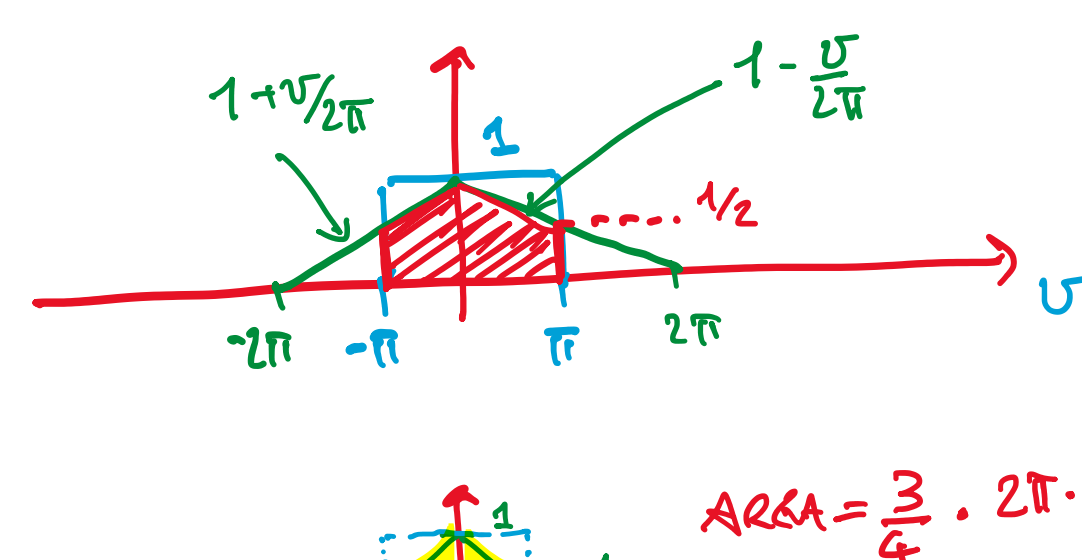
$x(t) = \text{sinc}(t) \xrightarrow{\mathcal{F}} X(j\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)$

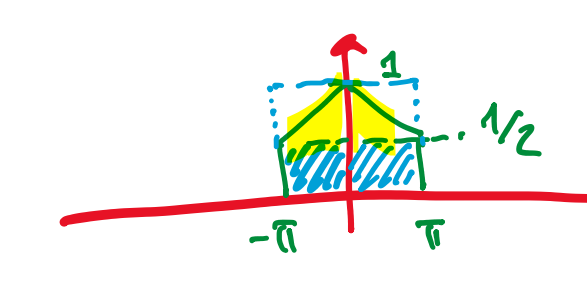
$y(t) = \text{sinc}^2(t) \xrightarrow{\mathcal{F}} Y(j\omega) = \text{triangle}\left(\frac{\omega}{2\pi}\right)$

$A_s = S(j\omega)\big|_{\omega=0}$

$A_s = S(j\omega)\big|_{\omega=0} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\upsilon)\,\underbrace{Y(j(\omega-\upsilon))}_{Y(-j\upsilon)}\,d\upsilon\,\Big|_{\omega=0}$

$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \text{rect}\left(\frac{\upsilon}{2\pi}\right)\,\text{triangle}\left(\frac{+\upsilon}{2\pi}\right) d\upsilon$ (PARI)





$\text{AREA} = \frac{3}{4}\cdot 2\pi\cdot 1$

$A_s = \frac{1}{2\pi}\cdot\frac{3}{4}\cdot 2\pi = \frac{3}{4}$